Regional Cooperation for
Limited Area Modeling in Central Europe

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ACCORD
A Consortium for COnvection-scale modelling
Research and Development

Dynamics in LACE - towards hectometric scales

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Outline

- Dynamical core in ACCORD
- SI time scheme
- Orographic terms in linear model (based on work of Jozef Vivoda and Fabrice Voitus)
  - Some equations
  - Idealised tests
  - Real simulations @200m
- Vertical velocity definition in nonlinear model (based on work of Fabrice Voitus)
  - Real simulations @200m

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Basic equations

- hydrostatic primitive equation system (HPE) or Euler equations (EE); recently implemented quasi elastic equation system (QE)
- prognostic variables $\vec{v}, T, q_s = \ln(\pi_s)$, in EE with $w, \hat{q} = \ln(\frac{p}{\pi})$

Discretization

- spectral transform method for horizontal direction
- hybrid vertical coordinate η based on hydrostatic pressure $\pi(\eta) = A(\eta) + B(\eta)\pi_s$; $A(top) = B(top) = 0, A(bottom) = 0, B(bottom) = 1$
- finite differences or finite elements for vertical direction discretization
- semi-implicit or iterative centred implicit scheme for time discretization
- semi-Lagrangian advection

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Semi-Implicit time scheme

System evolution

$$\frac{dX}{dt} = \mathcal{M}X$$

Linearization

$$X = \mathbf{X}^* + \mathbf{X}', \quad \mathcal{M} \longrightarrow \mathcal{L}^*$$

Using linear model $\mathcal{L}^*$ we get

$$\frac{dX}{dt} = \mathcal{L}^*\overline{[X]}^t + (\mathcal{M} - \mathcal{L}^*)X$$

and discretize in time to obtain

Semi-implicit scheme

$$\frac{\mathbf{X}^+ - \mathbf{X}^0}{\Delta t} = \mathcal{L}^*\left(\frac{\mathbf{X}^+ + \mathbf{X}^0}{2}\right) + (\mathcal{M} - \mathcal{L}^*)\mathbf{X}^{+\frac{1}{2}}$$

or

Iterative centered implicit scheme

$$\frac{\mathbf{X}^{+(n)} - \mathbf{X}^0}{\Delta t} = \frac{\mathcal{L}^*\mathbf{X}^{+(n)} + \mathcal{L}^*\mathbf{X}^0}{2} + \frac{(\mathcal{M} - \mathcal{L}^*)\mathbf{X}^{+(n-1)} + (\mathcal{M} - \mathcal{L}^*)\mathbf{X}^0}{2}$$

We know that both can be second order accurate in time when some care is taken (averaging along semi-Lagrangian trajectory).

Full model

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Temperature

$$\frac{dT}{dt} = \frac{\kappa T}{\kappa - 1}(D + d)$$

Horizontal momentum

$$\frac{d\vec{v}}{dt} = -RT\frac{\nabla\pi}{\pi} - \nabla\phi - RT\nabla\hat{q} - \frac{1}{m}\frac{\partial(p-\pi)}{\partial\eta}\nabla\phi$$

Vertical momentum

$$\frac{dw}{dt} = \frac{g}{m}\frac{\partial(p-\pi)}{\partial\eta}$$

Pressure departure

$$\frac{d\hat{q}}{dt} = \frac{1}{\kappa - 1}(D + d) - \frac{1}{\pi}\frac{d\pi}{dt}$$

Surface pressure

$$\frac{dq_s}{dt} = -\frac{1}{\pi_s}\int_0^1 \nabla\cdot(m\vec{v})d\eta$$

Diagnostic relations

$$\frac{d\pi}{dt} = \vec{v}\cdot\nabla\pi - \int_0^\eta \nabla\cdot(m\vec{v})d\eta'$$

$$\phi = \phi_s - \int_\eta^1 \frac{mRT}{p}d\eta'$$

$$d = -\frac{p}{mRT}\left(\nabla\phi\frac{\partial\vec{v}}{\partial\eta} - g\frac{\partial w}{\partial\eta}\right)$$

Definitions

$$D = \nabla\cdot\vec{v}$$

$$\kappa = \frac{c_p}{R}$$

$$m = \frac{\partial\pi}{\partial\eta}$$

Full model

Temperature

$$\frac{dT}{dt} = \frac{\kappa T}{\kappa - 1}(D + d)$$

Horizontal momentum

$$\frac{d\vec{v}}{dt} = -RT\frac{\nabla\pi}{\pi} - \nabla\phi - RT\nabla\hat{q} - \frac{1}{m}\frac{\partial(p-\pi)}{\partial\eta}\nabla\phi$$

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Definitions

$$D = \nabla\cdot\vec{v}$$

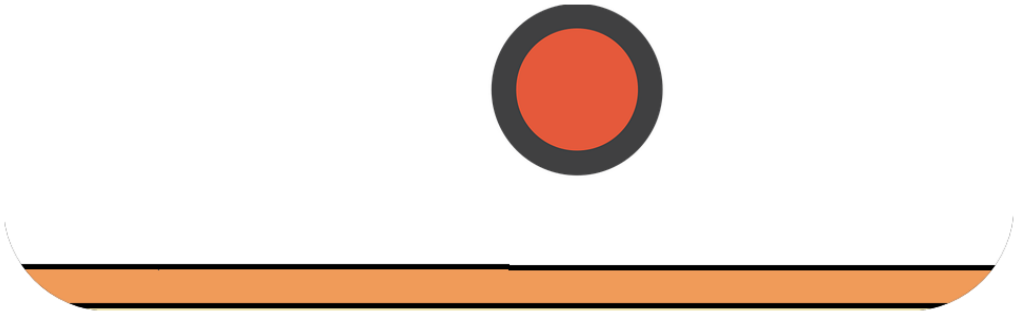
$$\kappa = \frac{c_p}{R}$$

$$m = \frac{\partial\pi}{\partial\eta}$$

Basic state

Current state:

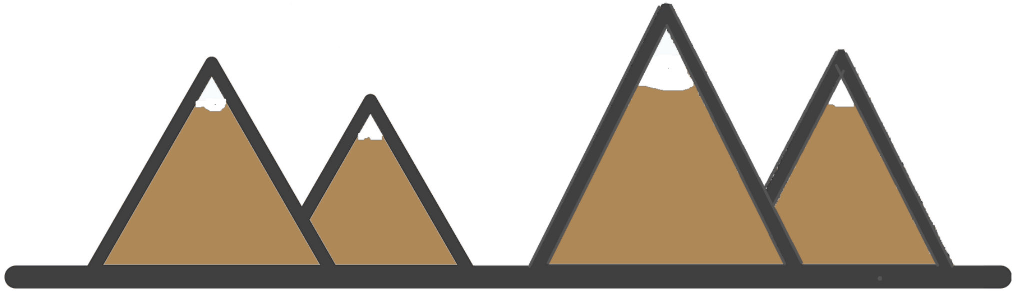
- stationary
- resting
- hydrostatically balanced (π_s^*)
- dry
- isothermal (T^*)
- with constant orography ($\nabla\phi^* = 0$)



Basic state

New:

- stationary
- resting
- hydrostatically balanced (π_s^*)
- dry
- isothermal (T^*)
- with constant orographic slope (in absolute value, $|\nabla \phi^*| \neq 0$)



Linear model

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Temperature

$$\frac{\partial T}{\partial t} = \frac{\kappa T^*}{\kappa - 1}(D + d)$$

Horizontal momentum

$$\frac{\partial \vec{v}}{\partial t} = -RT^*\frac{\nabla \pi}{\pi^*} - \nabla \phi - RT^*\nabla \hat{q} - \frac{1}{m^*}\frac{\partial \pi^* \hat{q}}{\partial \eta}\nabla \phi^*$$

Vertical momentum

$$\frac{\partial w}{\partial t} = \frac{g}{m^*}\frac{\partial \pi^* \hat{q}}{\partial \eta}$$

Pressure departure

$$\frac{\partial \hat{q}}{\partial t} = \frac{1}{\kappa - 1}(D + d) + \frac{1}{\pi^*}\int_0^\eta m^* D d\eta'$$

Surface pressure

$$\frac{\partial q_s}{\partial t} = -\frac{1}{\pi_s^*}\int_0^1 m^* D d\eta$$

Diagnostic relations

$$\nabla \phi = \nabla \phi_s - \int_\eta^1 \nabla\left(\frac{mRT}{p}\right) d\eta'$$

$$\nabla \phi^* = g\Lambda^* S^*(\eta)$$

$$d = -\frac{p}{mRT}\left(\nabla \phi \frac{\partial \vec{v}}{\partial \eta} - g\frac{\partial w}{\partial \eta}\right)$$

Definitions

$$\Lambda^* = \frac{1}{g}||\nabla \phi_s||^*$$

$$S^*(\eta) = \frac{B(\eta)\pi_s^*}{\pi^*(\eta)}$$

$$m^* = \frac{\partial \pi^*}{\partial \eta}$$

Linearized slope in SI

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Modified vertical divergence

$$d = \frac{p}{mRT}\left(\nabla\phi\,\frac{\partial\vec{v}}{\partial\eta} - g\frac{\partial w}{\partial\eta}\right)$$

Time evolution in linear model

$$\frac{\partial\vec{v}}{\partial t} = \mathbb{A} - \Lambda^* S^*(\eta)\mathbb{B}$$

$$\frac{\partial w}{\partial t} = \mathbb{B}$$

$$\frac{\partial d}{\partial t} = \frac{1}{RT^*}\left[\nabla\phi^*\ \partial^*\left(\frac{\partial\vec{v}}{\partial t}\right) - g\partial^*\left(\frac{\partial w}{\partial t}\right)\right]$$

$$= \frac{1}{RT^*}\left[g\Lambda^* S^*(\eta)\partial^*\mathbb{A} - g\Lambda^{*2} S^*(\eta)\left(S^*\partial^*\mathbb{B} + \mathbb{B}\partial^* S^*\right) - g\partial^*\mathbb{B}\right]$$

where

$$\partial^* X = \frac{\pi^*}{m^*}\frac{\partial X}{\partial\eta}$$

Linearized slope in SI

We omit the first order terms in Λ^* and then $\frac{\partial \vec{v}}{\partial t}$ is unchanged and all operators of the RHS of $\frac{\partial d}{\partial t}$ apply on $\hat{q}$.

Time evolution in linear model

$$\frac{\partial \vec{v}}{\partial t} = \mathbb{A} - \cancel{\Lambda^* S^* (\eta) \mathbb{B}}$$

$$\frac{\partial d}{\partial t} = \frac{1}{RT^*}\left[\cancel{g\Lambda^* S^* (\eta) \partial^* \mathbb{A}} - g\Lambda^{*2} S^* (\eta) \left(S^* \partial^* \mathbb{B} + \mathbb{B} \partial^* S^*\right) - g\partial^* \mathbb{B}\right]$$

Finally, since $\mathbb{B} = g\left(\partial^* + 1\right)\hat{q}$

Time evolution in linear model

$$\frac{\partial \vec{v}}{\partial t} = \mathbb{A} \rightsquigarrow \frac{\partial D}{\partial t} = \frac{\partial(\nabla \cdot \vec{v})}{\partial t} = \nabla \cdot \mathbb{A}$$

$$\frac{\partial d}{\partial t} = \mathcal{L}^*_{new} \hat{q}$$

Linearized slope in SI

We can define

New vertical Laplacian operator

$$\mathcal{L}^*_{new} = \alpha\, \partial^* (\partial^* + 1) + \beta\, (\partial^* + 1)$$

$$\alpha = 1 + \Lambda^{*2} S^{*2}(\eta)$$
$$\beta = \Lambda^{*2} S^*(\eta)\, \partial^* S^*(\eta)$$

$$\Lambda^* = 0 : \mathcal{L}^*_{new} \to \mathcal{L}^*_v = \partial^* (\partial^* + 1)$$
$$S^*(\eta) = 0 : \mathcal{L}^*_{new} \to \mathcal{L}^*_v$$

How to discretize the proposed solution?

New discretized vertical Laplacian operator

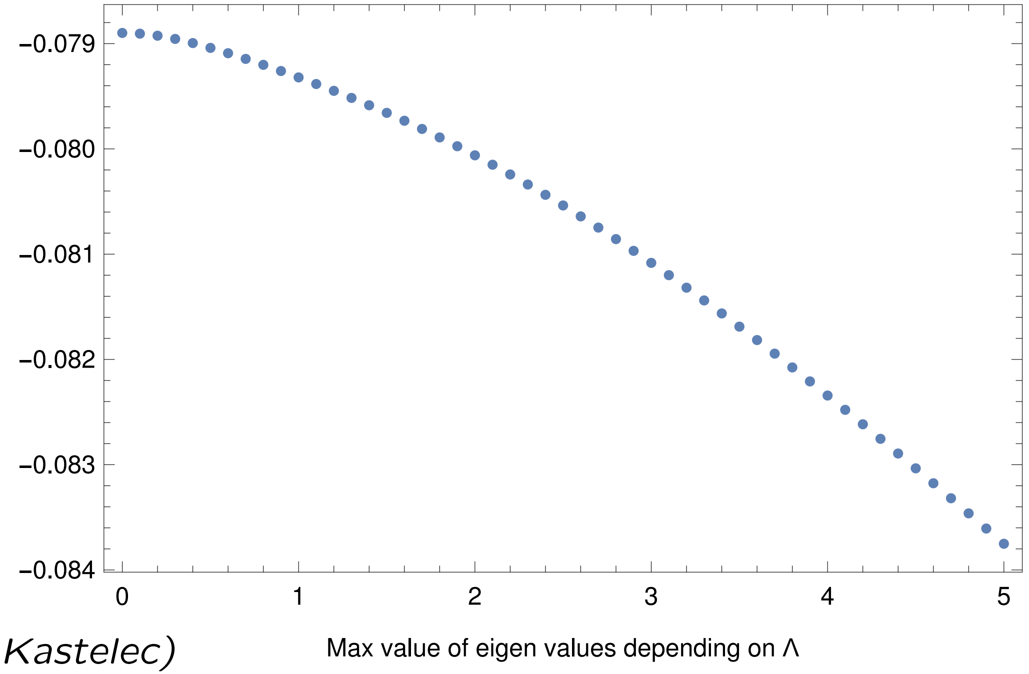
$$[\partial^*(\partial^* + 1)X]_l = \ldots$$
$$[(\partial^* + 1)X]_l = \ldots$$
$$S^*(\eta_l) = \ldots$$
$$\partial^* S^*(\eta_l) = \ldots$$

How to set boundary conditions?

Linearized slope in SI

Does $\mathcal{L}^*_{new}$ have only real and negative eigenvalues?

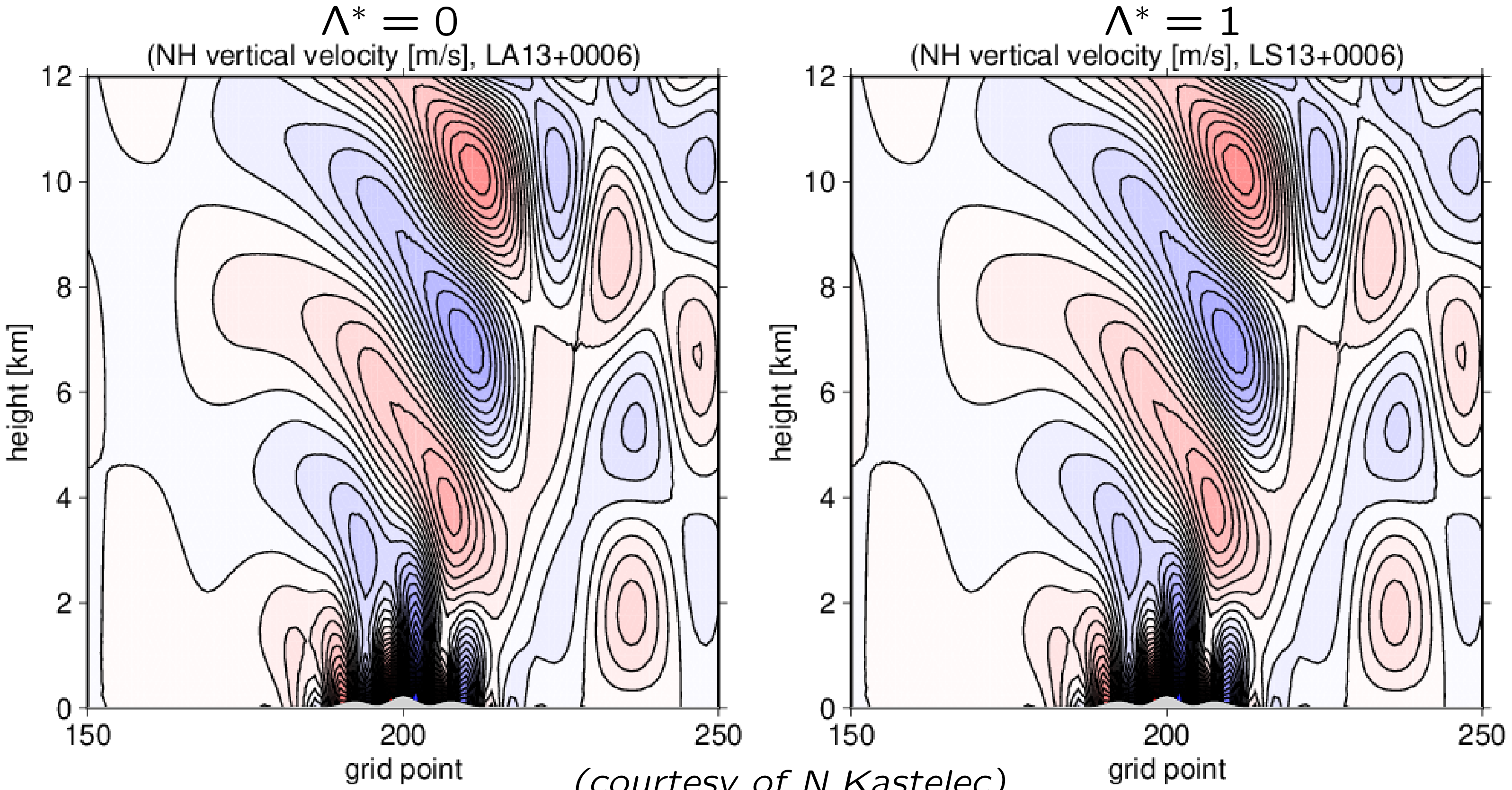
For an example of 87 vertical levels used in Czech operations we are safe.



(courtesy of N.Kastelec)

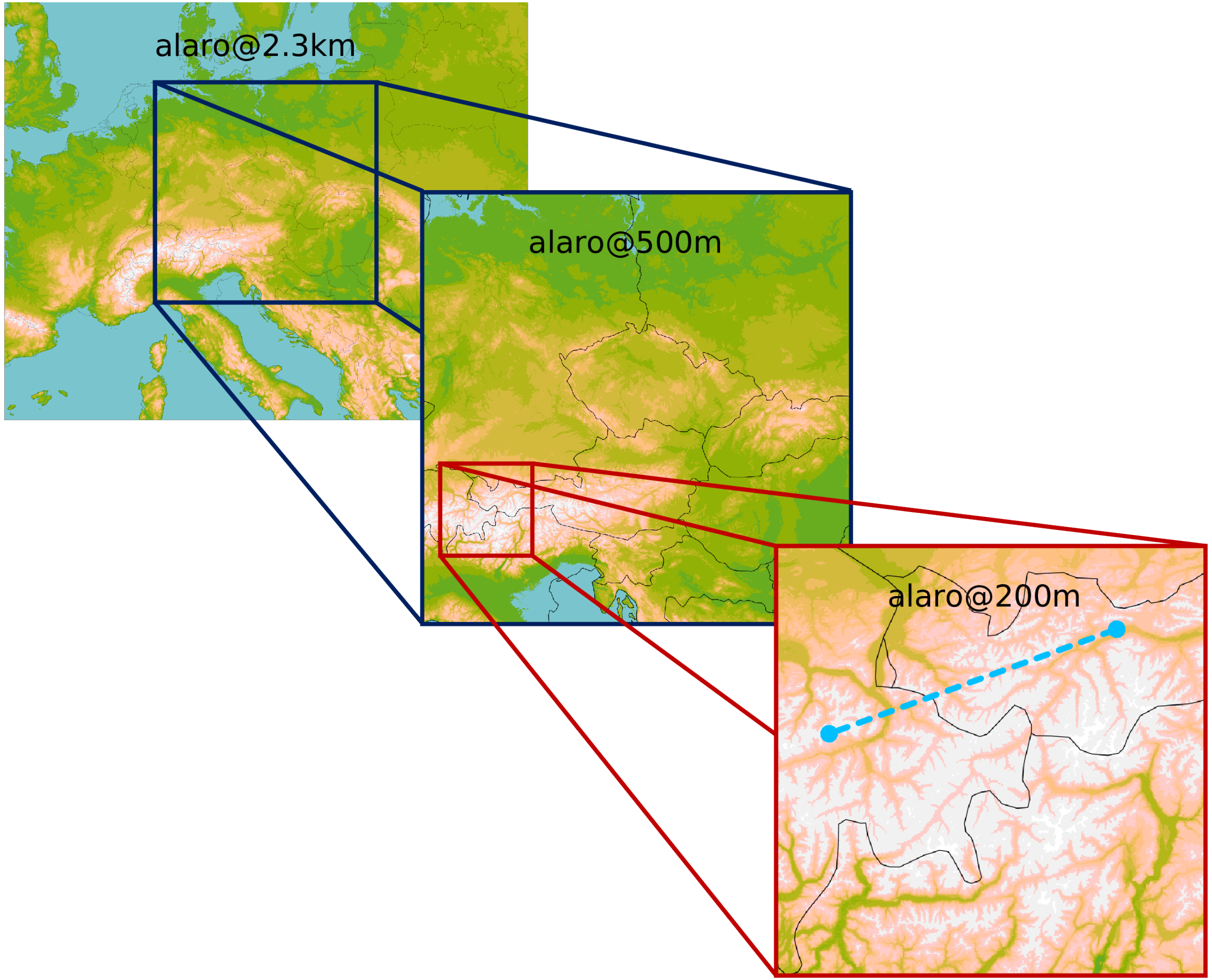
Then we can eliminate the discretized equations up to horizontal divergence D and solve the Helmholtz equation for D.

Vertical velocity for the Schär mountain case depending on Λ^*. ($\Delta x = 500\,m, \Delta z = 250\,m$, mountain height $h = 250\,m, T_0 = 288K, u_0 = 10\,m/s, \Delta t = 32\,s$)



(courtesy of N.Kastelec)

Real simulations @200m



We show results of 200m simulation over Alps.

Real simulations @200m

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The basic algorithmic choices for ALARO configurations @200m are:

Dynamical core

- semi-Lagrangian advection scheme with 4 iterations for trajectory calculation
- PC time scheme with one iteration, cheap variant (SL trajectories are not recalculated in corrector)
- modified vertical divergence d4 for vertical motion, transformation to vertical velocity w in the non-linear model
- reference values of the linear model: SITR=300K, SITRA=100K, SIPR=900hPa
- no decentering
- semi-Lagrangian horizontal diffusion applied on all model variables + TKE,TTE, hydrometeors
- linear truncation for all spectral fields except orography; quadratic truncation of orography

Real simulations @200m

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ALARO physics

- radiation scheme ACRANEB2
- turbulence and shalow convection scheme TOUCANS, model 2
- scale aware deep convection and microphysics scheme 3MT

Initialization

- initialization with 3DVAR + surface DA (canari) for 2.325km run; dynamical adaptation + DFI for 500m and 200m runs

Particular choices for ALARO@200m:

- cubic truncation of orography
- SITRA=50K
- no 3MT (deep convection), only STRAPRO (stratiform precipitation)

Real simulations

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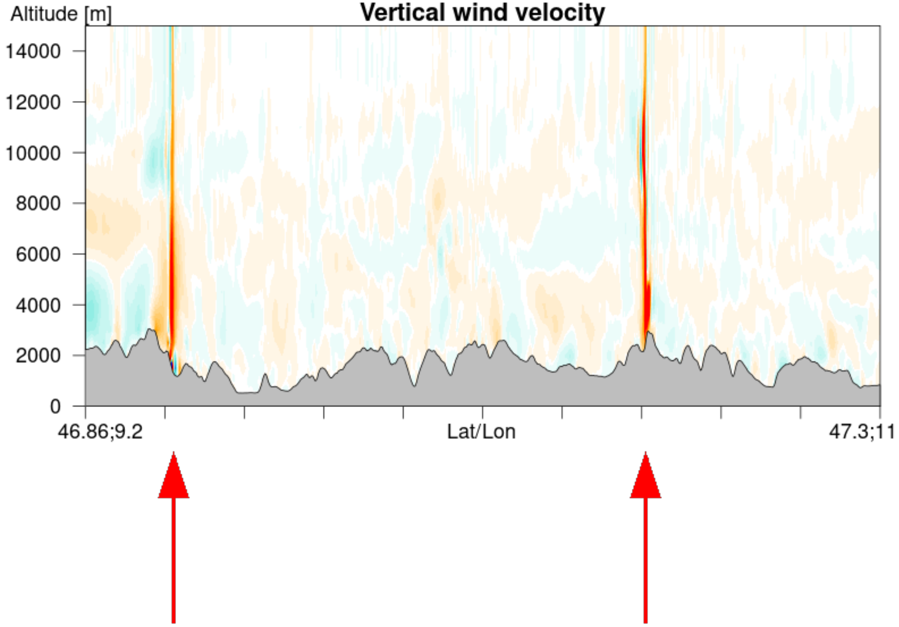
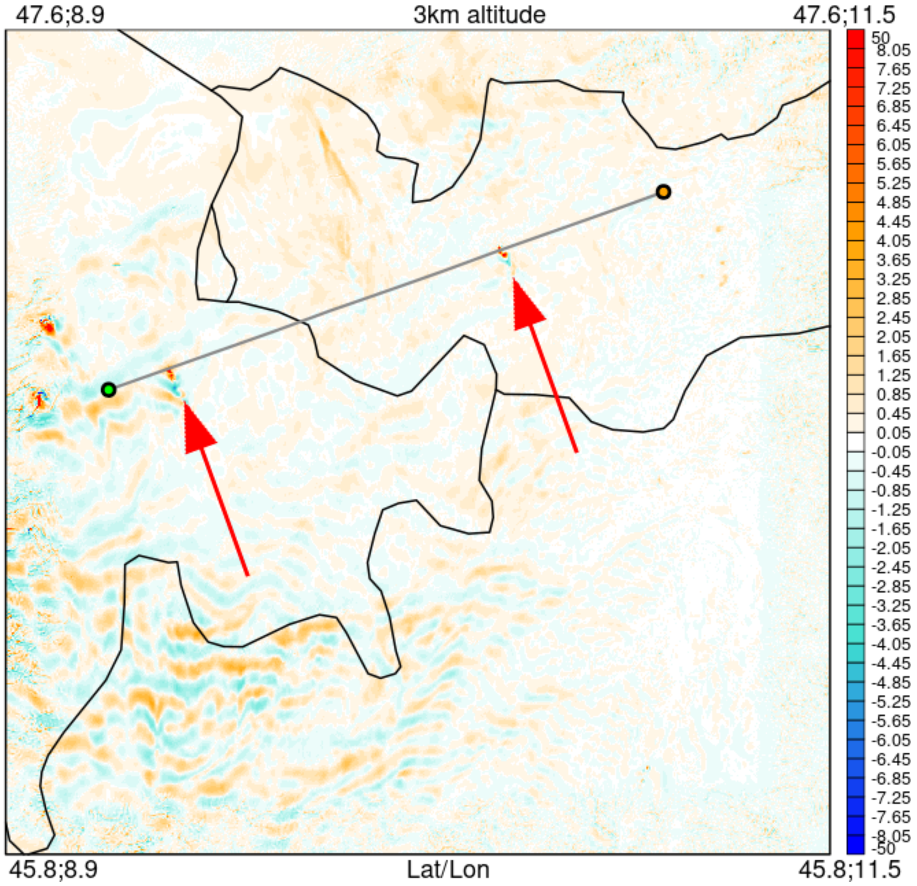
Vertical velocity for the alpine case 19 August 2022 OUTC + 24hours.

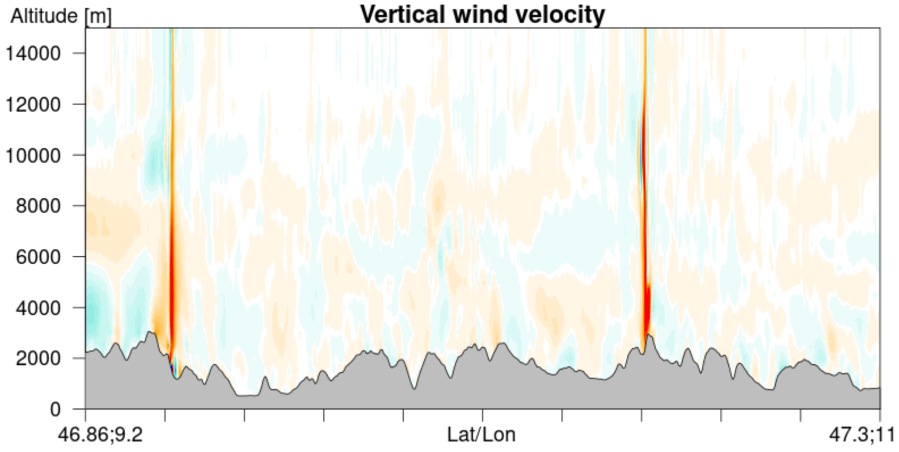




Real simulations

Vertical velocity depending on Λ^* for the alpine case 19 August 2022 0UTC + 24hours.

$\Lambda^* = 0$



With additional iterations of the SI scheme, the integration crashes.

Real simulations

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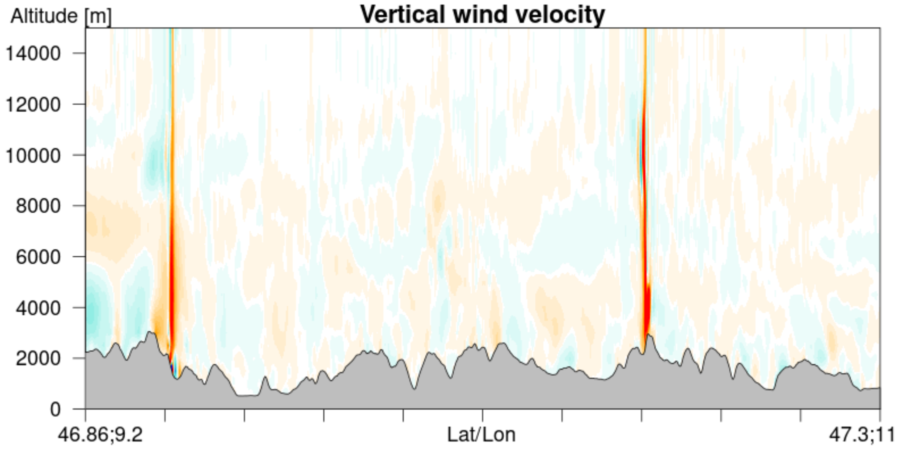
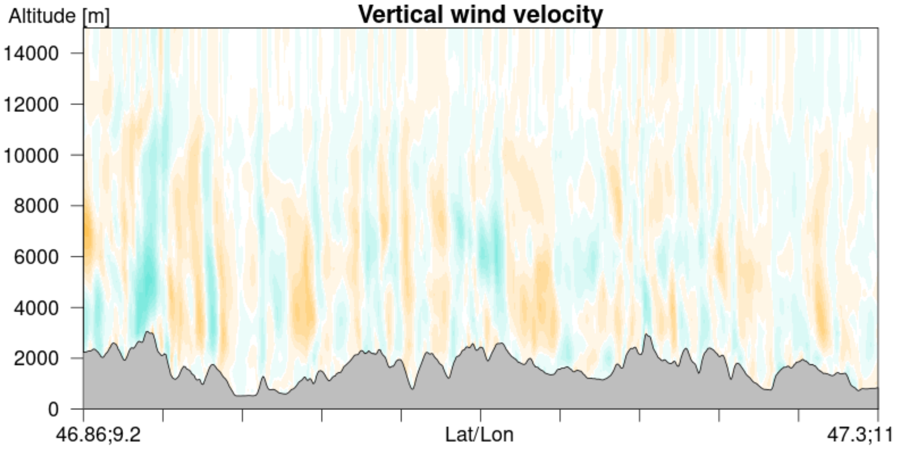
Vertical velocity depending on Λ^* for the alpine case 19 August 2022 0UTC + 24hours.

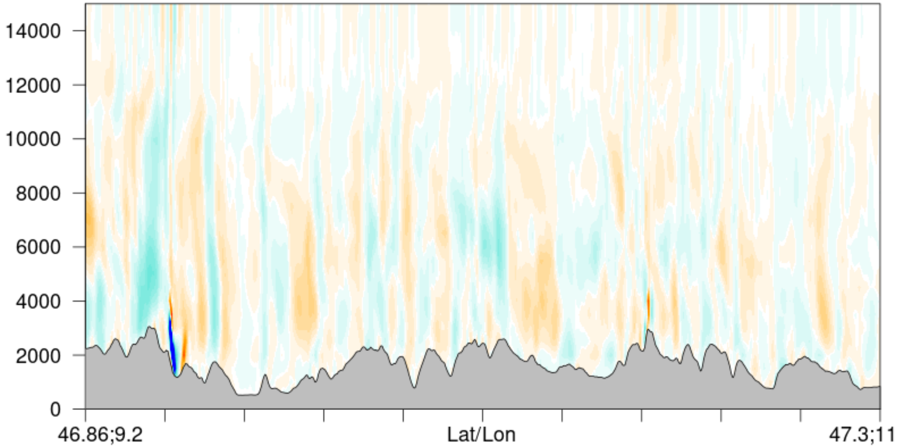
$\Lambda^* = 0$



$\Lambda^* = 1$



PC+1



PC+2

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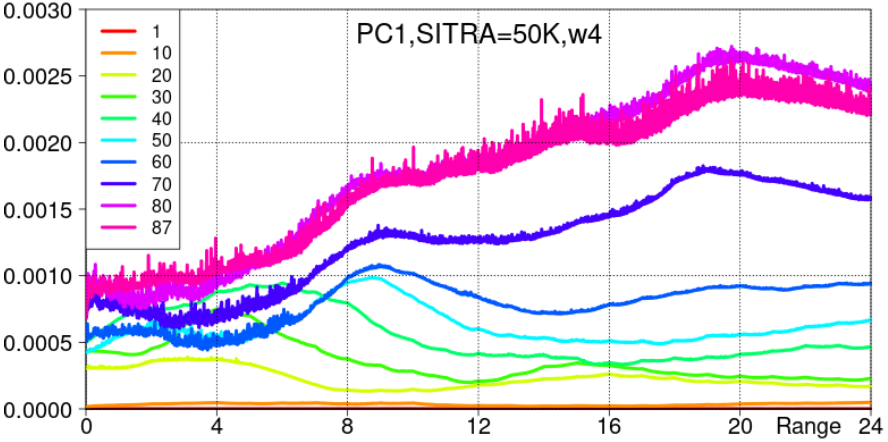
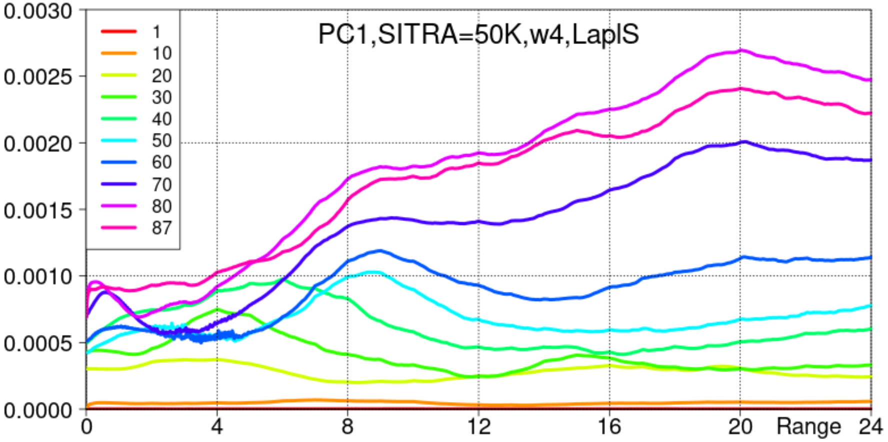
Real simulations

Averaged spectral norms of vertical divergence for the alpine case 19 August 2022 0UTC + 24hours.

$\Lambda^* = 0$ $\Lambda^* = 1$





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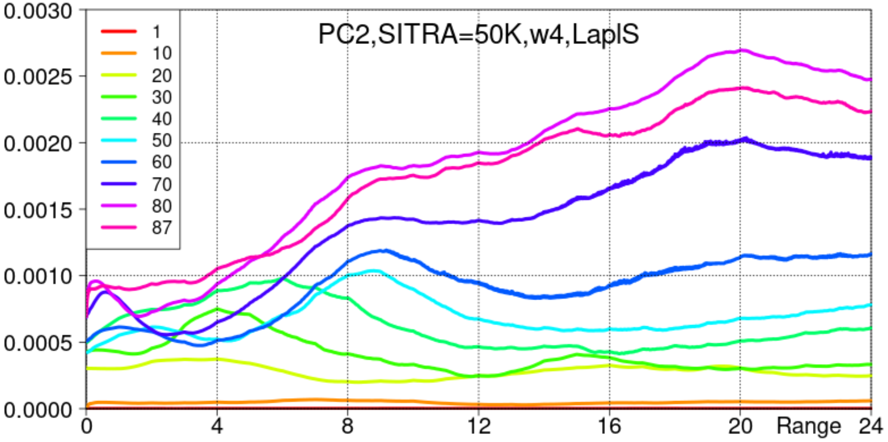

PC+2

Conclusions

- We must continue our efforts.
- We plan to test various possible definitions of vertical function $S^{*}(\eta)$.
- We plan to test various possible discretizations including boundary conditions.
- We plan to make further idealised tests and real simulations.
- If the time scheme allows a source of a noise, further time iterations may not help to stabilize the scheme. To the contrary, the scheme with further iterations may show even less stability.

Vertical motion variable

Fabrice Voitus proposed a modification of the vertical velocity defined in the model to simplify the bottom boundary condition and allow more precise calculation.

https://events.ecmwf.int/event/167/contributions/1379/attachments/794/1401/AS2020-Voitus.pdf

Current state

$$
\begin{aligned}
w &= \frac{dz}{dt} \\
d &= -g\frac{p}{mRT}\frac{\partial w}{\partial \eta} + X \\
X &= \frac{p}{mRT}\frac{\partial \mathbf{V}}{\partial \eta}\nabla\phi \\
\frac{\partial d}{\partial t} &= -\frac{g^2}{RT^*}\mathcal{L}_v^*\widehat{q} \\
\mathcal{L}_v^* &= \partial^*\left(\partial^* + 1\right)
\end{aligned}
$$

Fabrice's definition

$$
\begin{aligned}
W &= w - \mathbf{V}\cdot S\left(\eta\right)\frac{1}{g}\nabla\phi_s \\
d &= -g\frac{p}{mRT}\frac{\partial W}{\partial \eta} + X \\
X &= \frac{p}{mRT}\frac{\partial \mathbf{V}}{\partial \eta}\nabla\left(\phi - S\left(\eta\right)\phi_s\right) - \frac{p}{mRT}\mathbf{V}\frac{\partial \nabla\phi}{\partial \eta} \\
\frac{\partial d}{\partial t} &= -\frac{g^2}{RT^*}\mathcal{L}_{mod}^*\widehat{q} \\
\mathcal{L}_{mod}^* &= \partial^*\left(\partial^* + 1 + \Lambda^{*2}S\left(\eta\right)\gamma^*\right)
\end{aligned}
$$

Then discretization of these terms involves interpolations between half levels (where w is represented) and full levels (where $\mathbf{V}$ is represented) and is cumbersome ...

Bottom boundary condition

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Current state

in the non-linear model

$$w_s = \mathbf{V}_s \frac{1}{g} \nabla \phi_s$$

$$\frac{dw_s}{dt} = \frac{d\left(\mathbf{V}_s \frac{1}{g} \nabla \phi_s\right)}{dt}$$

while in the linear model explicit guess of d is calculated consistently and used in the implicit part

Fabrice's definition

in the linear and non-linear model

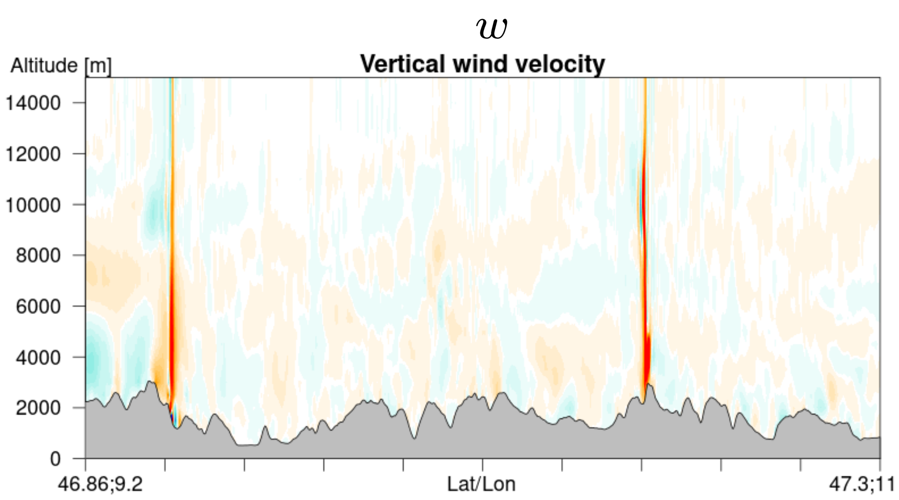
$$W_s = 0$$

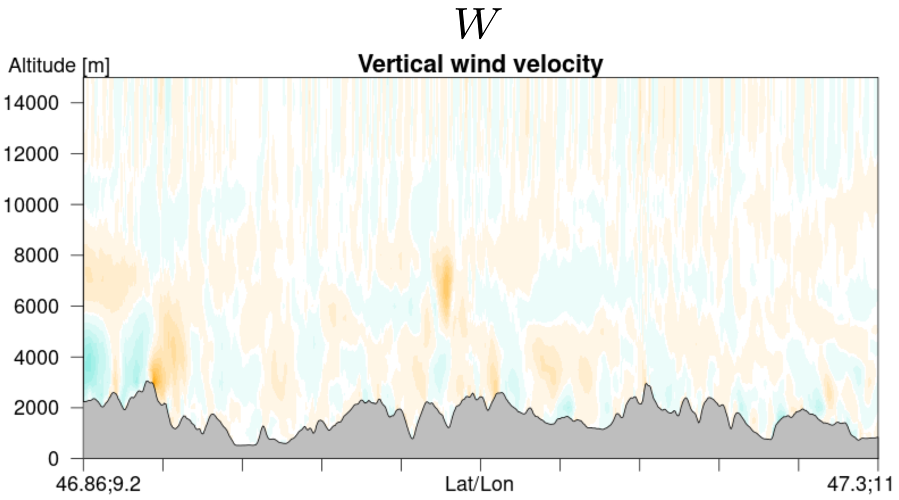
$$\frac{dW_s}{dt} = 0$$

Complicated and not exact! It is a source of noise which may grow.

Easily applicable!

Vertical velocity for the alpine case 19 August 2022 0UTC + 24hours.





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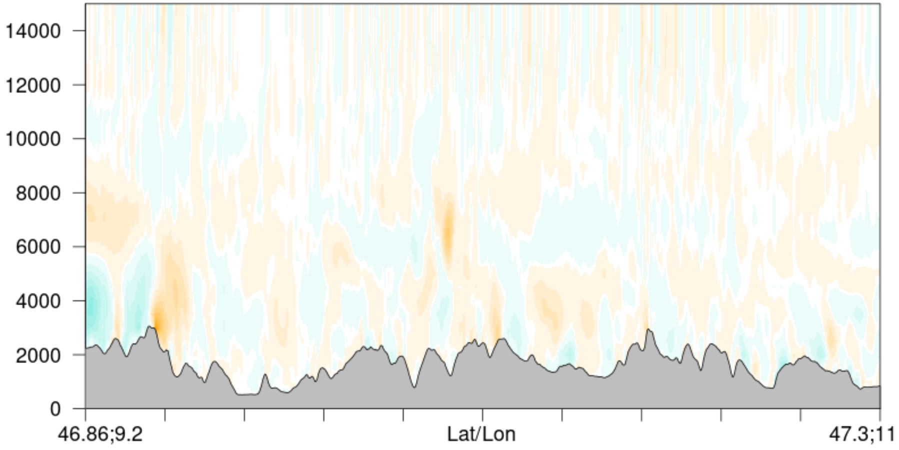

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Real simulations

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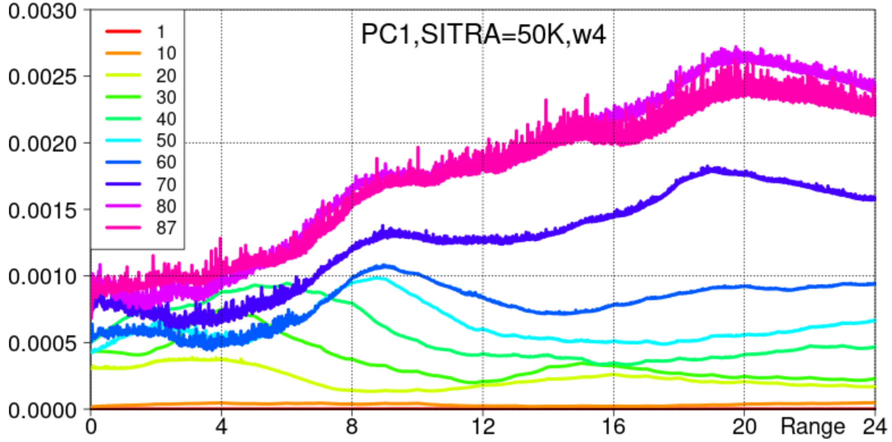
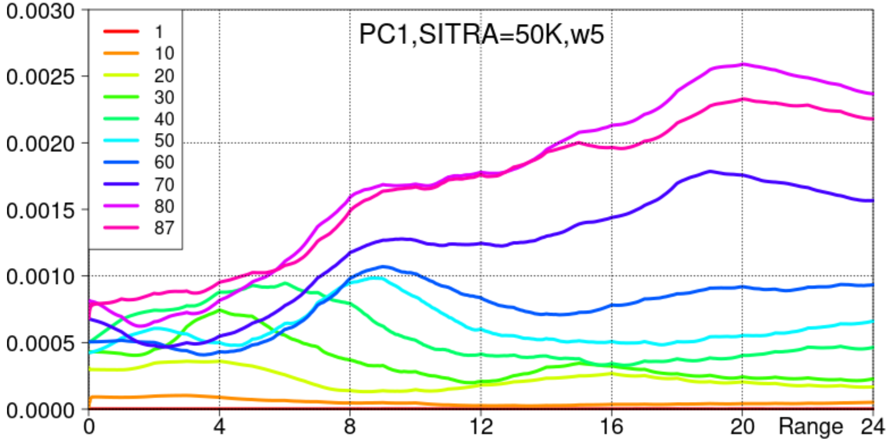
Averaged spectral norms of vertical divergence for the alpine case of 19 August 2022 0UTC + 24hours.

w



W



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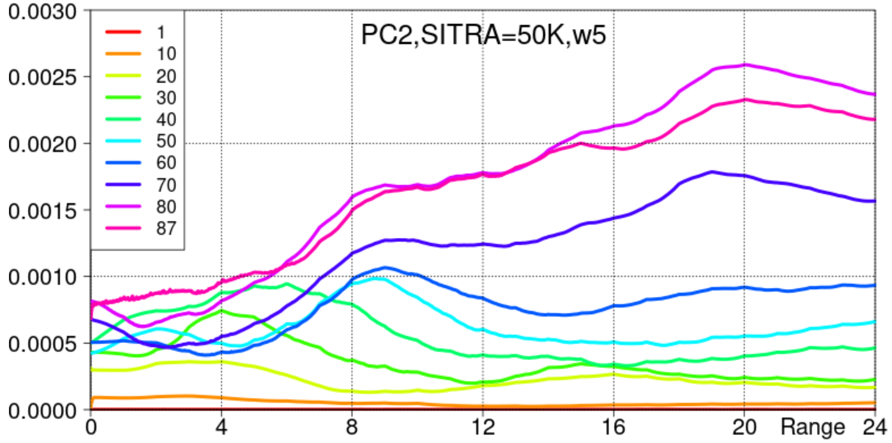

PC+2

Conclusions and advertisements

- The modification is available in cycle CY49t1 under namelist option NVDVAR=5, thanks to Fabrice Voitus and Karim Yessad.
- The new formulations may help to further reduce the non-linear residual of the ICI time scheme and to get rid of the noise coming from steep orography, especially in high resolutions.


Tack för din uppmärksamhet!