# RC LACE research stay report

**Topic:** Mixing length computation in TOUCANS

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## 1. Introduction

### 1.1. Mixing length in TOUCANS

Mixing length is one of crucial parameters in turbulence parametrization as it directly influences the magnitude of exchange coefficients for momentum, heat and moisture. On the other hand, it also affects the magnitude of production/destruction terms of the prognostic Turbulence Kinetic Energy (TKE) equation, and thus the TKE itself.

Unlike many previous parametrizations, TOUCANS (**T**hird **O**rder moments (TOMs) **U**nified **C**ondenstation **A**ccounting and **N**-dependent **S**olver for (for turbulence and diffusion)) turbulence parametrization differs between length scale for exchange processes ($L_K$) and dissipation length scale ($L_\epsilon$). According to Redelsperger et al. (2001) the relationship between $L_K$ and $L_\epsilon$ depends on stability. Following their approach and adapting it to TOUCANS framework (Bastak Duran et al. 2014, Bastak Duran 2015), the conversion equations between Prandtl type mixing length ($l_m$) and TKE-based length scales ($L_K$ and $L_\epsilon$) are obtained:

$$L_K = \frac{C_\varepsilon}{\nu^3} \frac{f(Ri)^{1/4}}{\chi_3^{1/2}} l_m \tag{1}$$

$$L_\varepsilon = \frac{C_\varepsilon}{\nu^3} \frac{\chi_3^{3/2}}{f(Ri)^{3/4}} l_m \tag{2}$$

where $C_\varepsilon$ and $\upsilon$ are closure constants controlling the intensity of turbulence dissipation and overall intensity of turbulence, $\chi^3$ is a stability function for momentum and $f(Ri)$ is a function of Richardson number given by:

$$f(Ri) = \chi_3(Ri) - Ri \cdot C_3 \cdot \phi_3 \tag{3}$$

where $C_3$ is inverse of Prandtl number at neutrality and $\phi_3$ is a stability function for heat. Finally, combining the equations (1) and (2), the conversion relation between $L_K$ and $L_\epsilon$ is obtained:

$$L_K = L_\varepsilon \frac{f(Ri)}{\chi_3^2} \tag{4}$$

However, due to consistency with previous turbulence parametrization (pTKE), a combined length scale (L) is used as the main length scale in the code:

$$L = (L_K^3 \cdot L_\epsilon)^{\frac{1}{4}} = \frac{C_\varepsilon}{\nu^3} l_m \tag{5}$$

The above set of equations allows us to compute the mixing length from Prandtl type formulations based on similarity theory or from TKE-based formulations (e.g. Bougeault-Lacarrere 1989, Deardorff 1980). Since the exchange coefficients computation expects Prandtl type input and TKE prognostic equation expects TKE-based input, conversion relations are needed in both cases.

In current operational setup of TOUCANS the Geleyn Cedilnik Prandtl type formulation is used:

$$l_{GC} = \frac{\kappa z}{1+\frac{\kappa z}{\lambda_m}\left[\frac{1+exp\left(-a_m\sqrt{\frac{z}{H_{ABL}}}+b_m\right)}{\beta_m+exp\left(-a_m\sqrt{\frac{z}{H_{ABL}}}+b_m\right)}\right]} \tag{6}$$

where $\kappa$ is Von Karman constant, $a_m$, $b_m$ and $\beta_m$ are values of tuning parameters from the namelist, and $H_{ABL}$ is the height of the atmospheric boundary layer (ABL). There are also several TKE-based formulations (and combinations among themselves or with the Geleyn-Cedilnik formulation) at disposal, but are not used operationally as they produce to low mixing in the lower layers (above the surface layer) which deteriorates the forecast (Bastak Duran 2015). Due to its nonlocal properties, we are here primarily interested in evaluating and improving the performance of Bougeault-Lacarrere (1989) TKE-based formulation (hereafter BL89). BL89 formulation determines the length-scale of the largest eddies (and the most energetic ones) at a given model level as a function of the stability profile of adjacent levels. The algorithm computes maximum upward ($L_{up}$) and downward ($L_{down}$) displacement of an air parcel having the mean kinetic energy of the level as initial kinetic energy. It is assumed that the parcel will stop when cumulated buoyancy effects ($N_v^2$ – moist Brunt-Väisälä frequency) equal the kinetic energy ($e$) (Bougeault and Lacarrere 1989, Vana et al. 2011):

$$\int_z^{z+L_{up}} N_v^2(z'-z)\,dz' = e(z) \tag{7}$$

$$\int_{z-L_{down}}^{z} N_v^2(z-z')\,dz' = e(z) \tag{8}$$

Once when maximum upward and downward displacement are determined special attention needs to be given to the way they are averaged. In regions where one of displacements is significantly larger than the other, the magnitude of an average will depend on the type of the averaging operator. Within the current TOUCANS code the averaging is done in the following way:

$$L_{BL89}(e, N^2) = \left(\frac{L_{up}^{-\frac{4}{5}} + L_{down}^{-\frac{4}{5}}}{2}\right)^{-\frac{5}{4}} \tag{9}$$

It is asumed that $L_{BL} = L$ which is then converted to Prandtl type mixing length ($l_m$) by using the inverted equation (5). Afterwarts $l_m$ is converted to $L_K$ or $L_\varepsilon$ by using equations (1) and (2).

### 1.2. Conversion from TKE-based formulations

It is considered that observed problems with the use of TKE-based length-scales are related to conversion among different type of scales within the code. Once when Prandtl type scale is determined, conversion from it to TKE-based scales (for usage in TKE prognostic equation) given by equations (1) and (2) seems correct, i.e. there is stability dependence included, as reported by Redelsperger et al. (2001). However, when TKE-based formulations are used, it is neccessary to make an inverse conversion to Prandtl type scale for the computation of exchange coefficients for momentum, heat and moisture. Due to $L_{TKE} = L$ assumption, there is no stability dependence between TKE-based output (TKE = BL89 or other) and Prandtl type scales. According to Bougeault-Lacarrere (1989) and Cuxart et al. (2000), an averaged value of upward and downward displacement of air parcel is associated either to production ($L_K$) or dissipation ($L_\varepsilon$) length scale, or eventually to $L$ (different than our $L$ in eq. (5)) where $L = L_K = L_\varepsilon$. Then (in our case), by using equations (1) and (2), this TKE-based length-scale should be converted to Prandtl type.

According to the above mentioned, we will release the assumption of $L_{TKE} = L$, where $L$ is given by equation (5). It will be assumed that $L_{TKE} = L_K$ where the subscript TKE stands for the BL89 formulation (it may be DE80 formulation or some combination of TKE-based length-scales as well). An averaging operator remains the one given by equation (9). We will also keep consistency with pTKE parametrization, i.e. the relationship between $L$, $L_K$ and $L_\varepsilon$ will remain as given by equation (5). Note that assumption given by equation (5) does not affect the conversion between scales given by equations (1) and (2) as definition of $L$ is hidden within them.

Technically, the conversion will be done as given by equations (1) and (2), but in two steps. First, we will convert the BL89 output ($L_{TKE} = L_K$) to computational scale $L$:

$$L = \frac{\chi_3^{1/2}}{f(Ri)^{1/4}} L_K \tag{10}$$

and then by using inverted equation (5) $L$ will be converted to Prandtl type scale ($l_m$):

$$l_m = \frac{\nu^3}{C_\varepsilon} L \tag{11}$$

### 1.3. List of assignments

Main assignments are to:

- Verifiy two versions of the **acmixelen.F90** subroutine (CY38T1 and CY38T1TR)
- Implement the stability dependent conversion from $L_{TKE}$ to $l_m$ (for BL89 method) as described in the introduction
- Perform the diagnostics of different mixing length formulations, focusing on currently operational Geleyn-Cedilnik and BL89
- Verify the performance of a modified formulation with stability dependent conversion from $L_{TKE}$ to $l_m$ for BL89 method

## 2. Results

In Table1. there is a short description of performed experiments and corresponding abbreviations used on the figures. For all simulations we have used the version CY38T1TR-op4 of the ALADIN-CZ configuration of the ALADIN system.

Table1. List of experiments with short description

| **Name of the experiment** | **Short description** |
|---|---|
| EL0a | Using currently operational Geleyn-Cedilnik mixing length given by (6). Conversion for TKE-budget equation is done according to (1) and (2). |
| EL1a | Using TKE-based "BL89" length-scale computed according to (7)-(9). It is assumed that $L_{BL89} = L$, which is then converted to $l_m$ by using (11). Conversion for TKE-budget equation is done according to (1) and (2). |
| EL1k | Similar to EL1a, but with assumption that $L_{BL89} = L_K$. $L_K$ is first converted to $L$ by using (10), and then $L$ is converted to $l_m$ by using (11). The same would be obtained with one-step conversion given by (1), i.e. assumption (5) does not effect the results. Conversion for TKE-budget equation is done according to (1) and (2). |

### 2.1. Verification of two versions of the acmixelen.F90 subroutine

After analysis of subroutines following differences were found:

- In CY38T1 version security checks were performed for Richardson gradient number and then Brunt-Väisälä frequency was calculated from "checked" values, while in CY38T1TR the security check was performed for Brunt-Väisälä frequency.
- In CY38T1TR version number of computations of mixing length for heat ($l_h$) are reduced, i.e. they are not calculated sepparately for each case, but at the end after the specific case is chosen.
- In CY38T1 version vertical variability of the $l_h/l_m$ ratio is allowed, while in CY38T1TR version $l_h/l_m$=C3TKEFREE=1.183.

Points 1) and 3) should be further discussed and tested later on (after the stay).

## 2.2. Implementation of stability dependent conversion from $L_{TKE}$ to $l_m$

The current TOUCANS code calls the **acmixelen.F90** subroutine twice per time-step, wherein the mixing length output of the first call is used as a first guess for moist antifibrilation (MAF) and moist gustiness correction (MGC). Modifications done in MAF and MGC schemes influence the values of mixing length (second call of the **acmixelen.F90** subroutine).

The stability computation is done in the **acmrip.F90** subroutine, which is also in-between two calls of the **acmixelen.F90** subroutine, but needs to be done before the first call to perform the EL1k experiment (cf. Table 1.). To make the implementation process as easy as possible, we have done the stability computation at the beginning of the **acmixelen.F90** subroutine, under the LDML switch. Within the first call dry values of neccessary stability parameters are calculated, while in the second call the outputs of the **acmrip.F90** subroutine are imported.

Code modifications include:

- **acmixelen.F90:**
    1) adding LDML switch for differences between two calls of the subroutine
    2) calculation of dry stability parameters (LDML=TRUE): $N_v^2$ (ZBVMO), $Ri$ (ZRIGD), $Ri_f$ (ZRIFD), $\chi_3$ (ZCHI3D), $\phi_3$ (ZPHI3D) and $f(Ri)$ (ZFRID) or importing **acmrip.F90** outputs (LDML=FALSE).
    3) implementation of additional conversion equations: (10) for $L_K \rightarrow L$ and adequate one for $L_\varepsilon \rightarrow L$:

$$L = \frac{f(Ri)^{3/4}}{\chi_3^{3/2}} L_\epsilon \tag{12}$$

    4) modifying the call of subroutine and preparing outputs for **aplpar.F90** subroutine (writing diagnostic parameters in each integration step)

- **acmrip.F90:**
  1) preparing additional outputs for the second call of the **acmixelen.F90** subroutine: $f(Ri)$ (PMFRI), $\chi_3$ (PCHI3TA) and $\phi_3$ (PPHI3TA), and performing the security check for $f(Ri)$.
- **aplpar.F90:**
  1) initialization of variables for the first call of the **acmixelen.F90** subroutine and modification of calls for **acmixelen.F90** and **acmrip.F90**.
  2) performing the diagnostics of mixing length by writing it to text file in each time step (link with **cnt3.F90**) or by writing it to GFL structure and passing to **mf_phys.F90**[1]**.**

## 2.3. Diagnostics of different mixing length formulations

Here we compare two TKE-based mixing length formulations (EL1a and EL1k) and currently operational Geleyn-Cedilnik formulation (cf. Table 1. for more details). The comparison is performed for the central point of the ALADIN-CZ domain, at several time steps during the 29.06.2009. 00 UTC forecast. Due to relatively large discrepancies between different formulations at 18 and 24 hours after the initialization, we added two more plots for the nearest 3 hour periods.

By including the stability dependence into conversion from TKE-based formulation output to Prandtl type mixing length (EL1k vs. EL1a), we increased the mixing at all model levels (Fig 1.). Within the ABL, the values of EL1k formulation resemble the values of operational Geleyn-Cedilnik formulation (EL0a). Daily cycle of the mixing length is also better simulated than for EL1a. However, there are some differences at higher model levels, as well as later afternoon and during night. As we will see later, overall (over entire domain) effect of increased mixing above the 850 hPa pressure level led to significant improvement of verification scores for temperature and relative humidity. Analysis of time series of mixing length at different model levels indicated that there are no random and unexpected variations for EL1k formulation when compared to others (not shown here).

The other stability dependent formulation which assumes $L_{BL89} = L_{\varepsilon}$ is also coded and will be tested later on, along with some other options like usage of different averaging operators for $L_{up}$ and $L_{down}$.

## 2.4. Verification of the performance of stability dependent conversion from $L_{TKE}$ to $l_m$

### 2.4.1. Temperature

Analysis of domain averaged vertical profiles of temperature and water vapour budget terms was performed with DDH. Since the verification scores provided by VERAL point out to significant improvement

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[1] Instructions taken from: https://hirlam.org/trac/wiki/HarmonieSystemDocumentation/40h1.1/PostPP/Diagnostics with the addition of point 5).

for EL1k experiment over EL1a, our further focus will be on the comparison of EL1k with the referent forecast (EL0a). Magnitude of temperature budget terms for the experiment is shown on Fig 2., while the difference from the reference is shown on Fig 3.

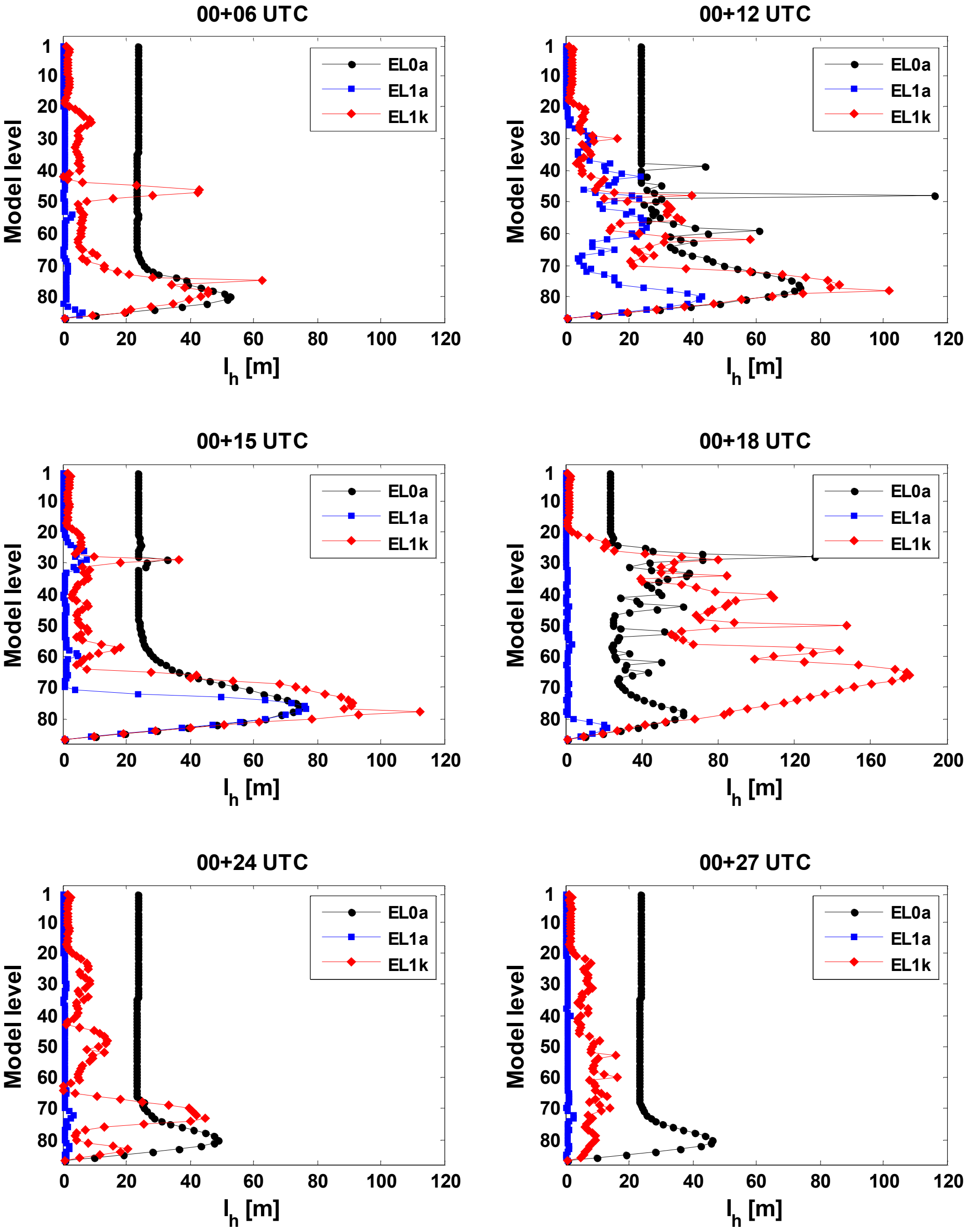

*Fig 1. Single point (centre of the domain) vertical profile of Prandtl type mixing length for heat and moisture for reference (EL0a), experiment 1 (EL1a) and experiment 2 (EL1k) at 06, 12, 15, 18, 24 and 27 hours after the initialization (29.06.2009. 00 UTC forecast).*

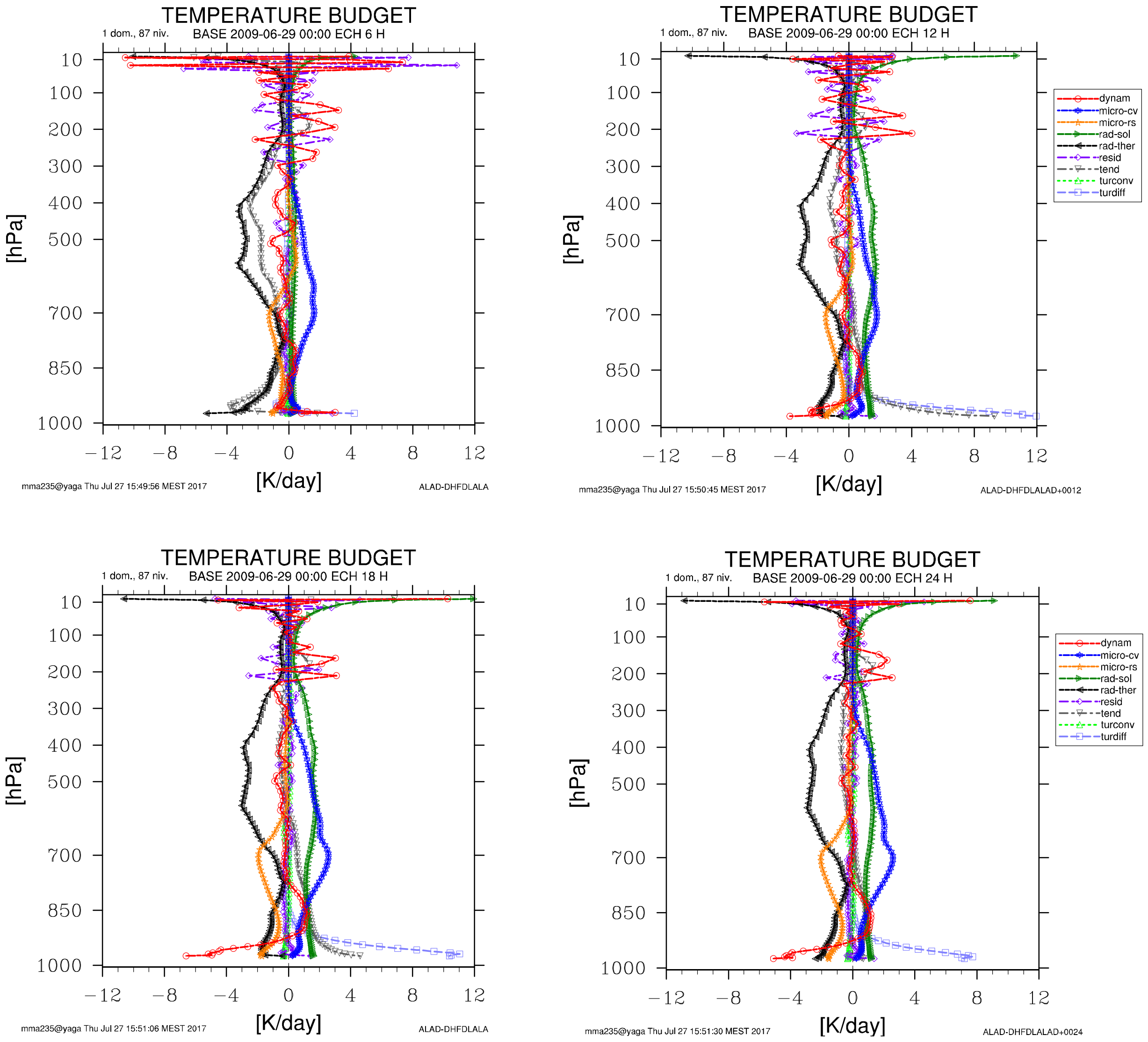

*Fig2. Domain averaged vertical profile of temperature budget terms for experiment 2 (EL1k) during the 29.06.2009. 00 UTC forecast: 06 hours (upper left panel), 12 hours (upper right panel), 18 hours (lower left panel) and 24 hours after the initialization (lower right panel).*

Turbulent diffusion of heat (light purple squares) is a dominant term of the temperature budget equation within the ABL (Fig 2.), but somewhat less in magnitude for experiment than the reference (Fig 3.). This results in relative cooling of the lowest few model levels and reducing the warm bias noticed in the referent forecast (Fig 4.). An exception is the screen level which is slightly warmer than for the referent forecast (Fig 5.). Above model levels 82-83, the magnitude of turbulent diffusion for experiment surpasses the reference, but this warming effect is dominated by cooling effect of microphysical processes (blue diamonds). This results in relative cooling and reducing the warm bias of the referent forecast up to about 500 hPa pressure level. As standard deviation (STDE) for the experiment is very similar to the reference, RMSE of the temperature is reduced almost everywhere below the 500 hPa pressure level (Fig 6.). Again, an exception is the screen level where STDE of temperature is relatively increased for the experiment, which leads to larger RMSE as well (Fig 7.).

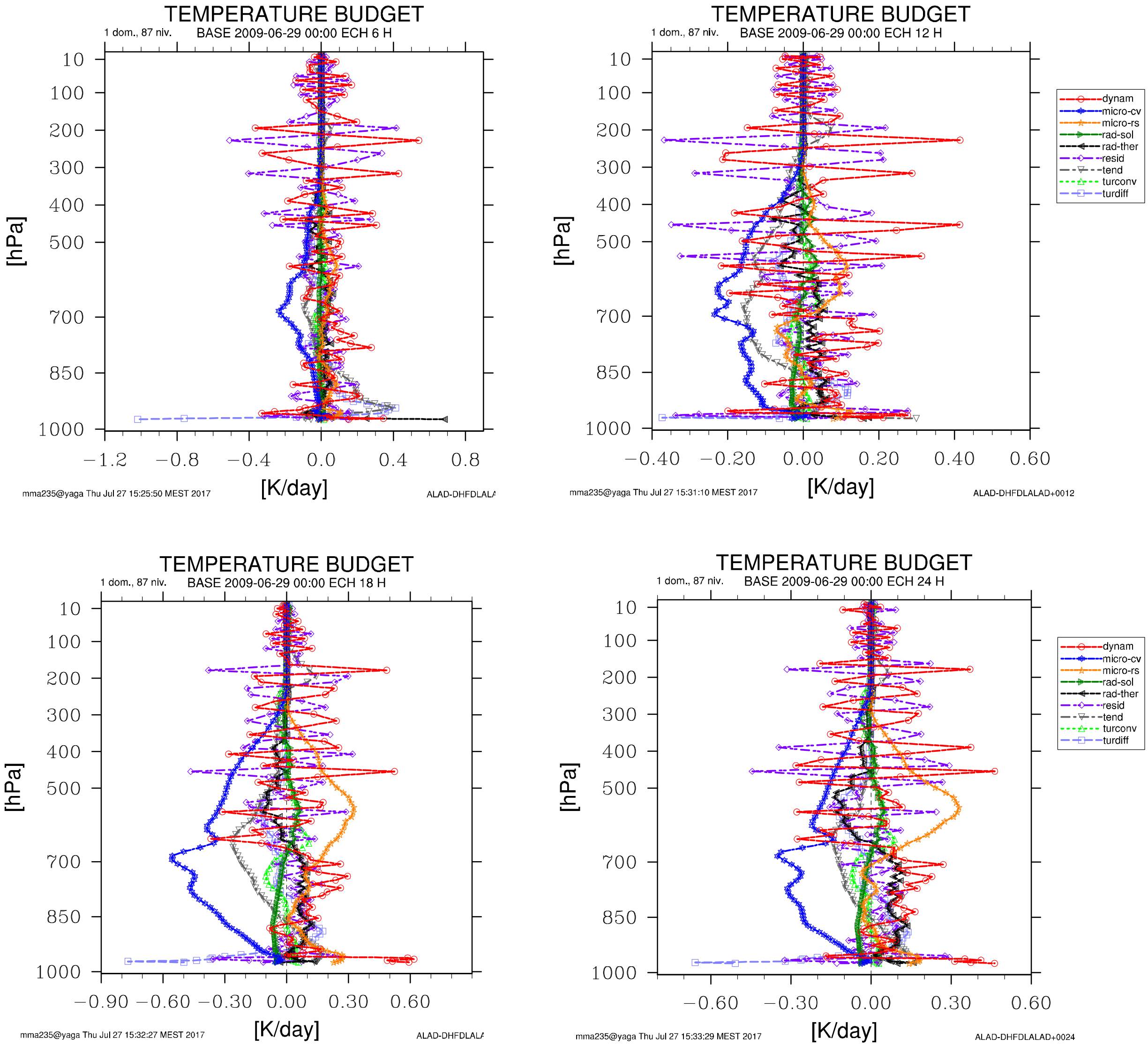

*Fig 3. Domain averaged vertical profile of differences of temperature budget terms between the reference (EL0a) and experiment 2 (EL1k) during the 29.06.2009. 00 UTC forecast: 06 hours (upper left panel), 12 hours (upper right panel), 18 hours (lower left panel) and 24 hours after the initialization (lower right panel). Where sign is "-", the experiment term is smaller in magnitude than the reference and opposite.*

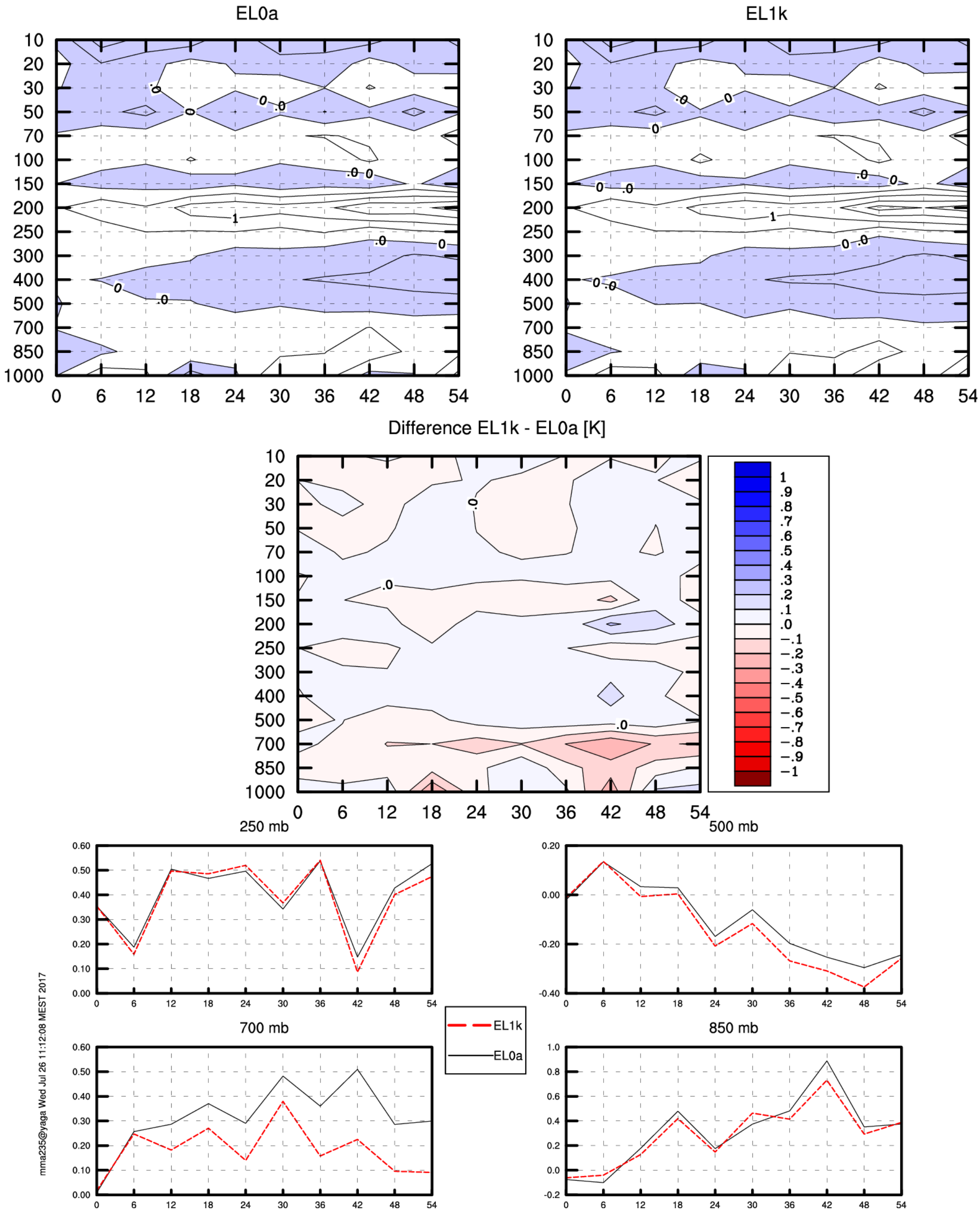

*Fig 4. Time evolution of the REAL BIAS of temperature for reference (EL0a) and experiment (EL1k) throughout the 54-hours forecast window during the 26-30.06.2009. period; (1) averaged[2] and height dependant – upper panels, (2) averaged and height dependent difference – middle panel and (3) at specific pressure levels – lower panels (for explanation of experiment abbreviations please check Table 1.).*

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[2] If not stated else under figure captions, by averaged we mean (spatially) averaged over entire domain.

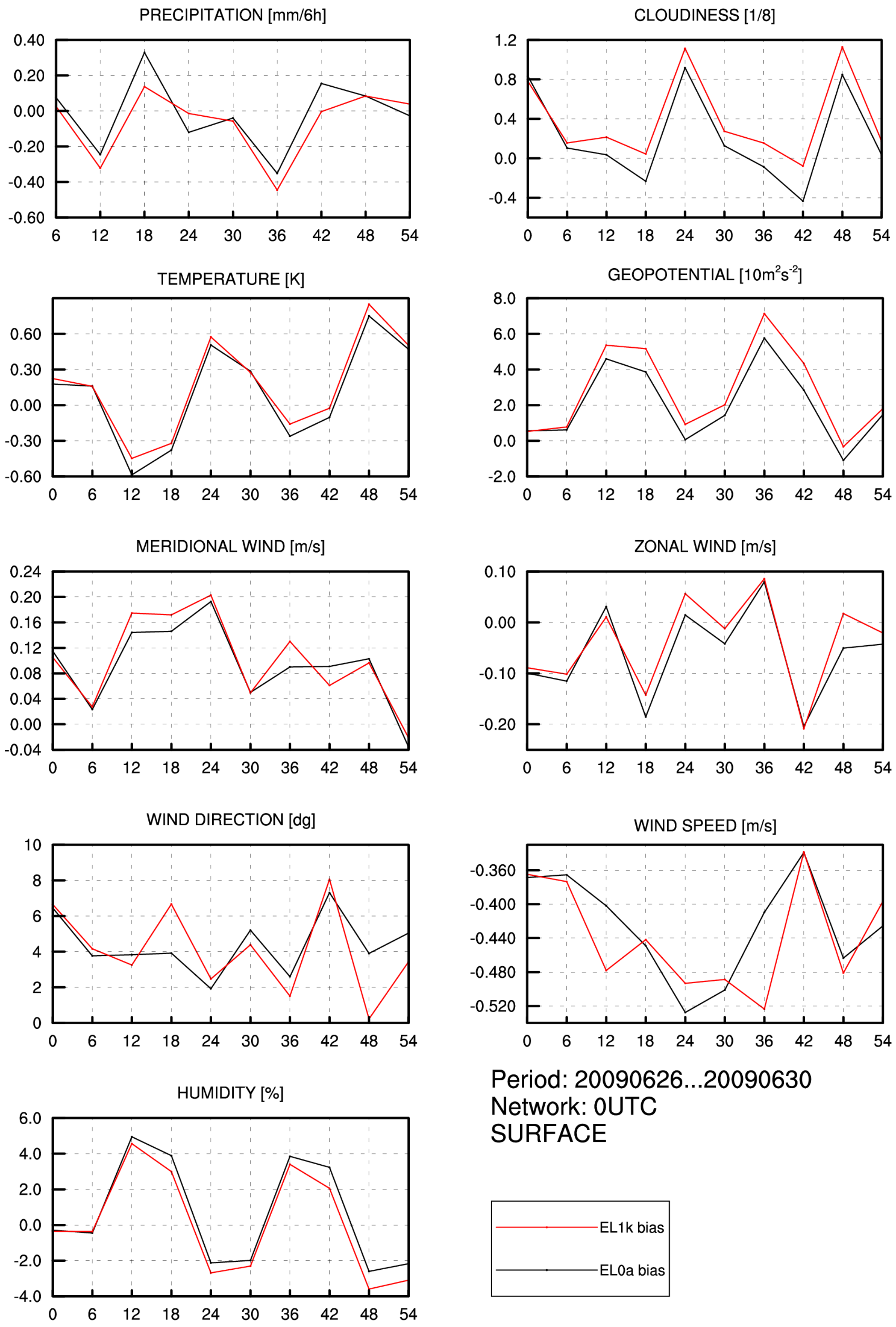

*Fig 5. Time evolution of averaged surface BIAS for basic meteorological parameters for reference (EL0a) and experiment (EL1k) throughout 54-hours forecast window during the 26-30.06.2009. period. Names and units of variables are given above each panel.*

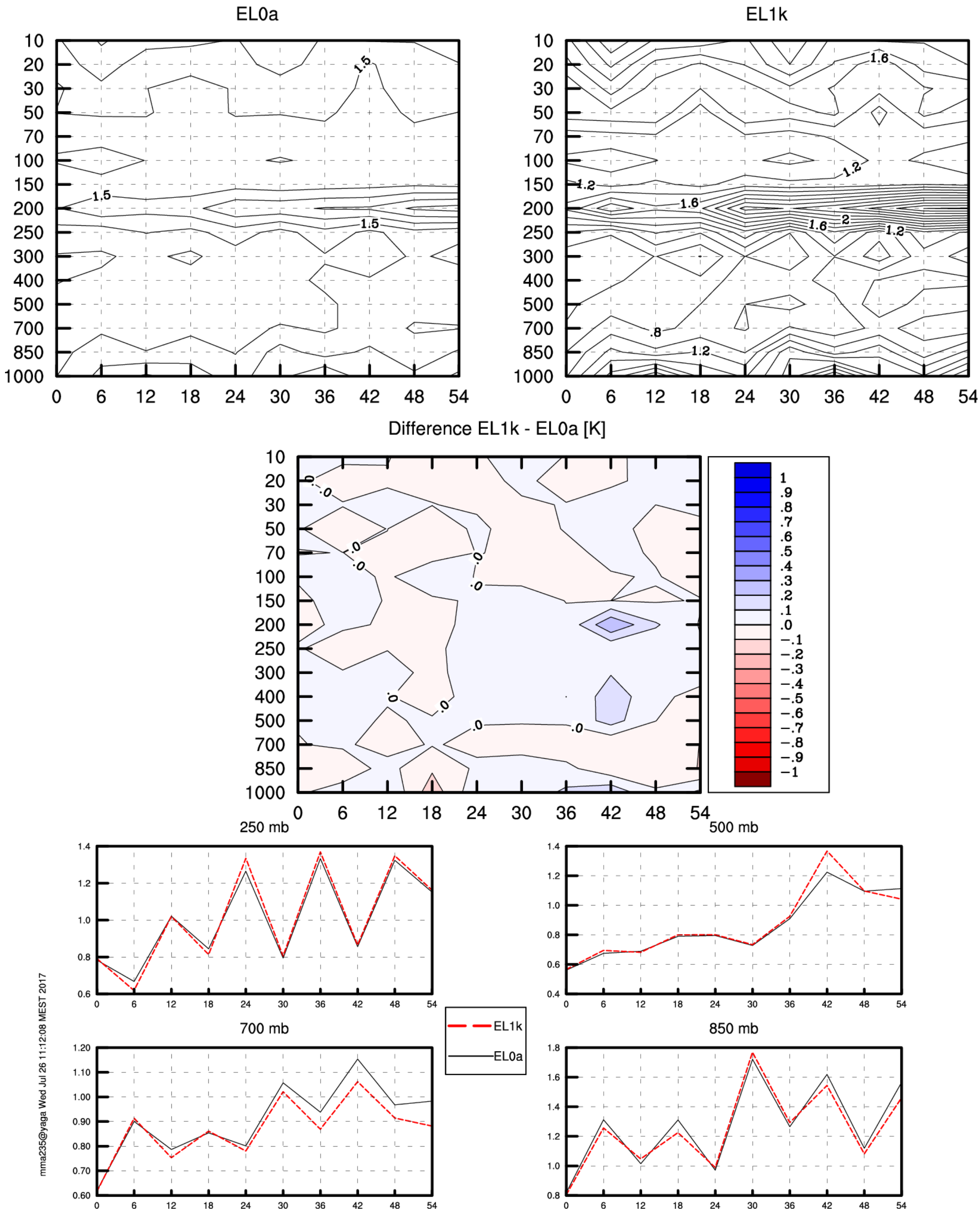

*Fig 6. Time evolution of the RMSE of temperature for reference (EL0a) and experiment (EL1k) throughout the 54-hours forecast window during the 26-30.06.2009. period; (1) averaged and height dependant – upper panels, (2) averaged and height dependent difference – middle panel and (3) at specific pressure levels – lower panels (for explanation of experiment abbreviations please check Table 1.).*

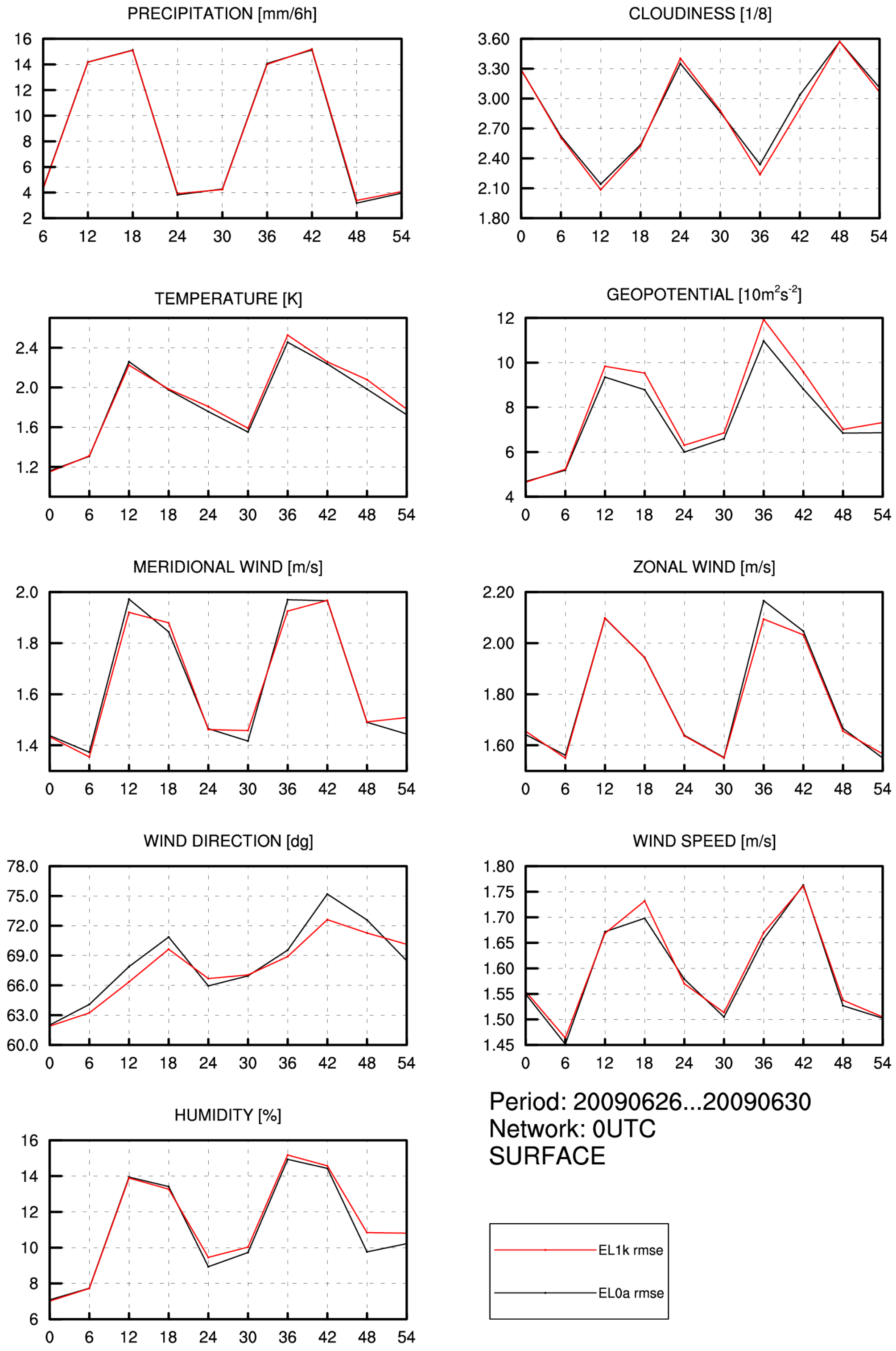

*Fig 7. Time evolution of averaged surface RMSE for basic meteorological parameters for reference (EL0a) and experiment (EL1k) throughout 54-hours forecast window during the 26-30.06.2009. period. Names and units of variables are given above each panel.*

### 2.4.2. Humidity

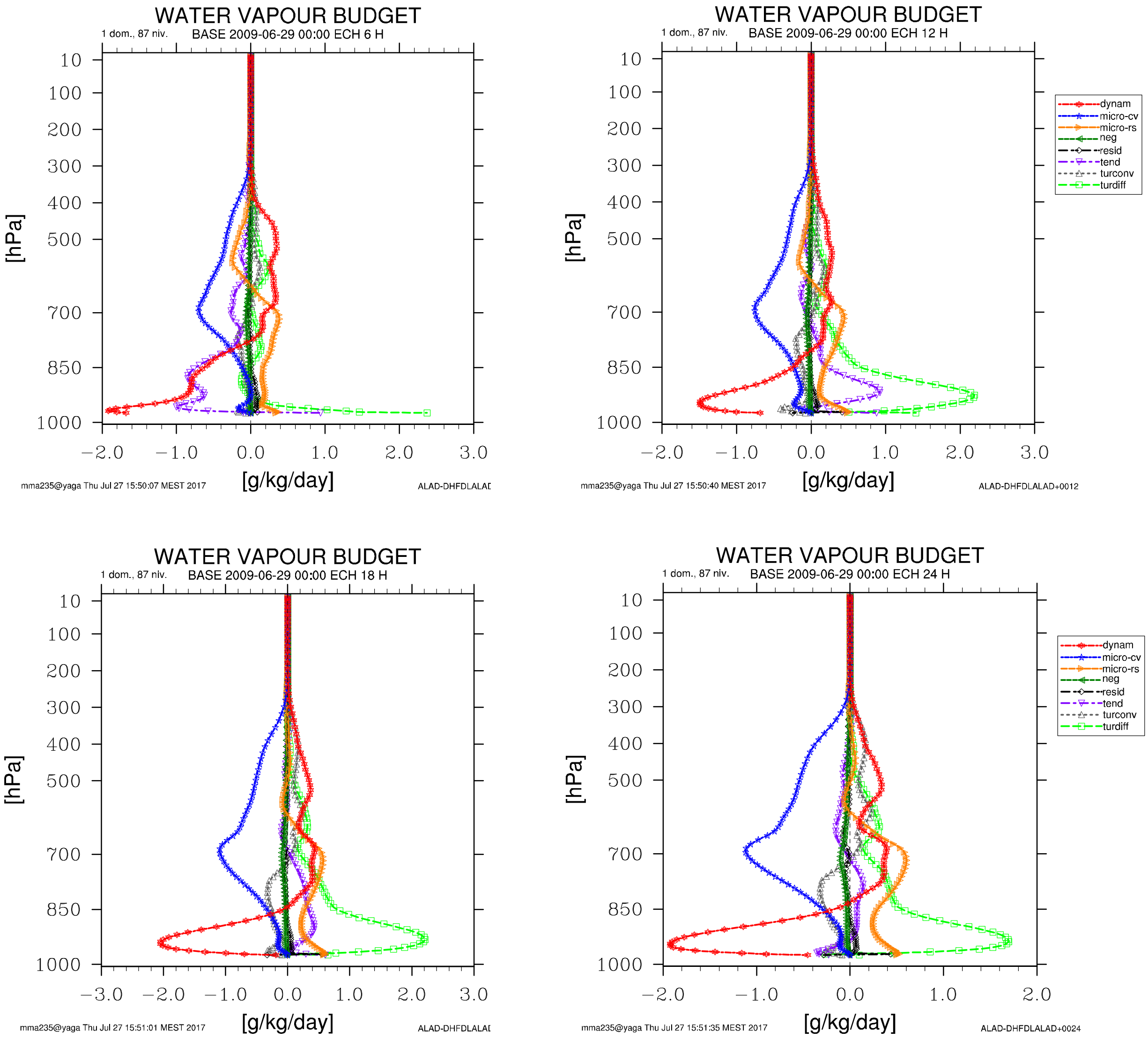

*Fig 8. Domain averaged vertical profile of water vapour budget terms for experiment 2 (EL1k) during the 29.06.2009. 00 UTC forecast: 06 hours (upper left panel), 12 hours (upper right panel), 18 hours (lower left panel) and 24 hours after the initialization (lower right panel).*

Turbulent diffusion of moisture (light green squares) and dynamics are two dominant terms of the water vapour budget equation within the ABL (Fig 8.). However, the magnitude of turbulent difussion is significantly smaller for EL1k experiment than for the reference (Fig 9.). This results in relative drying of the ABL (including the screen level; Fig 10.) compared to the reference. Above the 850 hPa pressure level the difference of magnitudes of turbulent diffusion changes, which results in relative moistening of this layer (up to about 500 hPa pressure level) within EL1k experiment. The effect of moistening is further supported by microphysical processes (blue stars). The overall results are decreasing the near surface positive relative humidity bias and decreasing the negative bias of relative humidity above the ABL (Fig 5. and Fig 10.). STDE is mostly neutral or improved, except near the surface and above the 250 hPa pressure level (not shown here). RMSE shows similar signal (Fig 11.). Regarding other forecast parameters, the positive bias of cloudiness is further increased and the amount of precipitation is decreased. Due to neutral STDE (not shown here), the RMSE of precipitation is very similar to the reference (Fig 7.). On the other hand, the STDE of cloudiness is reduced as it is the case for the RMSE (Fig 7.).

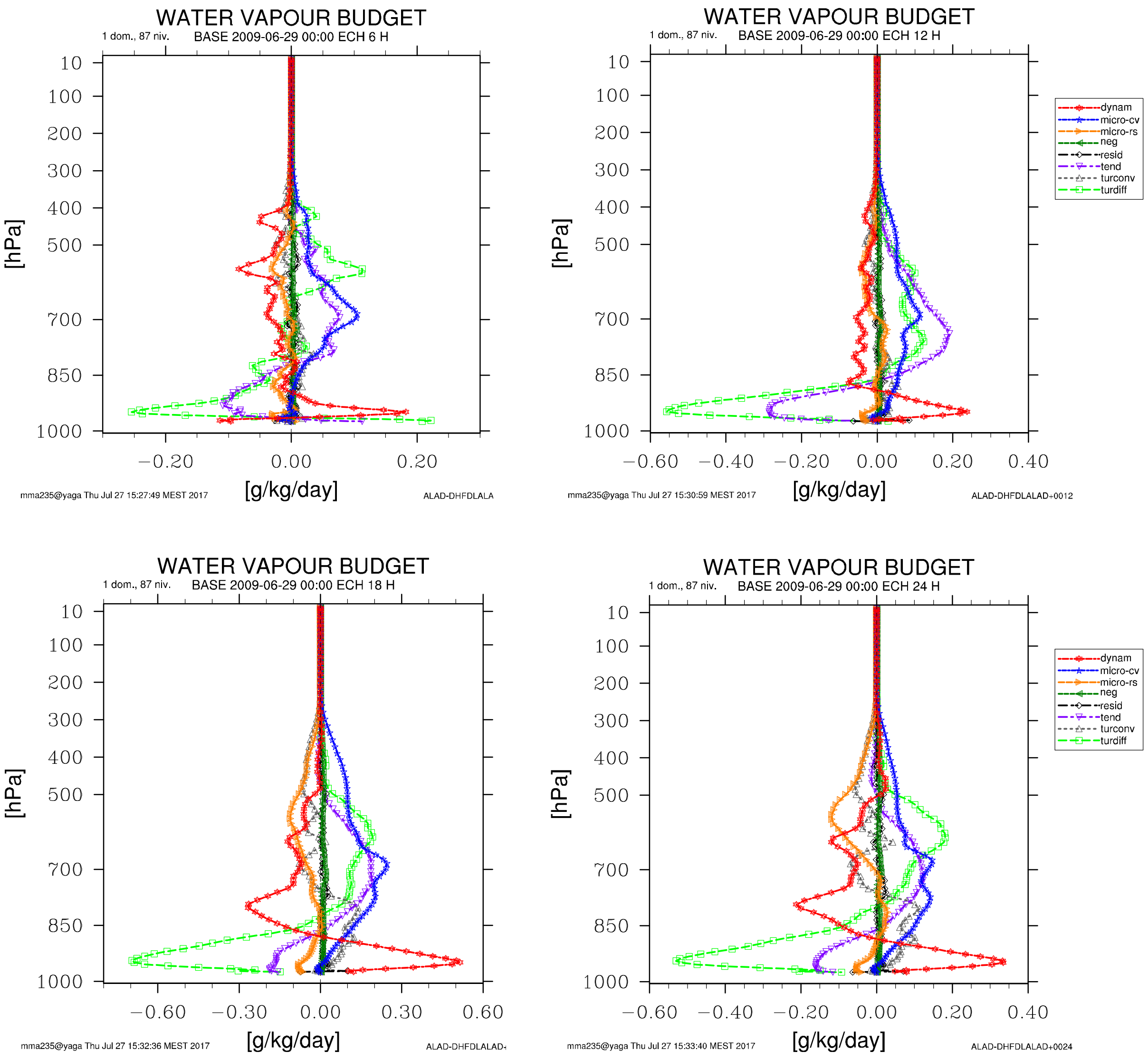

*Fig 9. Domain averaged vertical profile of differences of water vapour budget terms between the reference (EL0a) and experiment 2 (EL1k) during the 29.06.2009. 00 UTC forecast: 06 hours (upper left panel), 12 hours (upper right panel), 18 hours (lower left panel) and 24 hours after the initialization (lower right panel). Where sign is "-", the experiment term is smaller in magnitude than the reference and opposite.*

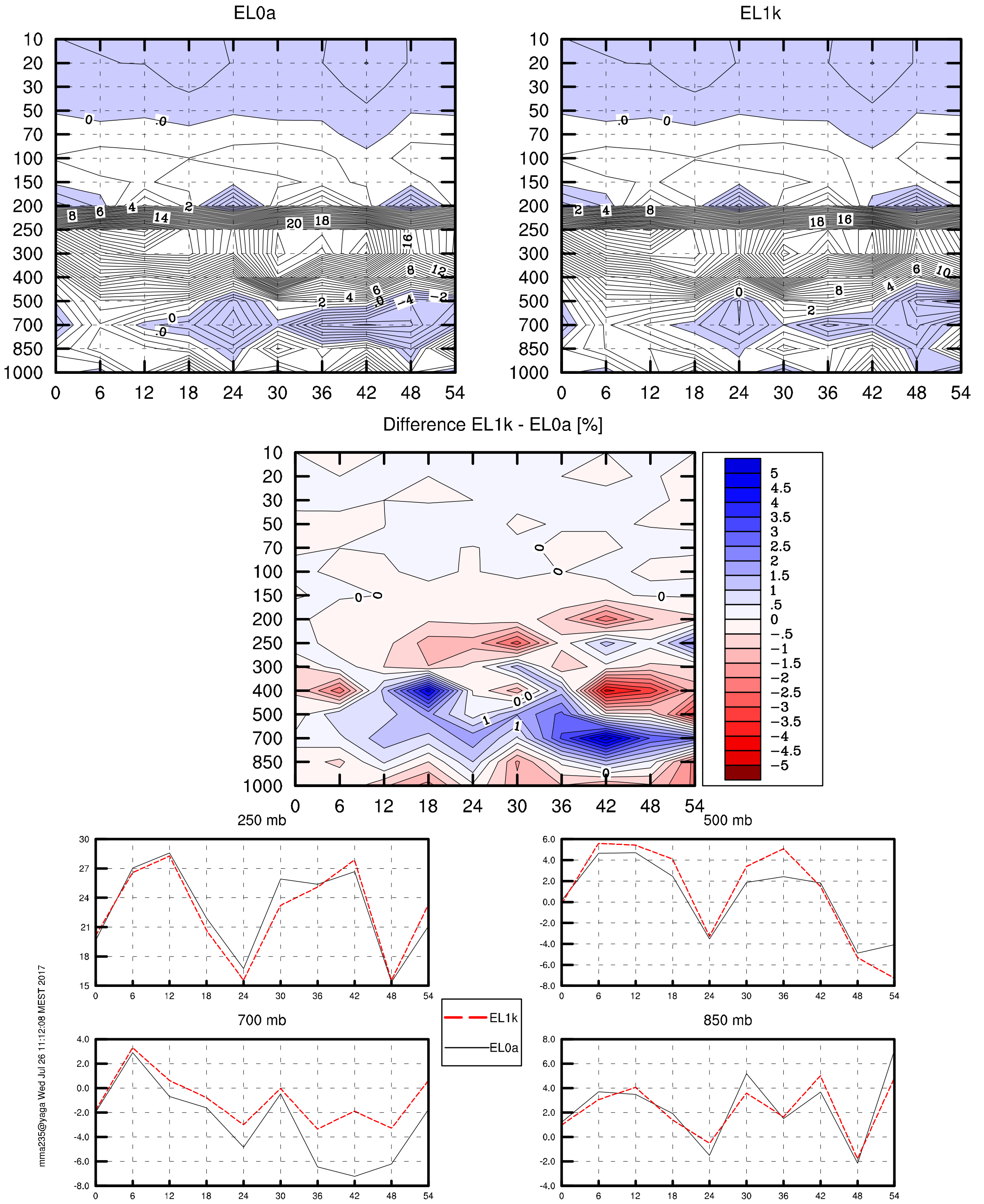

*Fig 10. Time evolution of the REAL BIAS of relative humidity for reference (EL0a) and experiment (EL1k) throughout the 54-hours forecast window during the 26-30.06.2009. period; (1) averaged and height dependant – upper panels, (2) averaged and height dependent difference – middle panel and (3) at specific pressure levels – lower panels (for explanation of experiment abbreviations please check Table 1.).*

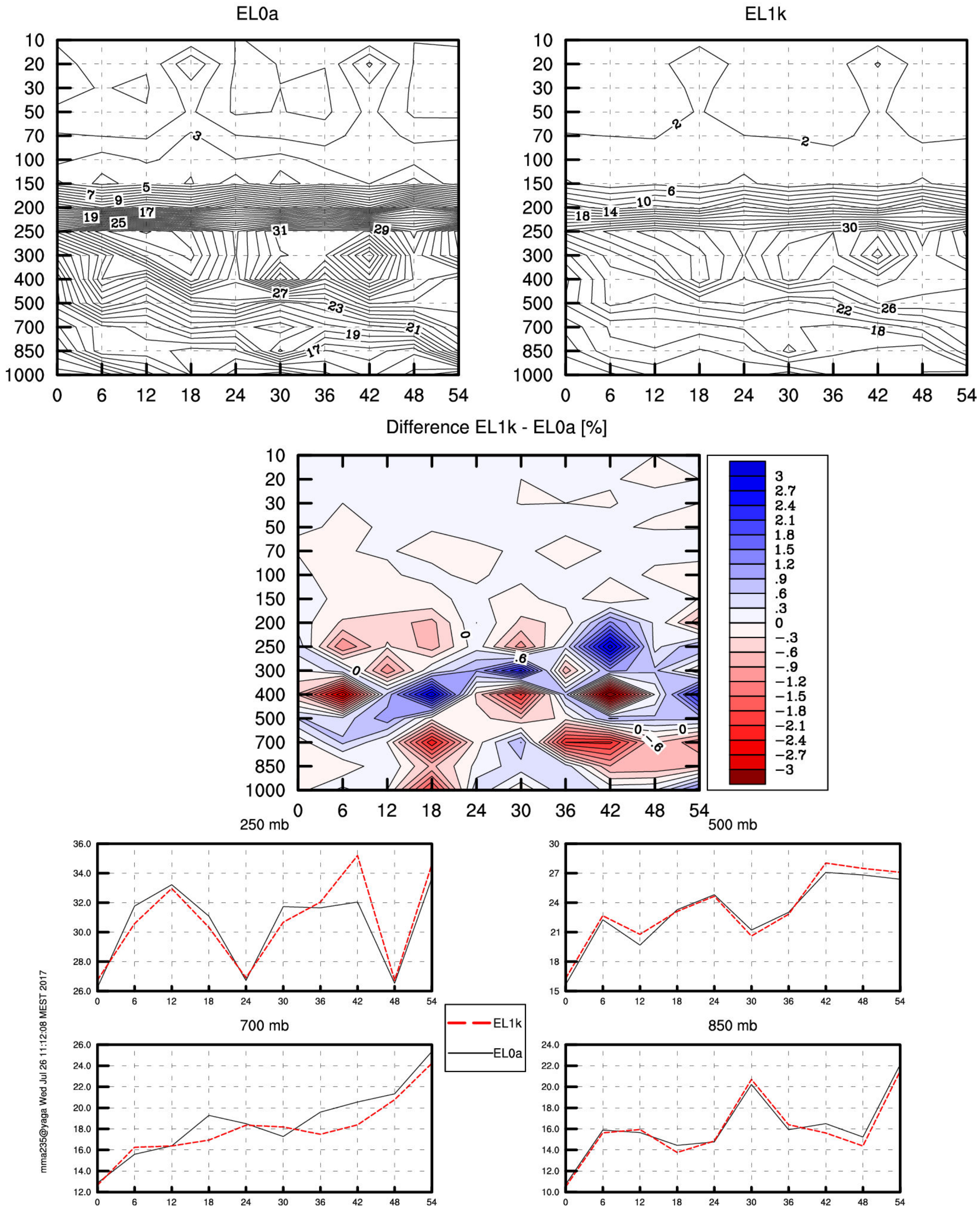

*Fig 11. Time evolution of the RMSE of relative humidity for reference (EL0a) and experiment (EL1k) throughout the 54-hours forecast window during the 26-30.06.2009. period; (1) averaged and height dependant – upper panels, (2) averaged and height dependent difference – middle panel and (3) at specific pressure levels – lower panels (for explanation of experiment abbreviations please check Table 1.).*

## 3. Conclusions and future plan

The conversion among different types of length scales for turbulence emerged as a major problem when TKE-based formulations were used within TOUCANS. Here we implemented and tested the stability dependent conversion (EL1k) between the output of the TKE-based formulation (BL89 in this case) and the Prandtl type mixing length, which is used for the computation of exchange coefficients. The stability dependence is a result of releasing the assumption that $L_{TKE} = L$ and stating a new one that $L_{TKE} = L_K$. The latter one is more or less consistent with Cuxart et. al (2000), except for the fact that TOUCANS differs from scales for exchange processes and dissipation.

Compared to the former constant conversion between TKE-based and Prandtl type scales(EL1a), the inclusion of stability dependence (EL1k) significantly improved the verification scores for all parameters. For this reason, the focus in this report is on comparison with the reference forecast (EL0a). The performance of the former constant conversion against the reference can be seen in Hrastinski (2016).

By implementation of EL1k formulation we increased the mixing length values obtained by TKE-based formulation within the ABL and above. Thus we improved the verification scores and in some aspects overcome the forecast based on currently operational Prandtl type mixing length. This especially stands for temperature and relative humidity below the 500 hPa pressure level. The exception is a screen level, where scores are mixed or slightly worse than for the reference. There is also a problem with the cloudiness whose positive bias within the reference is even more increased, while the STDE and RMSE are decreased. These problems should be solved by retuning the surface and microphysics schemes.

In further work, we should:

- test another assumption (i.e. formulation) of $L_{TKE} = L_\varepsilon$ (already coded)
- use different averaging operators for $L_{up}$ and $L_{down}$ (perhaps including different operators throughout the vertical)
- extend the verification period and include other, non-convection, cases
- combine BL89 with DE80 formulation or some other formulations, e.g. Grisogono and Belušić (2008)
- modify the $l_m = kz$ formulation at the lowest level like e.g. Lenderink and Holtslag (2004)

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## 4. References

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