Regional Cooperation for Limited Area Modeling in Central Europe

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ACCORD

A Consortium for COnvection-scale modelling Research and Development

LACE novelties: Stability of the SI time scheme with modified linear model

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Outline

- Dynamical core in ACCORD
- SI time scheme
- Control parameters of the linear model
- Stability analysis
- Idealised test cases
- Real simulations

Dynamical core in ACCORD

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- hybrid vertical coordinate η based on hydrostatic pressure π
- hydrostatic primitive equation system (HPE) or Euler equations (EE); recently implemented quasi elastic equation system (QE)
- prognostic variables $\vec{v}, T, q_s = \ln(\pi_s)$, in EE with $w, \widehat{q} = \ln(\frac{p}{\pi})$
- moisture included

Discretization

- spectral method for horizontal direction
- finite differences or finite elements for vertical direction
- semi-implicit or iterative centred implicit scheme for time
- semi-Lagrangian advection

Total time derivative

$$\dot{\psi} = \frac{\partial \psi}{\partial t} + \vec{v} \cdot \vec{\nabla} \psi + \dot{\eta} \frac{\partial}{\partial \eta}$$

Semi-Implicit time scheme

System evolution

$$\frac{\partial X}{\partial t} = \mathcal{M}X$$

Linearization

$$\frac{\partial}{\partial t}\mathcal{M} \longrightarrow \mathcal{L}$$

Using linear model $\mathcal{L}$ we divide

$$\frac{\partial X}{\partial t} = \mathcal{L}\overline{[X]}^t + (\mathcal{M} - \mathcal{L})X$$

and discretize in time to obtain

Semi-implicit scheme

$$\frac{X^+ - X^0}{\Delta t} = \mathcal{L}\left(\frac{X^+ + X^0}{2}\right) + (\mathcal{M} - \mathcal{L})X^{+\frac{1}{2}}$$

or

Iterative centered implicit scheme

$$\frac{X^{+(n)} - X^0}{\Delta t} = \frac{\mathcal{L}X^{+(n)} + \mathcal{L}X^0}{2} + \frac{(\mathcal{M} - \mathcal{L})X^{+(n-1)} + (\mathcal{M} - \mathcal{L})X^0}{2}$$

We know that both can be second order accurate in time when some care is taken (averaging along semi-Lagrangian trajectory).

Linearization

We can linearize $\mathcal{M}$ around a basic state X^* or we can use any linear preconditioner $\mathcal{L}$, allowing the convergence of our iterative algorithm.

We introduce a set of control parameters $\alpha, \beta, \gamma, \delta, \epsilon, \tau$ in $\mathcal{M}$ with the following properties:

1. if all control parameters = 0 we obtain HPE system
2. if all control parameters = 1 we obtain EE system

and get $\mathcal{M}_{control}$.

Then we linearize $\mathcal{M}_{control}$ around a basic state X^* to obtain $\mathcal{L}_{control}$, and afterwards we use

$$\begin{aligned} \mathcal{M} &= \mathcal{M}_{control}(\alpha, \beta, \gamma, \delta, \epsilon, \tau = 1) \\ \mathcal{L} &= \mathcal{L}_{control}(\alpha, \beta, \gamma, \delta, \epsilon, \tau \neq 1). \end{aligned}$$

Following the way how the control parameters were chosen, we can as well generally take

$$\begin{aligned} \mathcal{M} &= \mathcal{M}_{control}(\alpha, \beta, \gamma, \delta, \epsilon, \tau \neq 1) \\ \mathcal{L} &= \mathcal{L}_{control}(\alpha, \beta, \gamma, \delta, \epsilon, \tau \neq 1) \end{aligned}$$

and get an approximation of the full EE system.

$\mathcal{M}_{control}$ model

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Temperature

$$\dot{T} = \kappa T \frac{\dot{\pi}}{\pi} - \alpha \left[\kappa T \frac{\dot{\pi}}{\pi} + \frac{RT}{C_v} D_3 \right]$$

Horizontal momentum

$$\dot{\vec{v}} = -RT \frac{\vec{\nabla}\pi}{\pi} - \vec{\nabla}\phi - \beta \left[RT \vec{\nabla}\hat{q} + \left(\frac{1}{m} \frac{\partial p}{\partial \eta} - 1 \right) \vec{\nabla}\phi \right]$$

Vertical momentum

$$\dot{w} = \frac{g}{m} \frac{\partial \gamma (p - \pi)}{\partial \eta}$$

Pressure departure

$$\dot{\hat{q}} = -\delta \left[\frac{1}{1-\kappa} D_3 + \frac{\dot{\pi}}{\pi} \right]$$

Surface pressure

$$\dot{q}_s = -\tfrac{1}{\pi_s} \int_0^1 \vec{\nabla}(m\vec{v}) d\eta$$

Diagnostic relations

$$\dot{\pi} = \vec{v} \cdot \vec{\nabla}\pi - \int_0^{\eta} \vec{\nabla}(m\vec{v}) d\eta'$$

$$\phi = \phi_s - \int_{\eta}^{1} \frac{mRT}{\pi} d\eta' - \int_{\eta}^{1} \epsilon \left(\frac{\pi}{p} - 1 \right) \frac{mRT}{\pi} d\eta'$$

$$D_3 = D + \frac{\pi + \tau(p - \pi)}{mRT} \left(\vec{\nabla}\phi \frac{\partial \vec{v}}{\partial \eta} - g \frac{\partial w}{\partial \eta} \right)$$

HYDROSTATIC SYSTEM

with $\alpha, \beta, \gamma, \delta, \epsilon, \tau = 0$

NH SYSTEM

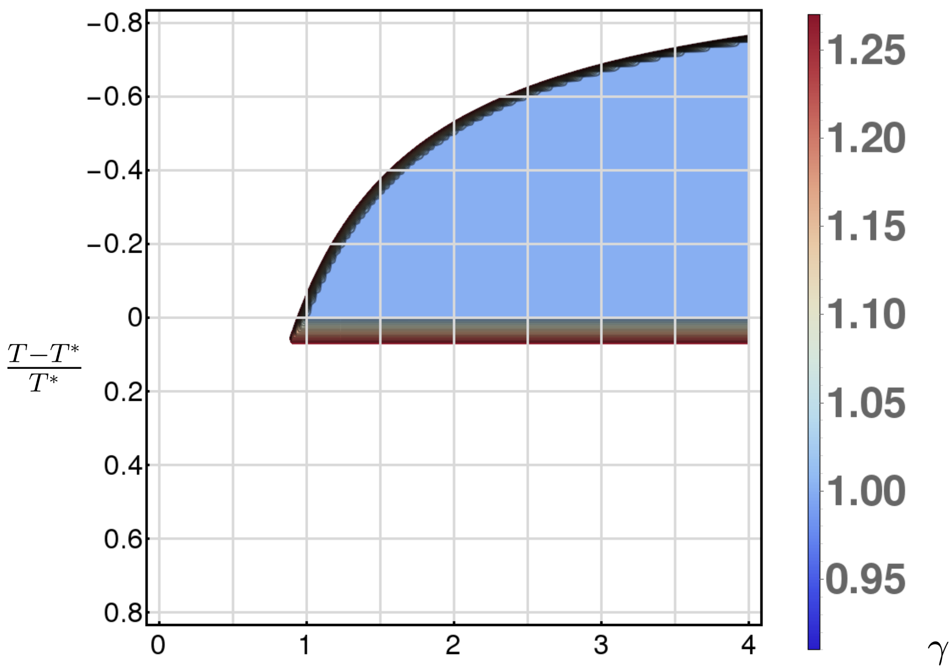
with $\alpha, \beta, \gamma, \delta, \epsilon, \tau = 1$

Parameter γ

Parameter $\gamma = SITR/SITRA$ in the ACCORD system.
Values $\gamma > 1$ were shown to increase stability of the SI time scheme.

[P. Bénard, 2004: On the use of a wider class of linear systems for the design of constant-coefficients semi-implicit time-schemes in NWP, doi=10.1175/1520-0493(2004)132<1319 OTUOAW>2.0.CO;2]

Amplification factor from the SHB type stability analysis: blue indicates stable region, red and white means very unstable



Control parameters relations

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Full elimination for the normal structure modes of the simplified equation system having the shape

$$X(x,\sigma) = \hat{X}\,\exp(ikx + i\omega t)\sigma^{(i\nu - \frac{1}{2})}$$

gives

the structure equation for given ω, k, ν

$$\omega^4 - c^2\omega^2 \left(\beta\delta \frac{\left(\nu^2 + \frac{1}{4}\right)}{H^2} + (1 - \frac{\epsilon\delta - \kappa\alpha}{(1-\kappa)})k^2 \frac{N^2H^2}{c^2\left(\nu^2 + \frac{1}{4}\right)} + \frac{\epsilon\delta - \kappa\alpha}{(1-\kappa)}k^2 \right) + \beta\delta c^2 N^2 k^2 = 0$$

Then

$$\left.\begin{array}{l} \beta = \epsilon \\ \frac{\epsilon\delta - \kappa\alpha}{(1-\kappa)} = \beta\delta \end{array}\right\} \Rightarrow \alpha = \beta\delta$$

to ensure the structure equation collapses to hydrostatic (if $\alpha, \beta, \gamma, \delta, \epsilon, \tau = 0$**) and fully elastic one (if** $\alpha, \beta, \gamma, \delta, \epsilon, \tau = 1$**). We apply only two possible settings with only one free parameter** h**:**

SET1

$$\alpha = \beta = \epsilon = h$$
$$\delta, \tau = 1$$

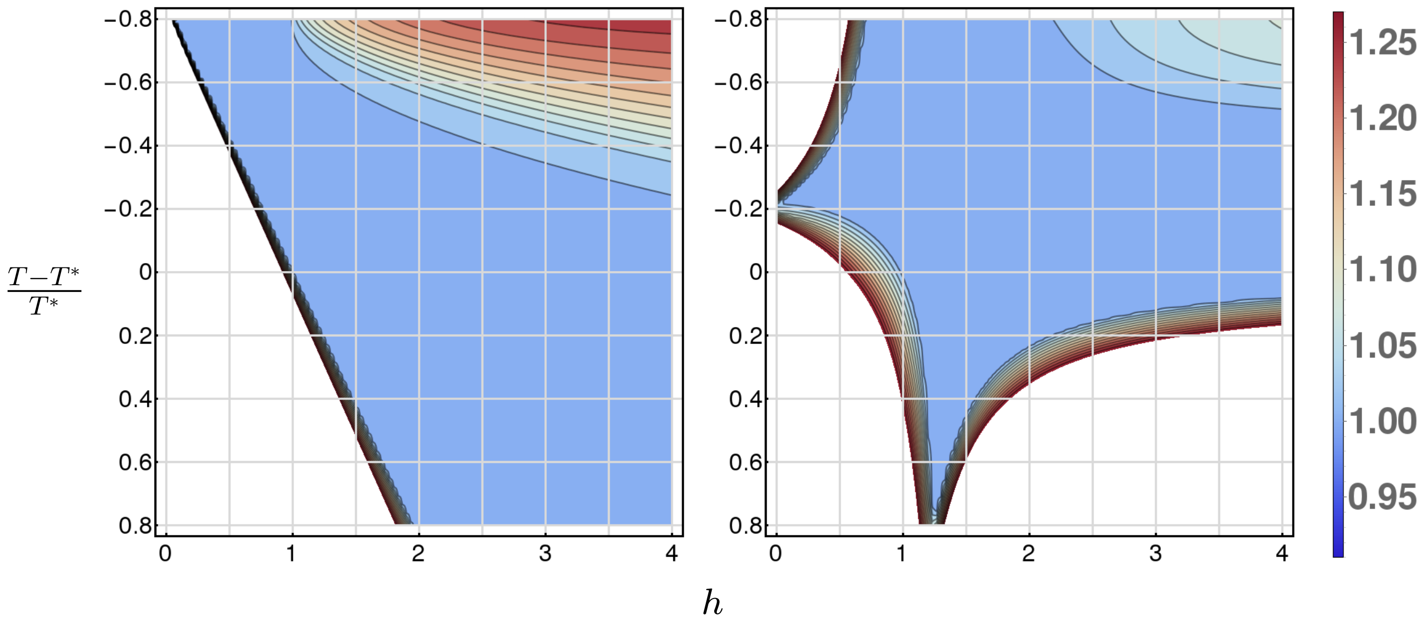
SET2

$$\alpha = \delta = h$$
$$\beta = \epsilon = \tau = 1$$

Stability analysis

Amplification factor from the SHB type stability analysis in the simpified context : isothermal atmosphere with $T \neq T^*$, one vertical mode, one wave number etc. $\Rightarrow$ we see one snapshot from a multiparametric space

blue indicates stable region, red and white means very unstable;



SI SET1 in linear model only

SI SET2 in linear model only

Stability analysis

Amplification factor from the SHB type stability analysis in the simpified context : isothermal atmosphere with $T \neq T^*$, one vertical mode, one wave number etc. $\Rightarrow$ we see one snapshot from a multiparametric space

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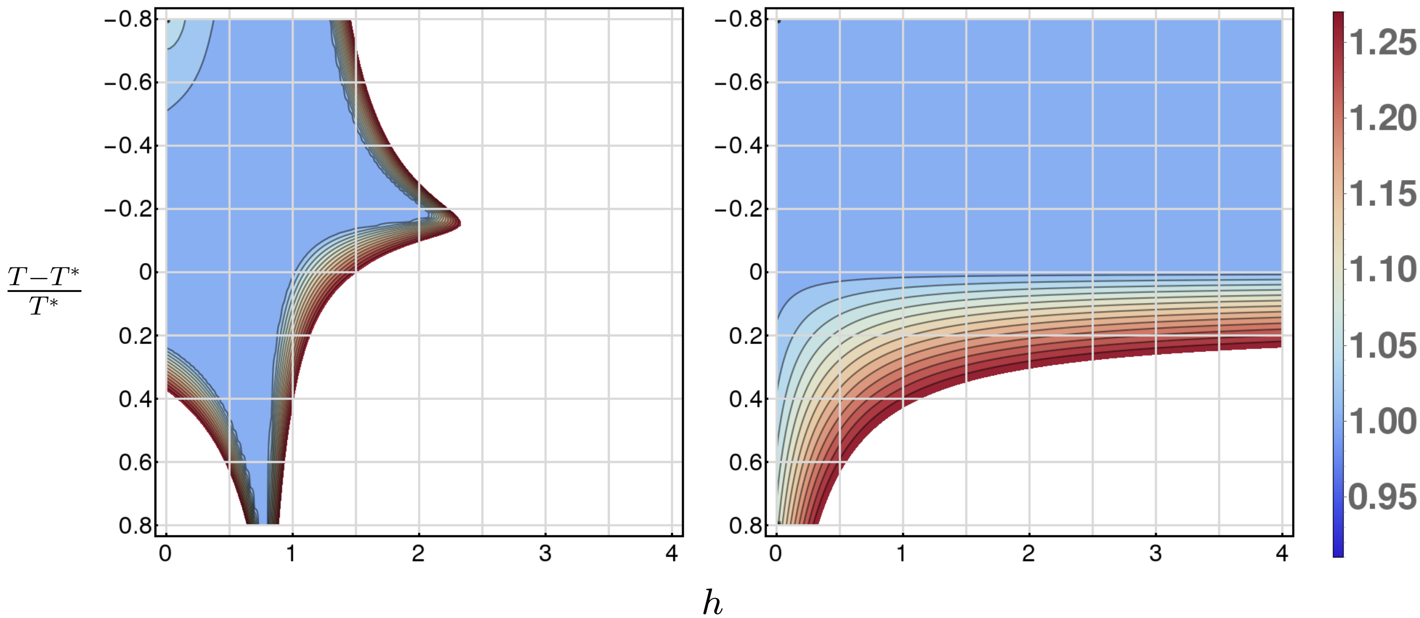
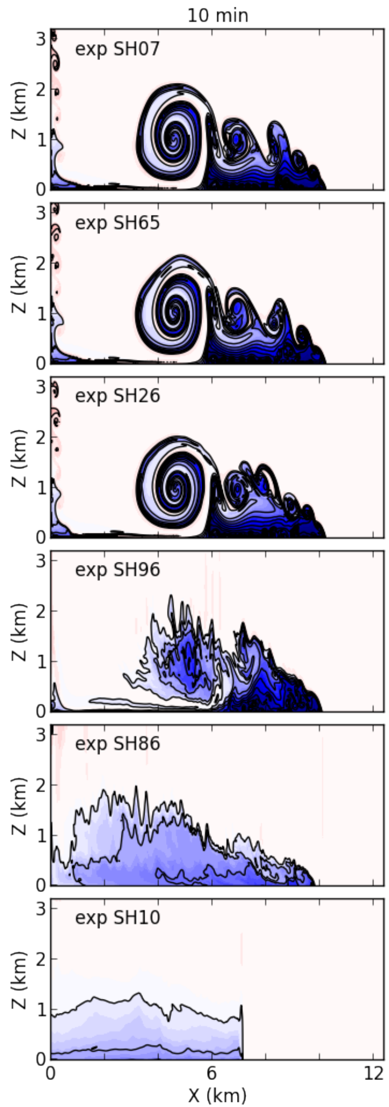


SI SET2 in full model only

SI SET2 in linear and full model

Idealised experiments

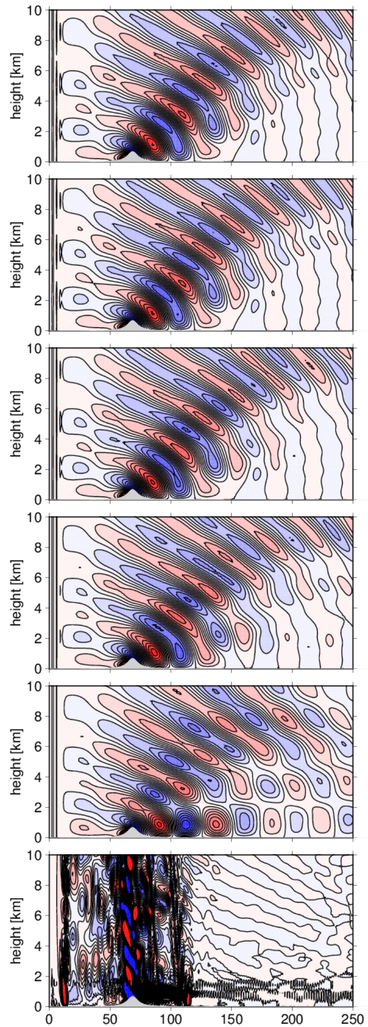
Straka density current with
$\Delta x = \Delta z = 25\,m$
$\Delta t = 1\,s$
after 600s,
the right part
of the potential
temperature field



- PC NH
- SI SET1 in linear model: $h = 5$
- SI SET2 in linear and full model: $h = 0.1$
- SI SET2 in linear and full model: $h = 0.01$
- SI SET2 in linear and full model: $h = 0.001$
- SI HYD

Idealised experiments

Non-linear non-hydrostatic flow over Agnesi shaped mountain with
$U = 4ms^{-1}$
$N = 0.01s^{-1}$
$\Delta x = 80\,m$
$\Delta z = 180\,m$
$\Delta t = 2\,s$,
after 6000 s
and with a sponge layer at the top, vertical velocity field



- PC NH
- SI SET1 in linear model: $h = 5$
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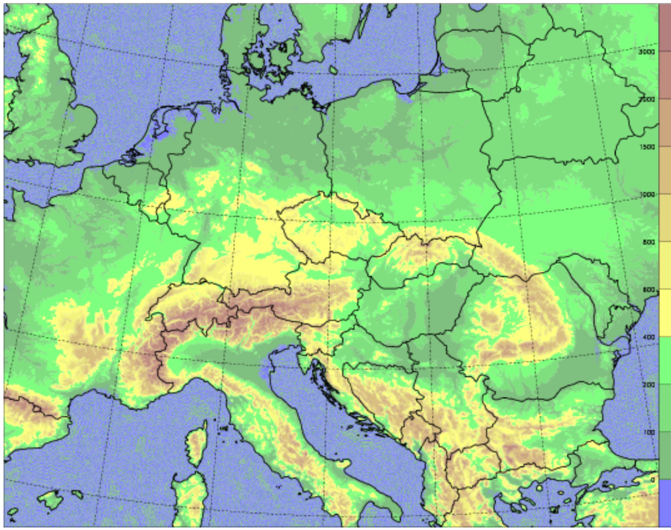
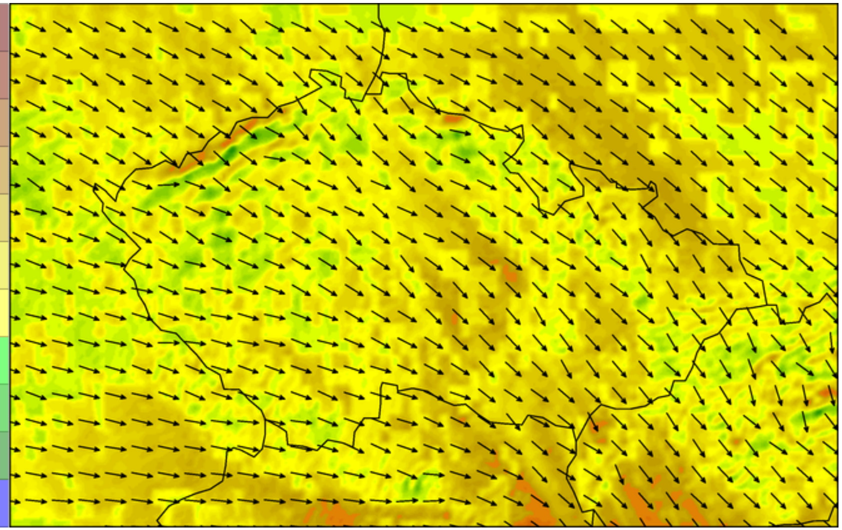
- SI SET2 in linear and full model: $h = 0.001$
- SI HYD

Real simulations

Lee waves behind Krusné hory, 19 February 2019 00 UTC + 11 hours
ALARO @ 2.325km, 87 vertical levels





orography **10m wind + wind direction**

Real simulations

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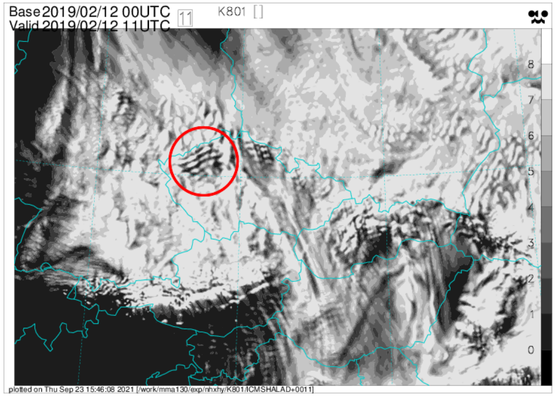
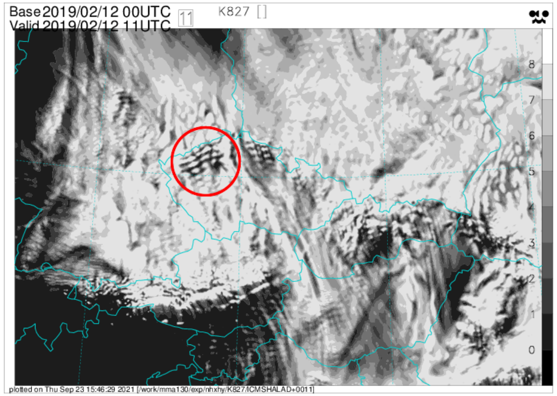
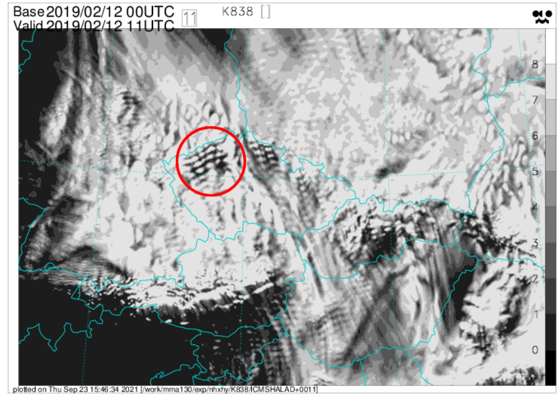
PC NH

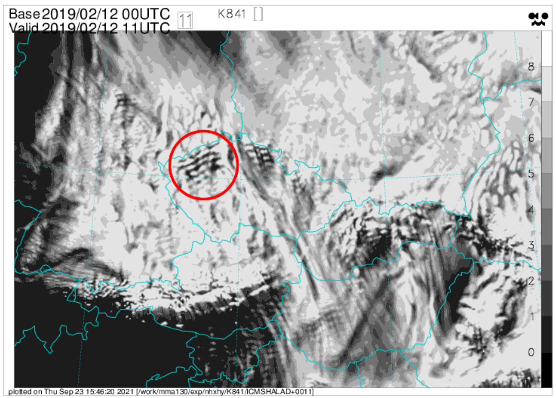


SI SET1 in linear model: $h = 5$

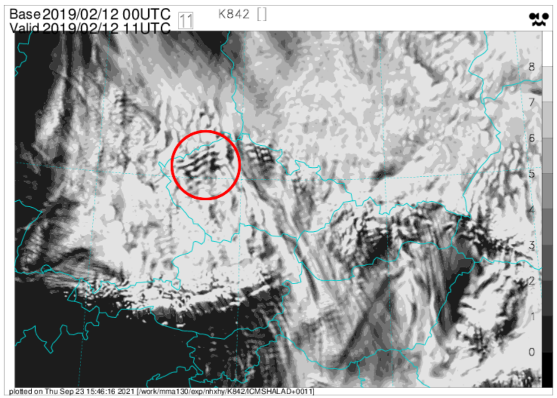


SI SET2 in linear and full model: $h = 0.1$

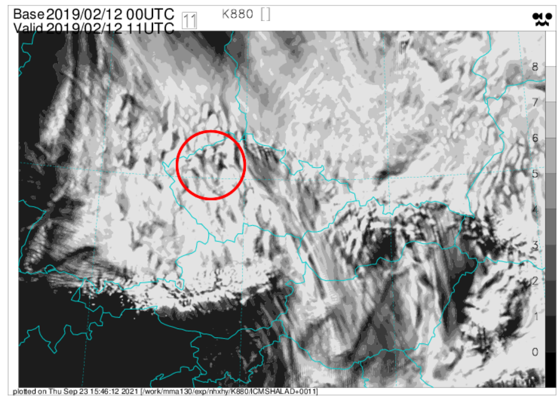




SI SET2 in linear and full model: $h = 0.01$



SI SET2 in linear and full model: $h = 0.001$



SI HYD

Conclusions

- the model $\mathcal{L}$ used in the SI time scheme for the separation of "linear"terms and "non-linear"residuum may be chosen arbitrarily, provided it ensures the convergence of the proposed iterative algorithm
- if $\mathcal{L}$ is defined as a linearization of $\mathcal{M} = \mathcal{M}_{control}(\alpha, \beta, \gamma, \delta, \epsilon, \tau \geq 1)$ around a basic state X^*, the stability of the SI time scheme may increase
- the stability of SI time scheme used with a modified equation set using $\mathcal{M} = \mathcal{M}_{control}(\alpha, \beta, \gamma, \delta, \epsilon, \tau \leq 1)$ may also be increased while the non-hydrostatic features of the flow are kept
- The SI time scheme algorithm of ACCORD would allow the vertical dependence of control parameters. We plan to investigate this in the near future.


Thank you for your attention!