Parameterisation of friction with respect to Ekman layer relationships and cyclogenesis

by

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Why Ekman layer ?

- Gives exact solutions for the friction force and its derivatives (although under very simplified conditions)
- Ekman pumping/suction effects have influence on cyclogenesis/anticyclogenesis (secondary circulation)
- Consequencies of Ekman-type of friction are at least qualitatively observed in the boundary layer (Ekman-Taylor spiral)

Main constraints

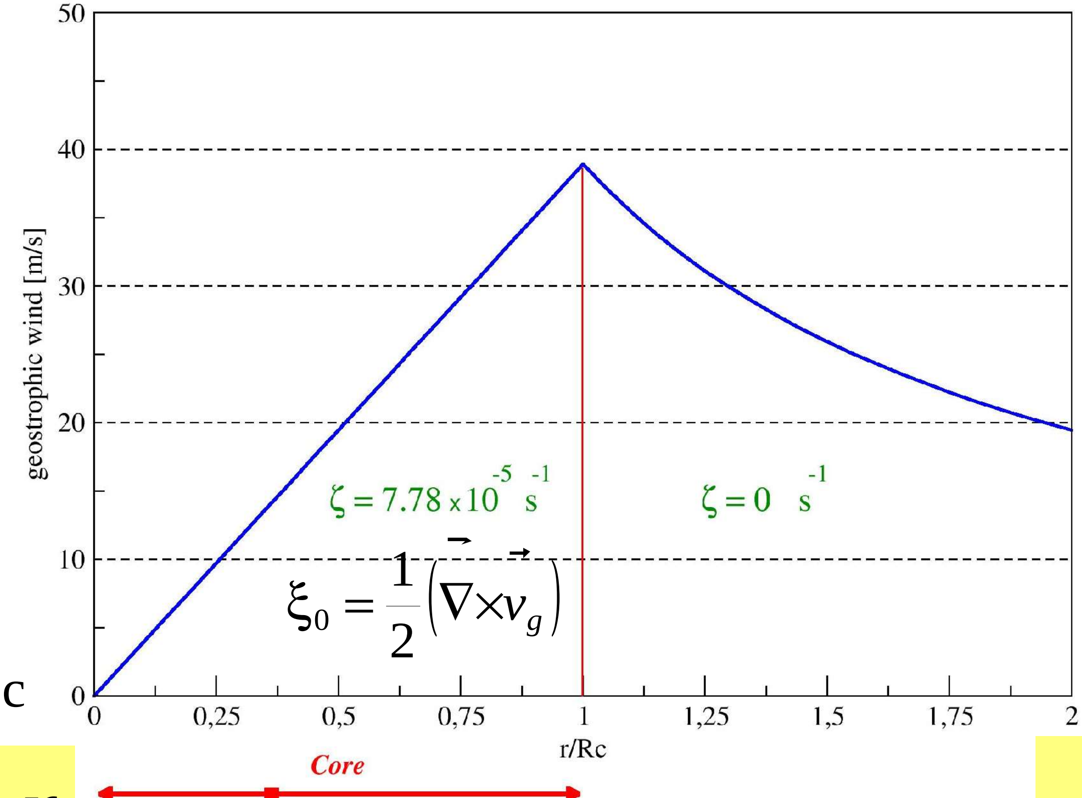
- Balance between the Coriolis, PG and friction forces
- Constant density (horizontally + vertically) $\Rightarrow$geostrophic wind is constant with height $\Rightarrow$atmosphere is barotropic
- Exchange (eddy viscosity) coefficients are constant
- Valid only for neutral stratifications !

Rankine vortex - simple example to evaluate effects on cyclogenesis

- Mixture of solid body rotation and irrotational axisymmetric vortex
- Core: uniform vorticity
- Outside of the core: shear vorticity cancels the curvature contribution
- Geostrophic adjustment

Construction of the vortex

Variation of the geostrophic wind velocity in Rankine vortex



$$\xi_0 = \frac{1}{2}\left(\vec{\nabla} \times \vec{v}_g\right)$$

Core: r≤ Rc

$$U_g = \xi_0 \, r$$

Outside:

$$U_g = \frac{R_c^{\,2} \xi_0}{r}$$

Ekman relationships for friction force

- Dependent on the angle between the geostrophic and actual wind: α (varies with geostrophic wind and latitude)
- Friction force is directly proportional to the geostrophic wind (generally)

$$F_x = -2\sqrt{2} K_e a^2 \sin\alpha\, e^{-az} \left[\sin\left(\alpha + \frac{3\pi}{4} - az \right) \right] U_g = a_{(Fx)} U_g$$

$$F_y = 2\sqrt{2} K_e a^2 \sin\alpha\, e^{-az} \left[\cos\left(\alpha + \frac{3\pi}{4} - az \right) \right] U_g = a_{(Fy)} U_g$$

Ekman relationships and vorticity evolution

- We apply the rotation of the Ekman kind of friction force to vorticity equation (while concentrating only to the friction term)

$$\frac{D\xi}{Dt} = k\left(\vec{\nabla}\times\vec{F}_{fric}\right) = \left(\vec{\nabla}\times\vec{v}_g\right)_z a_{(Fx)}$$

$$\frac{D\xi}{Dt} = 2\xi_0\, a_{(Fx)} \quad \text{(Core)} \qquad\qquad \frac{D\xi}{Dt} = 0 \quad \text{(Outside)}$$

- The effect of friction is directly proportional to vorticity (supposing $a_{(Fx)}$ horizontally uniform)

What about our parameterisation ?

- K – theory: Friction force depends on vertical variation of K- coefficient and wind shear

$$\vec{F} = \frac{1}{\rho}\left[\frac{\partial}{\partial z}\left(\rho K \frac{\partial \vec{v}}{\partial z}\right)\right]$$, where $$K = l_m^2 \left|\frac{\partial v}{\partial z}\right| F(Ri) \approx l_m^2 \left|\frac{\partial v}{\partial z}\right|$$

- We apply the wind shear from the Ekman relations (simulating the Ekman atmosphere)

K-parameterisation vs. Vorticity equation

- We get a quadratic dependence on geostrophic wind and vorticity:

$$F_x = \frac{l_m \sin\alpha\, e^{-az}}{K}\left[\left(a l_m - 2\frac{\partial l_m}{\partial z}\right)\left(a_{(Fy)} - a_{(Fx)}\right) + 2 a l_m\, a_{(Fx)}\right] U_g^2 = C_x U_g^2$$

$$\frac{D\xi}{Dt} = \vec{k}\left(\vec{\nabla}\times\vec{F}_{fric}\right) = 3 C_x \xi_0^2 r$$ Core

$$\frac{D\xi}{Dt} = \vec{k}\left(\vec{\nabla}\times\vec{F}_{fric}\right) = -C_x \xi_0^2 \frac{R_c^4}{r^3}$$ Outside of the core

Consequencies

- Spurious hodographs of the friction force for K- parameterisation
- Exaggerated friction force mainly for big geostrophic winds
- Non-proportional vorticity changes (cancelation of the Rankine vortex)
- Creation of vorticity outside of the core

Treatment

- Elimination of the first term in the Cx expression

$$F_x = \frac{l_m \sin\alpha \, e^{-az}}{K}\left[\underbrace{\left(a l_m - 2\frac{\partial l_m}{\partial z}\right)}_{=\,0}\left(a_{(Fy)} - a_{(Fx)}\right) + 2 a l_m \, a_{(Fx)}\right] U_g^2 = C_x \, U_g^2$$

- Solving a differential equation for the mixing length and specification of the boundary conditions with respect to the desired Ekman solution …

Solution

- An exponential profile for the mixing length !

$$l_m = l_{m\,(z_0)}\, e^{\frac{a}{2}(z - z_0)}$$

- From boundary conditions:

$$l_{m\,(z_0)} = \sqrt{\frac{K_0}{\left|\frac{\partial v}{\partial z}\right|_0}} \approx \kappa z_b$$

, where Zb represents the top of the surface layer

Model implementation

- Purely exponential solution possible only in academic situations
- We have to specify « a » and « Zb »
- Possible way: exp. solution until the top of the Ekman layer + keeping the present formula above +

$$a = \sqrt{\frac{f}{2K}} \approx \sqrt{\frac{f}{2K_b}}$$

$$K_b = l_m^2(z_b) \left| \frac{\partial v}{\partial z} \right| (z_b)$$

keeping in mind, that K is vertically uniform in the Ekman layer, thus, equal to the value at arbitrary height of the surface layer Zb

Two sets of mixing lengths

- Goal: to have smooth transition at the top of the Ekman layer (vertical derivations are there equal for both sets)
- Thus:

$$l_m = \kappa z_b e^{\frac{a'}{2}(z - z_b)} \qquad Z <= Zpe$$

$$l_m = l_{m\,(z_{pe})} + \left[\frac{C_{pe}(z - z_{pe})}{1 + \frac{C_{pe(z - z_{pe})}}{\lambda_m}} \right] \left[\beta + \frac{1-\beta}{1 + \left(\frac{z - z_{pe}}{H - z_{pe}} \right)^2} \right] \qquad Z > Zpe$$

, where $C_{pe} = \left(\frac{\partial l_m}{\partial z} \right)_{z_{pe}} = \frac{a' l_{m(z_{pe})}}{2}$

and $z_{pe} = \frac{\pi}{a'}$ is the parameterized height of the Ekman layer

Properties of the new scheme

- Not an arbitrary mixing length profile, but dependent on the latitude and wind shear
- Small vertical wind shears: parcel keeps longer its properties, hence the mixing length is longer and vice versa
- Small vertical wind shears: close to the present parameterisation, big shears: more uniform profile with height

Results

- Scheme is rather suppressing rapid cyclogenesis and the impact on false cyclogenesis is small and ambiguous
- The impact of Ekman friction on cyclogenesis is smaller and takes more time as secondary effects of turbulent transport of momentum and heat
- The global means give expected increase of static stability (due to suppression of the PBL top maxima of K coefficients)
- Scheme is stable and not much more CPU consuming ….

Future tests:

- Academic tests: more complicated models with « a », « f » latitude dependent, simulation of the barotropic decay of the cyclone
- Model: Tuning of the surface layer height and wind shears at this level (or of the limits for the height of the Ekman layer and of the PBL top)