# Report on stay at ZAMG

23/05/2016 – 17/06/2016

## Stochastic pattern generators

Stochastically Perturbed Parameterized Tendencies (SPPT) is a scheme which has been operationally and successfully used in global IFS model by ECMWF (Buizza et al., 1999, Palmer et al., 2009). In the previous years there was a growing interest around model error representation also in limited area ensemble systems, especially in convection-permitting ensembles. That was a motivation inside ALADIN community to implement the scheme in the limited area version of ARPEGE-IFS code which was done by Francois Bouttier, Météo France, and tested in an AROME-EPS framework (Bouttier et al., 2012).

Last year as a part of a similar LACE stay a problematic issue was reported which is connected to the random number fields used by the SPPT (Szűcs, 2015). This issue was presented on ALADIN-HIRLAM workshop (Szűcs, 2016a) and SRNWP-EPS workshop (Szűcs, 2016b), as well.

The main topic of my stay was an investigation around this problem and the test of two possible solution. The first possibility is the modification of the current stochastic pattern generator and the second one is the implementation of a brand new method.

Author:
Mihály Szűcs (OMSZ)
szucs.m@met.hu

Supervisor:
Martin Belluš (SHMU)
martin.bellus@shmu.sk

Christoph Wittmann (ZAMG)
christoph.wittmann@zamg.ac.at

## 1. Problematic issues with the current spectral pattern generator

Stochastic physics schemes need random numbers to generate perturbations. Perturbations can be added on the top of the total tendencies like in the operational and default implementations or to partial tendencies like in many tests. Whatever is the applied perturbation method the characteristics of the used random numbers are crucial. In the original stochastic physics scheme of ECMWF (BMP representation, Buizza et al. 1999) there were boxes where uniformly distributed random numbers stayed constant for a given time period. Recalculated random numbers were independent from each other after this time period and outside the boxes. That meant problematic jumpiness in space and time which resulted unphysical behave in many cases.

In the revised version of the scheme (SPPT representation, Palmer et al., 2009) a spectral pattern generator was introduced which can produce a field of random numbers which changes smooth in space and time to avoid the above-mentioned jumpiness. Theoretically the pattern generator produces Gaussian distributed numbers which standard deviation ($\sigma$) can be controlled from namelist. Horizontal ($L$) and temporal ($\tau$) correlation of the random numbers can be also set from there.

This kind of random number generation was extended to limited area models (LAMs) in accordance with the different model geometry (Bouttier et al., 2012). In a previous LACE stay report (Szűcs, 2015) it was already underlined that from the above-mentioned three important control parameters at least two can not work exactly on the expected way. Demonstrating this problem a random pattern is visualized with the default values ($\sigma$=0.5, L=500000) on the Hungarian AROME domain (Fig. 1)

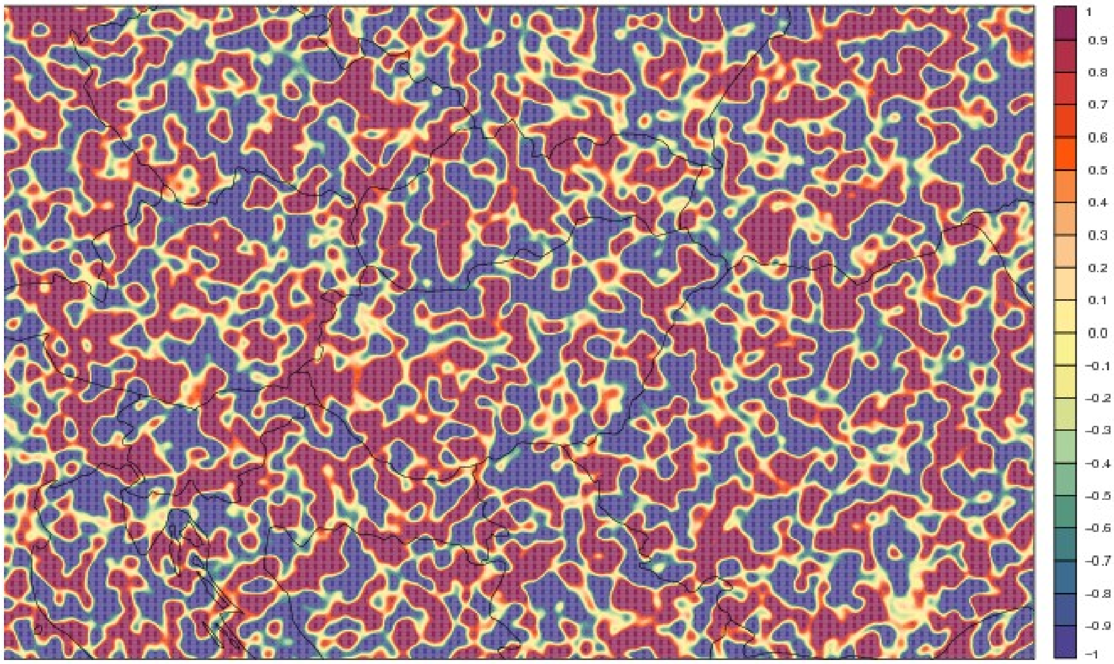

**Fig.1:** Spectral pattern in AROME model if $\sigma$=0.5 and L=500km

Looking on Fig. 1 two problematic issue can be realized:
- horizontal correlation is smaller than set and pattern seems “noisier” than expected,
- there are a lot of spots where random numbers have +1 or -1 value.

The second issue can be also visualized by a histogram of the pointwise values (Fig. 2)

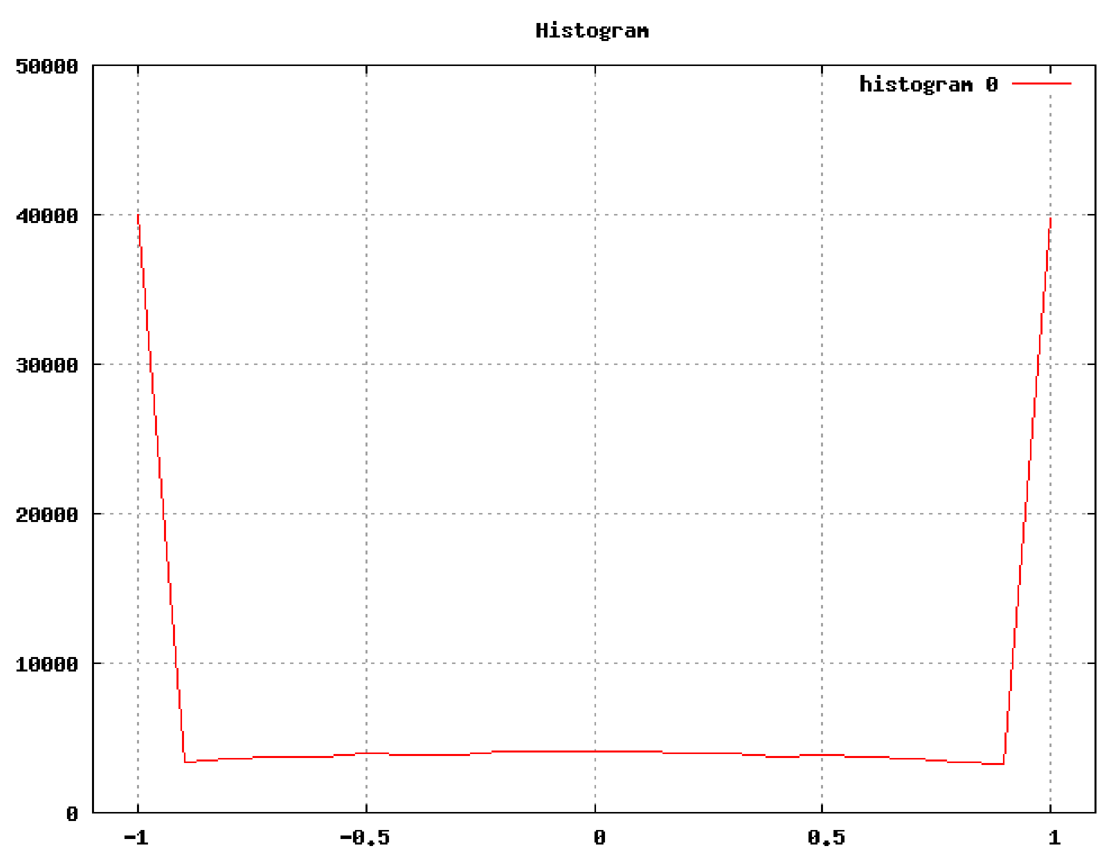

**Fig.1:** Histogram of random numbers if σ=0.5 and L=500

The reason why there is no number bigger than +1 or smaller than -1 is that there is a clipping ratio (X=2 as default) which helps to bound random values into the [-(X*σ);+(X*σ)] intrval. So it looks that real standard deviation is much bigger than σ and the distribution of the random numbers is quite strange after the additional clipping.

The above-mentioned issues can be handled on a very pragmatic way. After testing many settings and looking on many patterns one can find empirically tuned values which gives back more or less the expected behave. Such a setting (σ=0.2, X=5.0, L=4000000) was used in earlier test to get early test results (Szűcs, 2016a). The problem is that no one can be sure that the used setting is correct without the theoretical background and ideal values can be very domain dependent.

In the following section the equations of the spectral pattern generator will be revised to investigate around the source of this interesting behave.

## 2. Equations of the current spectral pattern generator

In ARPEGE-IFS code the SPPT scheme is applied in spectral models, so random fields ($r$) are generated in spectral space and then transformed to grid point space where the actual parameterization computations are performed. Therefore $r$ is described by spherical harmonics ($r_{mn}$) in a spectral global model (Palmer et al. 2009) and by bi-Fourier functions in a spectral limited area model (Bouttier et al. 2012). The $r_j$ field is evolved by a so-called spectral pattern generator where its spectral coefficients $(r_j')_{mn}$ are described by a first order auto-regressive (AR(1)) process which ensures the temporal correlation.

$$r_{mn}(t+\Delta t) = \varphi\, r_{mn}(t) + \sigma_n \mu_{mn}(t)$$
$$\varphi = \exp(-\Delta t/\tau) \tag{1}$$

In every timestep new $r_{mn}$ values are calculater from an AR(1) process described by eq. 1. $\varphi$ denotes the one-timestep correlation set by $\tau$ decorrelation-timescale. $\mu$ values are independent random numbers picked from a Gaussian distribution with 0 mean, 1 variance and bounded into the [-2; 2] interval. These values are multiplied by the $\sigma_n$ parameters which are defined via the variance values according to different wavenumbers.

The initialization of the variance spectrum is not a trivial question while it should satisfy two expectations. First, the whole variance of the the process should be well-controlled by $\sigma$ standard deviation namelist parameter which is usually set to 0.5. Second, the so-called space correlation length ($L$) should play a role here which controls the "smoothness" of $r$ fields. The initialization of the variance spectrum takes place in suspsdt subroutine.

The initialization process is described by Palmer et al., 2009, which refers to Weaver and Coutier, 2001. This paper describes a method how to define the horizontal correlation of background error with a generalized diffusion-type equation.

The equations of $\sigma_n$ parameters are the following:

$$\sigma_n = F_0 \exp(-\kappa T\, n(n+1)/2) \tag{2}$$

In eq. 2. the $F_0$ constant parameter denotes the following:

$$F_0 = \left( \frac{var(r)(1-\varphi^2)}{2\sum_{n=1}^{N} (2n+1)\exp(-\kappa Tn(n+1))} \right)^{1/2} \tag{3}$$

In eq. 3. the variance depends according to the one-timestep correlation:

$$\sigma^2 = var(r) = \frac{{\sigma_n}^2}{1-\varphi^2} \tag{4}$$

Eq. 4. also makes the relationship between the $\sigma$ standard deviation set from namelist and the spectrum of $\sigma_n$. In eq. 2. and 3. $\kappa$ and $T$ are the diffusion coefficient and the time in the original diffusion equation introduced in Weaver and Coutier, 2001. In this case they can be replaced by a measure of the horizontal correlation length normalized by the Earth radius in the following equation:

$$\kappa T = \frac{1}{2}\left(\frac{L}{R_E}\right)^2 \tag{5}$$

If we want to summarize the problematic issues again in accordance with the equations we can highlight three major points:

- The values of $\sigma_n$ spectrum look simply too large as it set via eq. 2-5 (see also the previous point).
- $\sigma_n$ is monotonic decreasing as a function of $n$, so the spectrum will always have its peak at the biggest wavenumber no matter how small the namelist defined $L$ is.
- In the normalization of $L$ we use $R_E$ even if we have a limited area model which size is definitely smaller than the globe.

## 3. Equations of a modified spectral pattern generator

After the revision of Weaver and Courtier, 2001, some slight but important differences have been discovered in the equations used to the definition of the variance spectrum. The source and the reason of these differences are not clear at the moment.
If we follow the original paper then eq. 2. can be rewritten as:

$$\sigma_n = F_0 (2n+1)^{1/2} \exp(-\kappa T n(n+1)) \tag{6}$$

Eq. 3. can be also modified in accordance of the source:

$$F_0 = \frac{(var(r)(1-\varphi^2))^{1/2}}{\sum_{n=1}^{N} (2n+1) \exp(-\kappa T n(n+1))} \tag{7}$$

Both in eq. 2-3 and in eq. 6-7 we can choose $var(r)(1-\varphi^2)=1$ and generate Fig.3., which is the comparison of the variance spectrum of the two different sets of equations. The reason of this special choice is that we can get back the Figure 1. of Weaver and Courtier, 2001 which verifies the corresponding calculations.

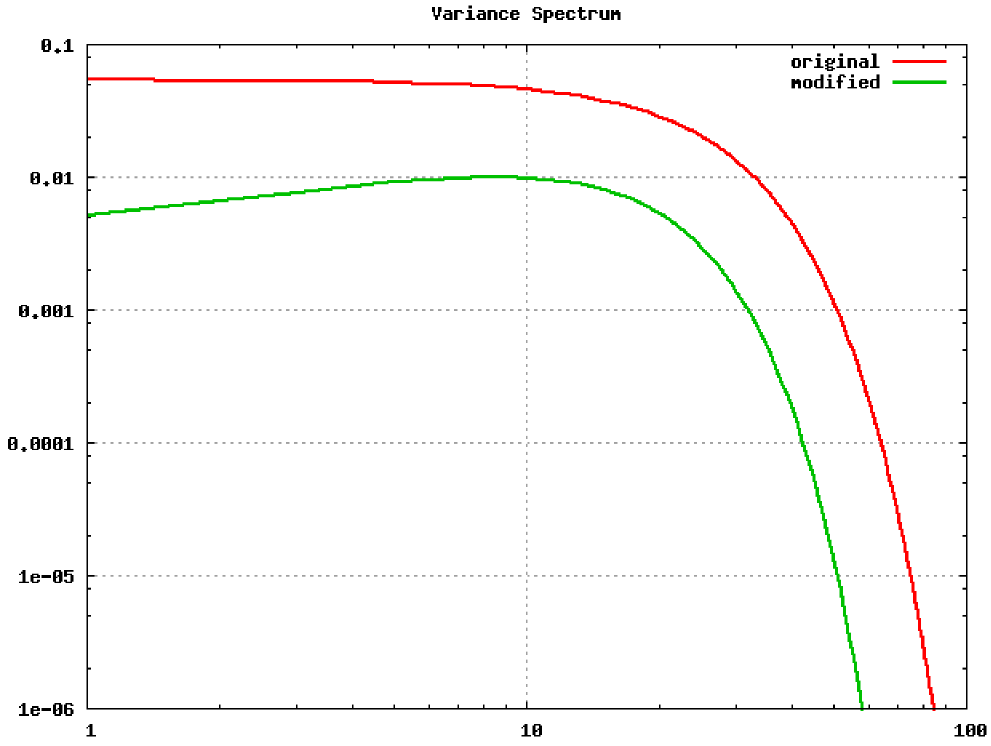

**Fig.3.:** Variance spectrum according to eq. 2-3 (original) and eq. 6-7 (modified, identical with Figure 1. of Weaver and Courtier, 2001) with choice of $var(r)(1-\varphi^2)=1$.

Fig.3. shows that the modified version has two favorable properties which is connected to the first two problematic issues of the previous section:

- The values of $\sigma_n$ spectrum can be smaller with this choice,
- While $exp(-\kappa Tn(n+1)/2)$ was monotonic decreasing $(2n+1)^{1/2}exp(-\kappa Tn(n+1))$ is not. It means that the new spectrum can have its maximum also at bigger wavenumbers.

To handle the third problematic issue, as well, eq.5. was also modified in a way to rescale horizontal correlation length on a domain size dependent way:

$$\kappa T = \frac{1}{2}\left(\frac{2\pi L}{W_{long}}\right)^2 \tag{8}$$

In eq.8. $W_{long}$ is longitudinal domain width. It has to be noted that in $L<<R_E$ was an important assumption in case of global models. This means also a limitation for LAMs and $2\pi L<<W_{long}$ assumption should be followed in the namelist settings. In practice e.g. Hungarian AROME domain has 1250km longitudinal width and it was found that L should not be bigger than 50km. If it is bigger then variance spectrum can have strange shape (Fig.4.) and random numbers can become too large again (not shown here).

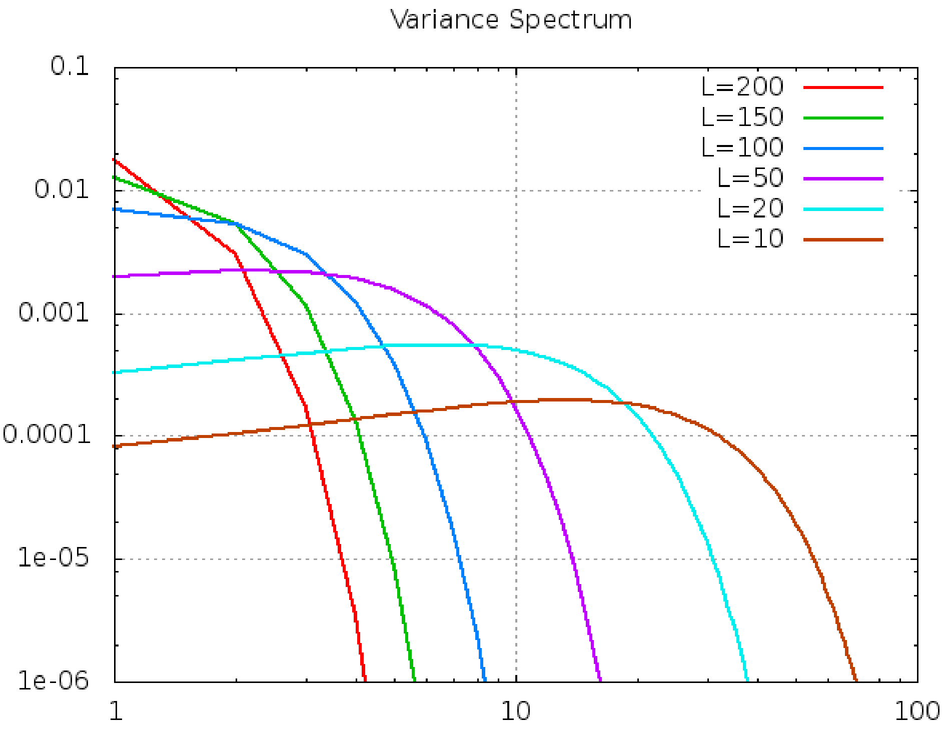

**Fig.4.:** $\sigma_n$ spectrum on Hungarian AROME domain with different horizontal correlation lengths.

At the same time SPPT always work more effectively with relatively bigger horizontal correlation length. E.g. 20<L<50 was found as an ideal range for Hungarian AROME domain. In another example 50<L<80 looks as the ideal range for LACE domain (where Hungarian ALARO-EPS runs). Unfortunately this setting stayed domain size dependent which can be interpret as deficiency of the current random field generator concept.

Finally Fig.5. shows the pattern itself if L=20km. Fig.6. shows the histogram of point-wise random numbers in four cases:

- Current spectral pattern generator, default values (σ=0.5, X=2.0, L=500000, purple line),
- Current spectral pattern generator, manually tuned values (σ=0.2, X=5.0, L=2000000, green line),
- Modified spectral pattern generator, long horizontal correlation length (L=50km, blue line),
- Modified spectral pattern generator, long horizontal correlation length (L=50km, orange line).

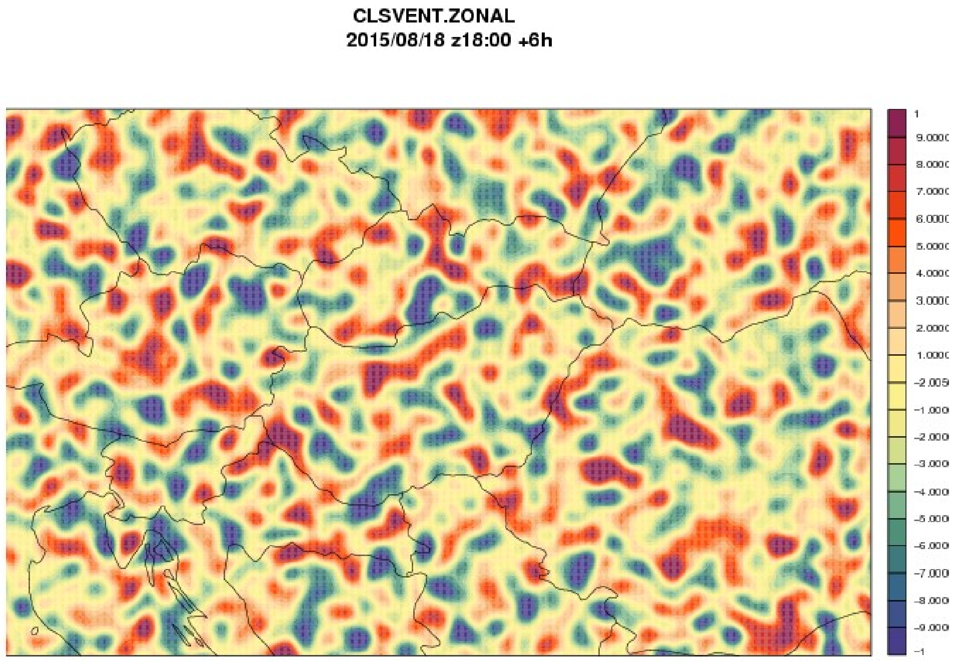

**Fig.5:** Spectral pattern in AROME model with the revised spectral pattern generator (σ=0.5 and L=20km)

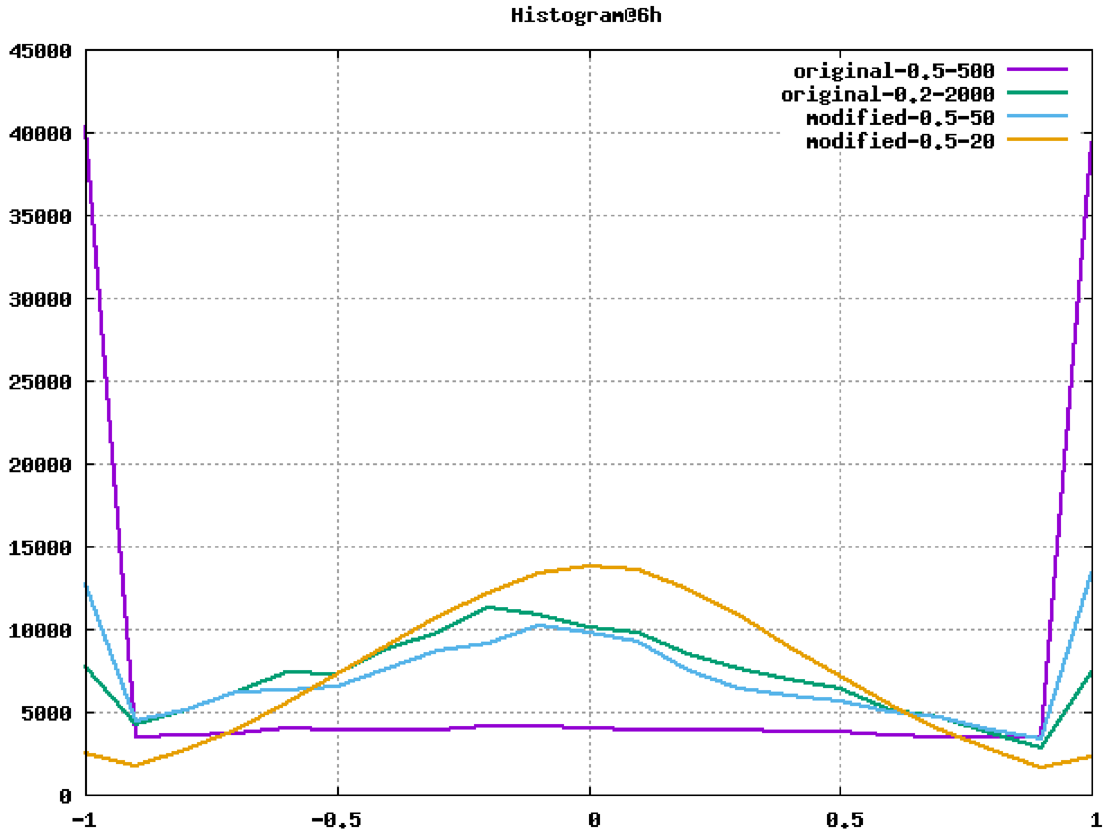

**Fig.6:** Histogram of point-wise random values with various spectral pattern generator versions and settings (see the text for more explanation).

## 4. Description of a Limited-Area Spatio-Temporal Stochastic Pattern Generator

During my LACE stay I had the possibility to get familiar with the work of Michael Tsyrulnikov and Dmitry Gayfulin (Tsyrulnikov and Gayfulin, 2016). The development of their Limited-Area Spatio-Temporal Stochastic Pattern Generator (SPG) was motivated mainly by the problem that usually the currently used random field generators have a given time and spatial horizontal correlation length but these values are independent from each other. An example can be eq.1. where $\tau$ is the decorrelation-timescale for all the $n$ values. In contrast SPG have the “proportionality of scales” property: large-scale (small-scale) in space field components have large (small) temporal length scales. They follow the “well-known -5/3” slope of the atmospheric spectra.

Other attractive properties of the SPG are:

- Its basic solver is also spectral-space based,
- It is developed to limited area models and the acceptable range of correlation values is wider than in our current pattern generator,
- It has 2D and 3D in space versions, as well,
- The generated noise is theoretically Gaussian.

The SPG scheme is implemented as a FORTRAN program and freely available from https://github.com/gayfulin/SPG

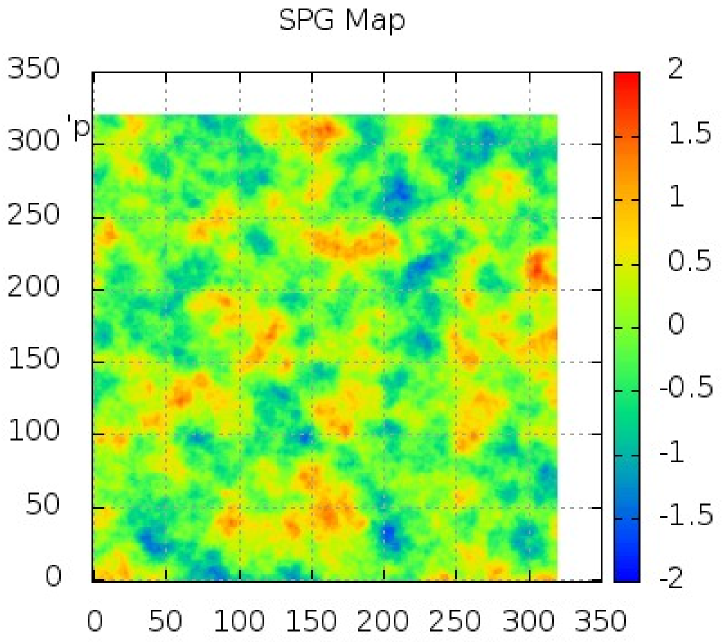

**Fig.7:** Random field generated by SPG (as an external program) with the LACE domain specific configuration.

This program can be run as an external one with a configuration file where model dependent parameters (e.g. model size and timestep) can be defined in advance. Some additional printout line can help visualize the pattern generated by this program (Fig.7.).

The main steps of the algorithm can be highlighted as the following:
- Read and initialization of settings
- Pattern generation
    - Initialization of some additional variables
    - Loop for the different wave-numbers (in 2D or 3D) and calling the solver separately
        - Calling for Gaussian noise
        - Loop for the different eps members (samples)
            - Initialization of the random numbers of SPG at time=0 using the Gaussian noise
        - Calling for Gaussian noise
        - Loop for the different eps members
            - Loop for the different timesteps
                - Evolving the random numbers in time using Gaussian noise
    - Fast Fourier Transformation (FFT)
- Calculation of gridpoint statistics

After the examination of these points it became clear that a quite big part of the code is responsible for the Gaussian noise generation and for the FFT. While such algorithms are also available in ALADIN code it became clear that the rest of it is easier to implement than generate thousands of fields with an external program and than read and use during a model integration.

## 5. Implementation of the SPG into the ALADIN code

As it was mentioned above there is no need to implement the whole SPG program into the ALADIN code. However there are two main related questions:

- Where can it be implemented?
- How can it be reorganized?

At the first test we would like to use the random fields of the SPG on the same way in the SPPT than it is with the current random pattern generator. So the easiest way was to implement everything at the same part of the model where the current pattern generator works. Technically speaking it means that the calculations in spectral space should be changed, which are the subroutines of spectral_arp_mod are responsible for. The way how these subroutines are called from the setup of SPPT (suspsdt) and from stepo should be just slightly modified while grid-point space calculations stay untouched.

The structure of the program needed a massive reorganization because of the order of the loops. Of course in our case all the eps members are independent model runs so their loop has to come on the highest level. What is even more interesting that the loop on the wavenumbers and on the timesteps has to be changed. An extra problem is that the SPG works with a more frequent time resolution so additional substeps had to be applied. The storage of some substep fields had to be also handled because they are needed to evolve values over model timesteps.

- 1st call of the generator from the suspsdt subroutine
    - Initialization of some additional variables
    - Calling for Gaussian noises (same way as in current pattern generator)
    - Loop on the different wavenumbers
        - Calling for the solver
            - Initialize the SPG random number of the given wavenumber
            - Loop on the substeps
                - Evolve SPG random number of the given wavenumber
            - Store the last three substeps of the scheme
- Fast Fourier Transformation (same way as in current pattern generator)
- Using grid-point values to perturb total tendencies (same way as in current pattern generator)
- Loop on the model timesteps
    - Further call of the generator from the stepo subroutine
        - Calling for Gaussian noises (same way as in current pattern generator)
        - Loop on the different wavenumbers
            - Calling for the solver
                - Restore the SPG random number of the last 3 substeps
                - Loop on the substeps
                    - Evolve SPG random number of the given wavenumber
                - Store the last three substeps of the scheme
- Fast Fourier Transformation (same way as in current pattern generator)
- Using grid-point values to perturb total tendencies (same way as in current pattern generator)

An additional challenge over the correct definition of substeps was that external SPG program works with rectangular truncation while ALADIN uses elliptic truncation. It needed a careful revision of wavenumbers, as well.

The very-first version of the implementation was able to generate a random field at timestep=0 which looks qualitatively similar to the one generated by the external program (Fig.8.).

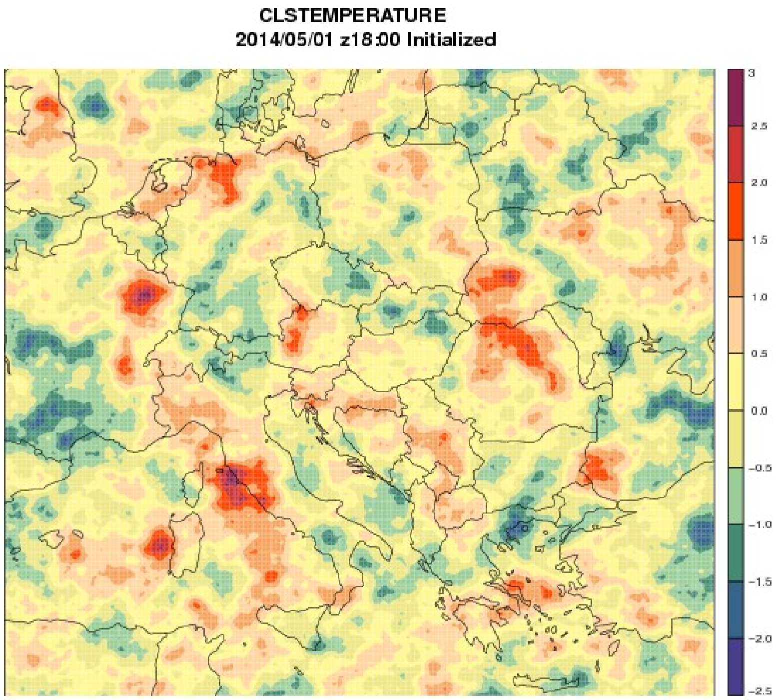

**Fig.8:** Random field generated by SPG (as an external program) with the LACE domain specific configuration.

The statistical features of SPG generated random field has to be checked. Especially if it is evolved over a longer period. These characteristics have to be compared with the statistics generated by the external program. All these controls were over the limitation of my stay.

The initialization of many variables is done by various subroutines in the original SPG code. Many of them is simply just read from the external runs and hardcoded to the ALADIN implementation. This is not a nice solution and has to be improved in the future versions of the implementation.

## 6. Conclusion and future plans

- In section 1. it was underlined that current spectral pattern generator does not work properly in LAM. It means that its settings (standard deviation, horizontal correlation length) can not give back the expected results. In section 2. its equations have been revised and some problematic issues have been underlined. In section 3. some modifications have been proposed in accordance with the widely-referred Weaver and Courtier, 2001 paper and with LAM geometry specifications. After these modifications the pattern has favorable characteristics but its ideal setting stays unfortunately model domain dependent.
- The proposed solution of section 3. has to be tested in the future. Such tests has been already started in Hungarian ALARO-EPS framework after my LACE stay. Results are going to be shared in LACE Predictability group.
- In section 4. a brand new pattern generator (SPG) was briefly described. In section 5. a very-first implementation of this scheme and its additional challenges have been detailed.
- While this implementation was able to produce a qualitatively similar pattern than the external source program many additional work is needed before a real test in SPPT:
    - In the second version the initialization of many variables has to be improved,
    - It has to be checked if the SPG implementation gives statistically similar fields than the external program,
    - It has to be checked if the time evolution of the pattern works properly over a longer period
    - In long-term it should be figured out how to use the scheme in 3D. At the moment it looks very challenging but it would give an additional exciting possibility to use in comparison with the current pattern generator.

## Acknowledgement

Finally I would like to say thank you for the whole modeling group of ZAMG, for their supportive attitude. They helped me in all my scientific, technical and personal troubles which made my work really smooth during my stay. I am also grateful to Martin Belluš for the inspiring discussions and for the patience what he showed as LACE Area Leader.
I am appropriate to Francois Bouttier, who supervises ECMWF's special project, called 'spfrbout', which SBU was used by me during my stay.
I am really thankful to Michael Tsyrulnikov and Dmitry Gayfulin from the Russian HydroMetCenter. I was allowed to examine their source program and documentations. Additionally they were very helpful and cooperative during our short mailing.

## References

Belluš, M., 2014: Stochastically perturbed physics tendencies of surface fields in ALADIN-LAEF system, Report on stay at ZAMG 12/05 - 20/06/2014, Vienna, Austria

Bouttier, F., Vié, B., Nuissier, O., Raynaud, L., 2012: Impact of Stochastic Physics in a Convection-Permitting Ensemble. Mon. Wea. Rev., **140**, 3706–3721.

Buizza, R., Miller, M., Palmer, T.N., 1999, Stochastic representation of model uncertainties in the ECMWF Ensemble Prediction System, Quart. J. Roy. Meteorol. Soc., **125**, 2887–2908.

Palmer, T., Buizza, R., Doblas-Reyes, F., Jung, T., Leutbecher, M., Shutts, G., Steinheimer, M., Weisheimer, A., 2009: Stochastic parametrization and model uncertainty. Tech. Rep., ECMWF Tech.

Memo. **598**, 42 pp. [Available online at http://www.ecmwf.int/publications/.]

Szűcs, M., 2014: Stochastically Perturbed Parameterized Tendencies in ALARO and AROME , Report on stay at ZAMG 06/10/2014 - 31/10/2014, Vienna, Austria

Szűcs, M., 2015: Tests of possible SPPT developments , Report on stay at ZAMG 28/09/2015 - 06/11/2015, Vienna, Austria

Szűcs, M., 2016a: Test of the SPPT scheme, Joint 26th ALADIN Workshop & HIRLAM All Staff Meeting 2016, 4-8/04/2016, Lisbon, Portugal

Szűcs, M., 2016b: The convection-permitting ensemble system of the Hungarian Meteorological Service, SRNWP-EPS II Workshop “Probabilistic prediction of severe weather phenomena” 17-19 May 2016, Bologna, Italy

Tsyrulnikov, M. and Gayfulin, D., 2016: A Limited-Area Spatio-Temporal Stochastic Pattern Generator for ensemble prediction and ensemble data assimilation, Meteorologische Zeitschrift, submitted.