Regional Cooperation for
Limited Area Modeling in Central Europe

LACE
nwp central europe

LACE : SL interpolations

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thanks to Alexandra Craciun, Ján Mašek and other colleagues

DHMZ SHMU OMSZ ZAMG ROMANIA ANM

Dynamical core of ALADIN/HIRLAM system

- fully compressible Euler equations (NH) or hydrostatic primitive equations (HPE)
- space discretization in horizontal: Fourier spectral method
- mass based vertical coordinate using Laprise hydrostatic pressure
- semi-implicit time scheme – direct solver for Helmholtz equation for one prognostic variable, vertical/horizontal direction separation
- semi-Lagrangian advection
- prognostic variables differ in grid-point space and in spectral space for stability and accuracy reasons; they are transformed every time step

Outline of current work

1. **Design of finite element scheme for vertical discretization of ALADIN-NH model**
2. **Application of ENO technique to semi-Lagrangian interpolations**
3. **Tuning and redesign of the horizontal diffusion depending on the scale**
4. **Evaluation of the model dynamical core in very high resolutions**
   - Clear comparison of existing time schemes (iterative or not)
   - Upper boundary conditions

ENO technique in SL interpolations

ENO (Essentially Non-Oscillatory) method

Motivation: to explore alternative interpolators which are

- less overshooting then Lagrange polynomials close to discontinuities
- more accurate than their quasi-monotonic versions

$\Rightarrow$ interpolation depending on the smoothness of the interpolated field

Previous work:

1) Tests in 1D - rectangular pulse in a periodic domain, a sinusoidal signal in a periodic domain

2) ENO/WENO interpolators of second (quadratic) order implemented in ALADIN and tested in 2D vertical plane tests

ENO technique in SL interpolations

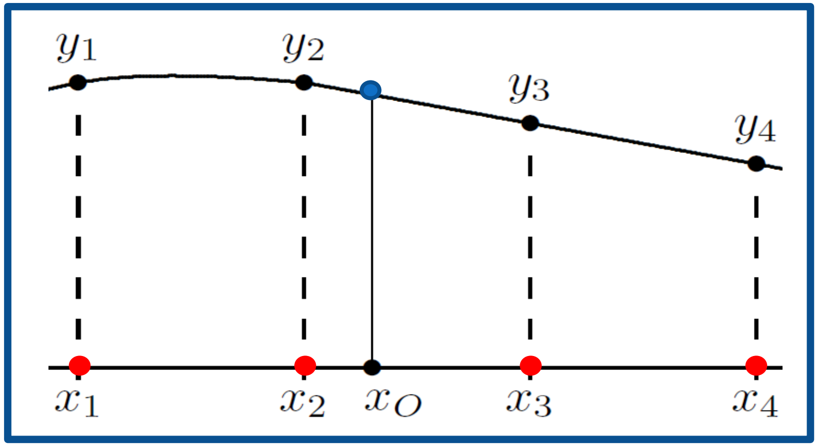
3) cubic ENO interpolations in SL scheme implemented in ALADIN – technically demanding, the stencil for SL 3D interpolation has to be extended from 32 points to 120 points

Current progress:

4) the implementation of ENO scheme in the SL interpolations redesigned for WENO (weighted approach) and tested in 2D vertical plane tests

ENO technique in SL interpolations

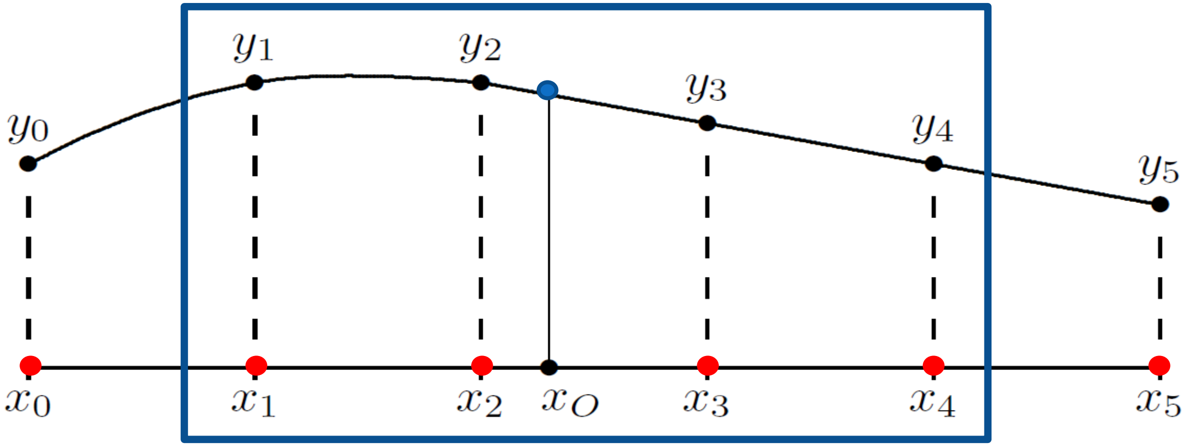
Third order interpolation scheme (cubic) needs 4 points to find ●



ENO technique in SL interpolations

Third order interpolation scheme (cubic) needs 4 points to find ●,

⇒ **6 points** needed for ENO/WENO interpolation



ENO technique in SL interpolations

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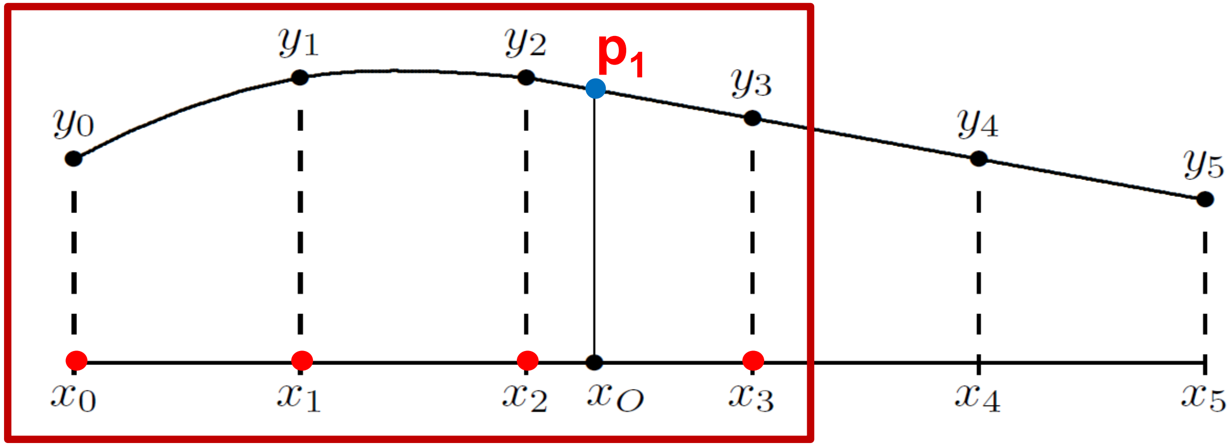

Stencil 1 =>
interpolated value p_1

ENO technique in SL interpolations

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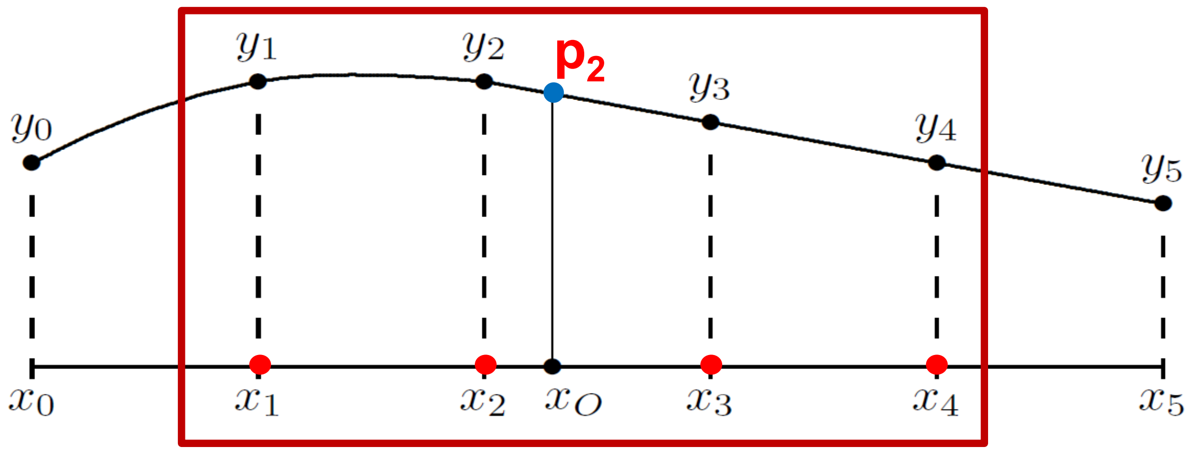
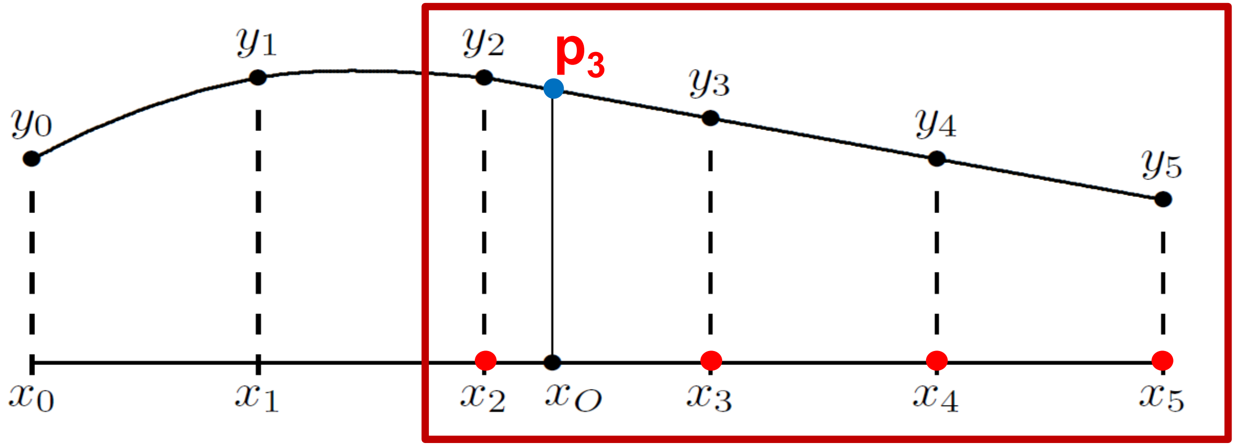


Stencil 2 =>
interpolated value p_2

ENO technique in SL interpolations

Third order interpolation scheme (cubic) needs 4 points to find ● :



Stencil 3 =>
interpolated value p_3

ENO technique in SL interpolations

p_1, p_2, p_3 … interpolated values on three stencils

Final interpolated value

$y = w_1 p_1 + w_2 p_2 + w_3 p_3$, where w_1, w_2, w_3 are weights with

$$w_1 + w_2 + w_3 = 1$$

ENO chooses the smoothest solution ($S_i = min(S_1,S_2,S_3) \Rightarrow w_i = 1$)

WENO weighted combination based on smoothness

WENO technique in SL interpolations

WENO = Weighted Essentially Non-Oscillatory Schemes

- Proposed by Liu, Osher and Chan, 1994
- Developed by many authors for different purposes

Motivation:

- the weighted sum of values interpolated on several stencils is used with weights based on smoothness evaluation
- in smooth regions certain optimal weights are approached to achieve a higher order of accuracy (for cubic polynomial we get 5th order)
- in regions near discontinuities, the stencils which contain discontinuities are assigned a nearly zero weight

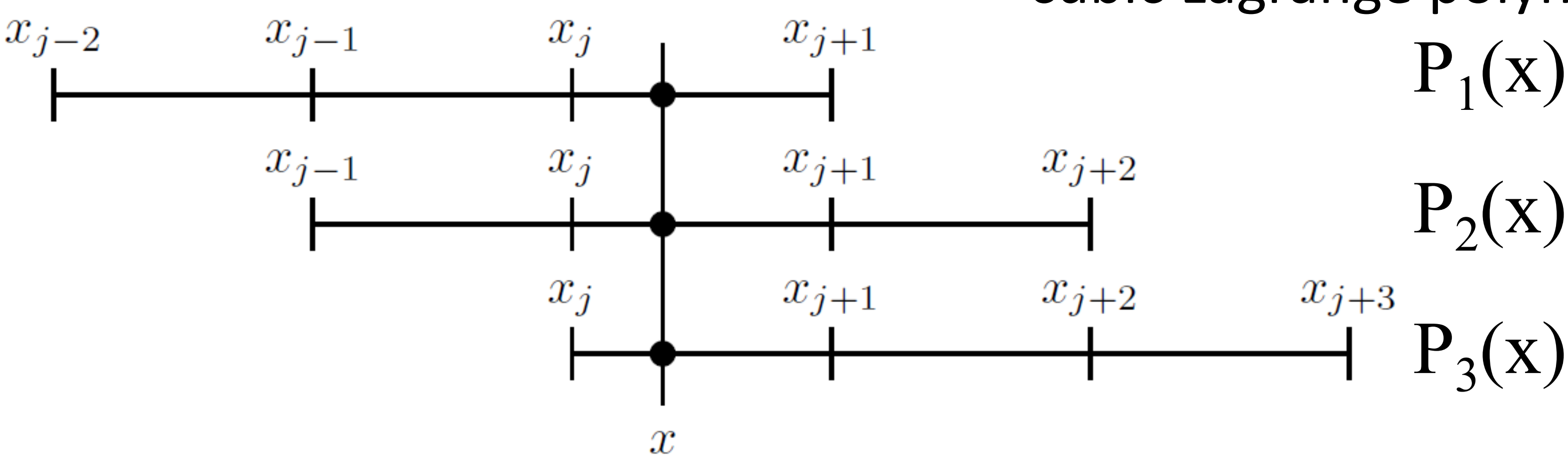
WENO technique in SL interpolations

- completely removes the logical statements that appear in the ENO stencil choosing step => WENO scheme appear to be much faster than ENO scheme on vector machines

WENO technique in SL interpolations

How it works:

cubic Lagrange polynomials



weighted combination of cubic Lagrange polynomials

$$P(x) = \sum_{k=1}^{3} \omega_k(x) P_k(x)$$

ENO technique in SL interpolations

$$P(x) = \sum_{k=1}^{3} \omega_k(x) P_k(x)$$

normalized weights:

$$\omega_k(x) = \frac{\tilde{\omega}_k(x)}{\tilde{\omega}_1(x) + \tilde{\omega}_2(x) + \tilde{\omega}_3(x)}, \quad \tilde{\omega}_k(x) = \frac{C_k(x)}{(\beta_k(x) + \varepsilon)^p}$$

with coefficients being a second order polynomials in $\xi = \frac{x - x_j}{\Delta x}$

$$C_1(x) = \frac{1}{20}(2 - \xi)(3 - \xi)$$

$$C_2(x) = \frac{1}{-10}(2 + \xi)(\xi - 3)$$

$$C_3(x) = \frac{1}{20}(2 + \xi)(1 + \xi)$$

ENO technique in SL interpolations

$$P(x) = \sum_{k=1}^{3} \omega_k(x) P_k(x)$$

normalized weights:

$$\omega_k(x) = \frac{\tilde{\omega}_k(x)}{\tilde{\omega}_1(x) + \tilde{\omega}_2(x) + \tilde{\omega}_3(x)}, \quad \tilde{\omega}_k(x) = \frac{C_k(x)}{(\beta_k(x) + \varepsilon)^p}$$

with smoothness indicators based on ℓ_2-norm of high-order variations of the reconstruction polynomials

$$\beta_k(x) = \sum_{l=1}^{3} \int_{x_j}^{x_{j+1}} (\Delta x)^{2l-1} (P_k^{(l)}(x))^2 dx$$

ENO technique in SL interpolations

$$P(x) = \sum_{k=1}^{3} \omega_k(x) P_k(x)$$

normalized weights:

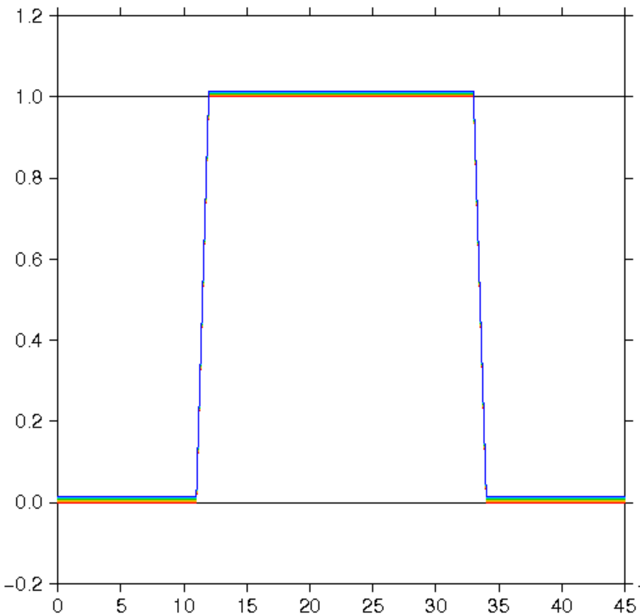
$$\omega_k(x) = \frac{\tilde{\omega}_k(x)}{\tilde{\omega}_1(x) + \tilde{\omega}_2(x) + \tilde{\omega}_3(x)}, \quad \tilde{\omega}_k(x) = \frac{C_k(x)}{(\beta_k(x) + \varepsilon)^p}$$

and with parameter p in the denominator serving to increase/decrease the dissipation of the scheme

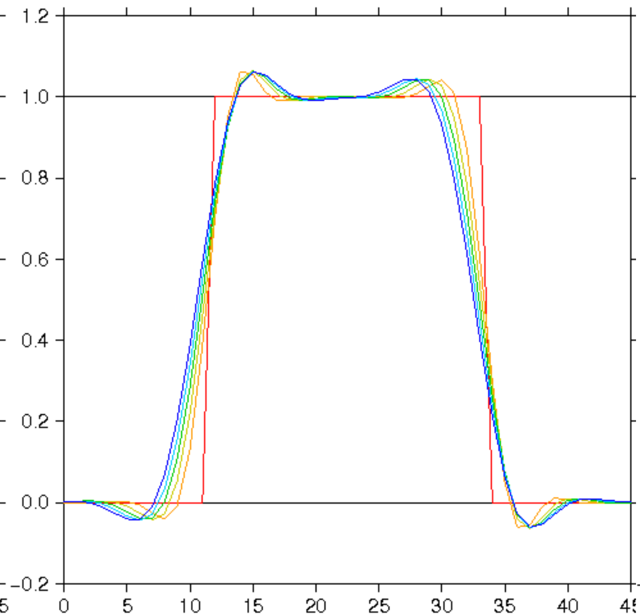
ENO technique in SL interpolations

Toy model:

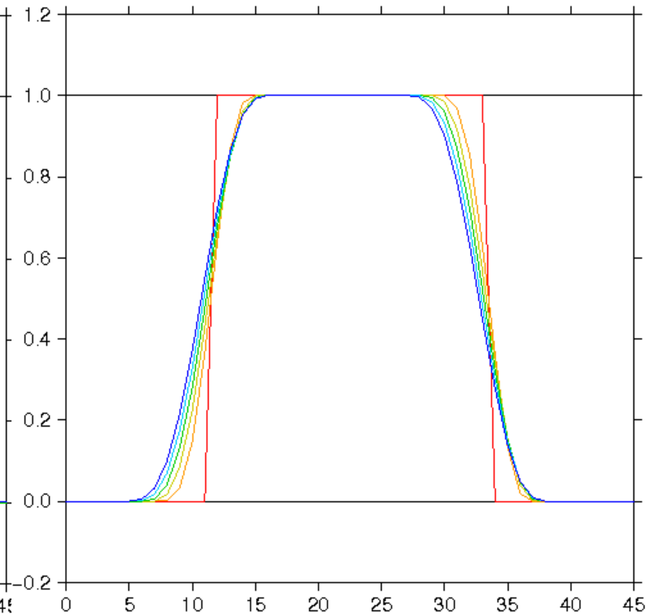
1D linear advection of a rectangular pulse in a periodic domain



ideal advection



cubic Lagrange



cubic Lagrange QM

Courtesy of Ján Mašek me, Italy

ENO technique in SL interpolations

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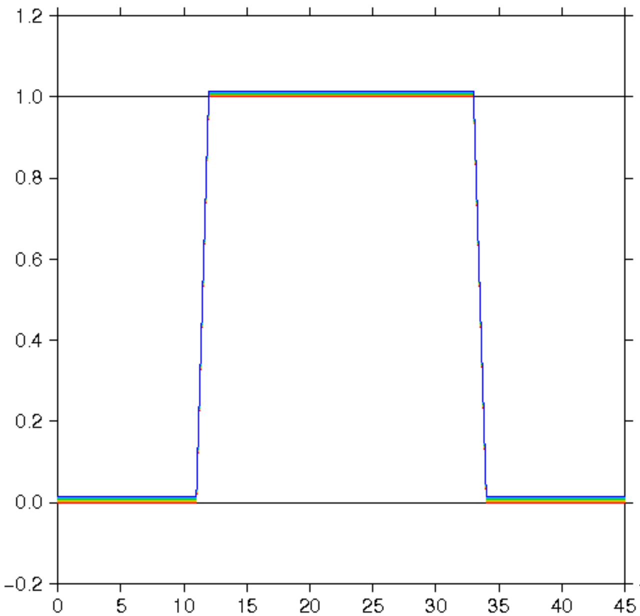
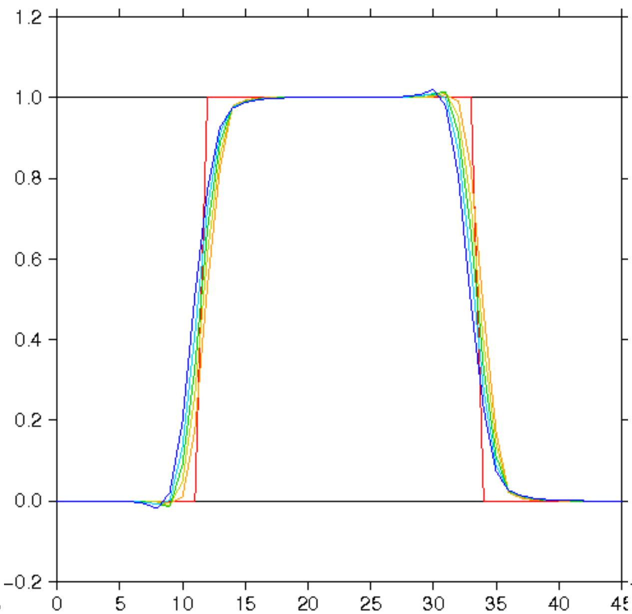
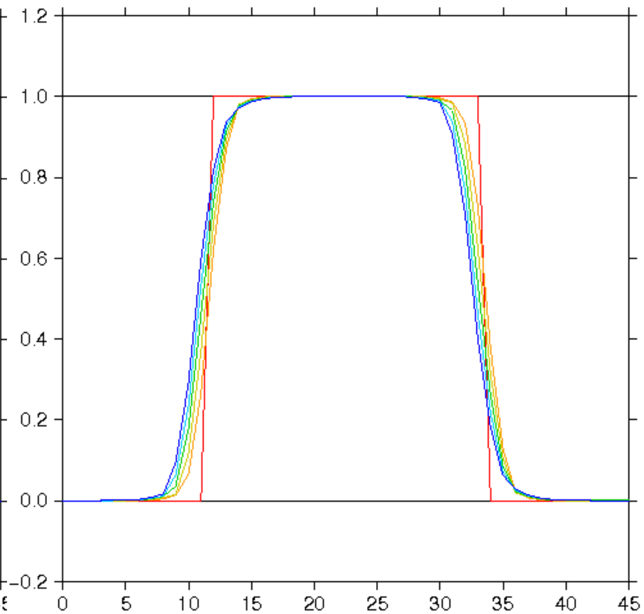


ideal advection



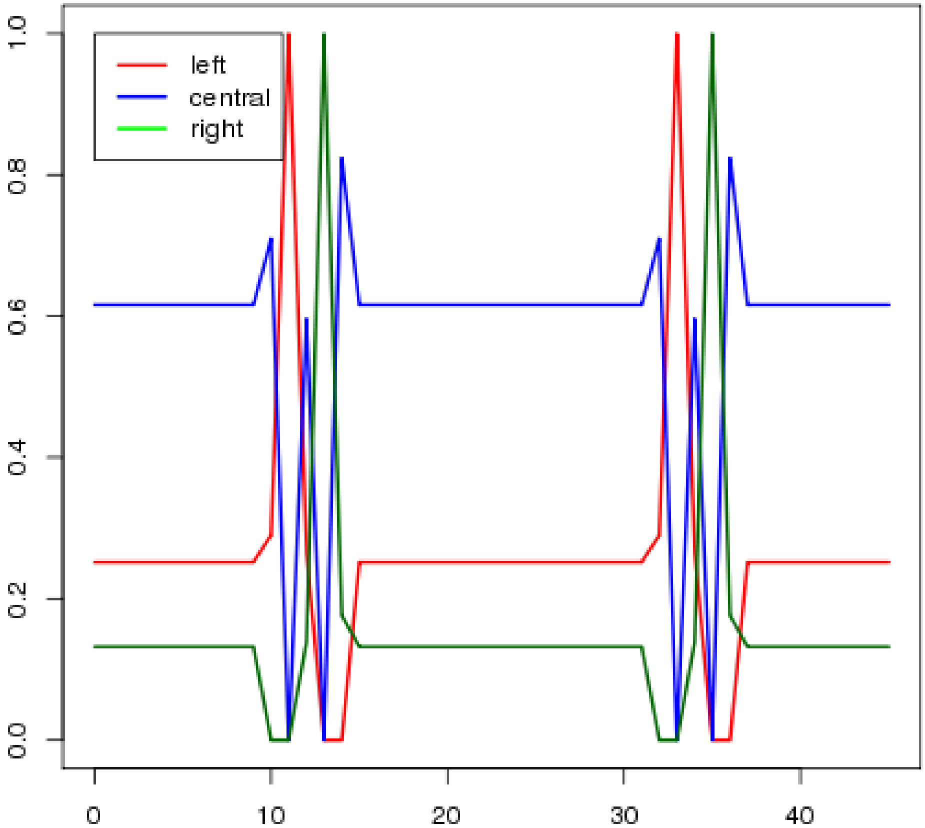
cubic ENO

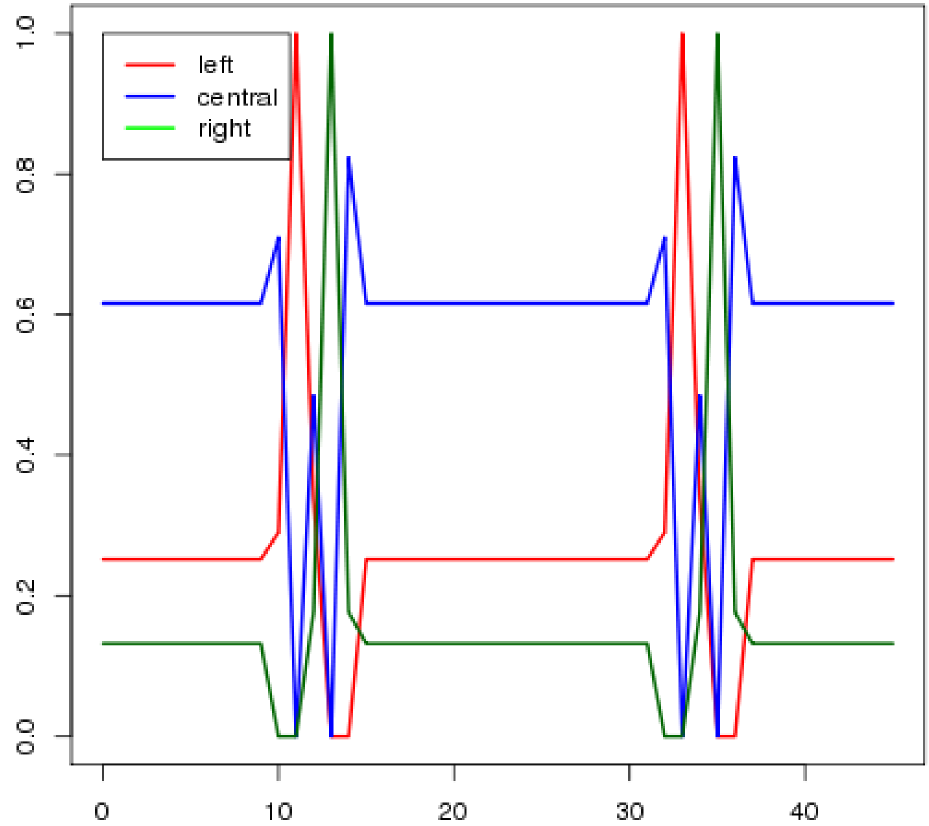


cubic ENO QM

ENO technique in SL interpolations

Toy model: calculated weights for 2 different definitions of smoothness indicators





ENO technique in SL interpolations

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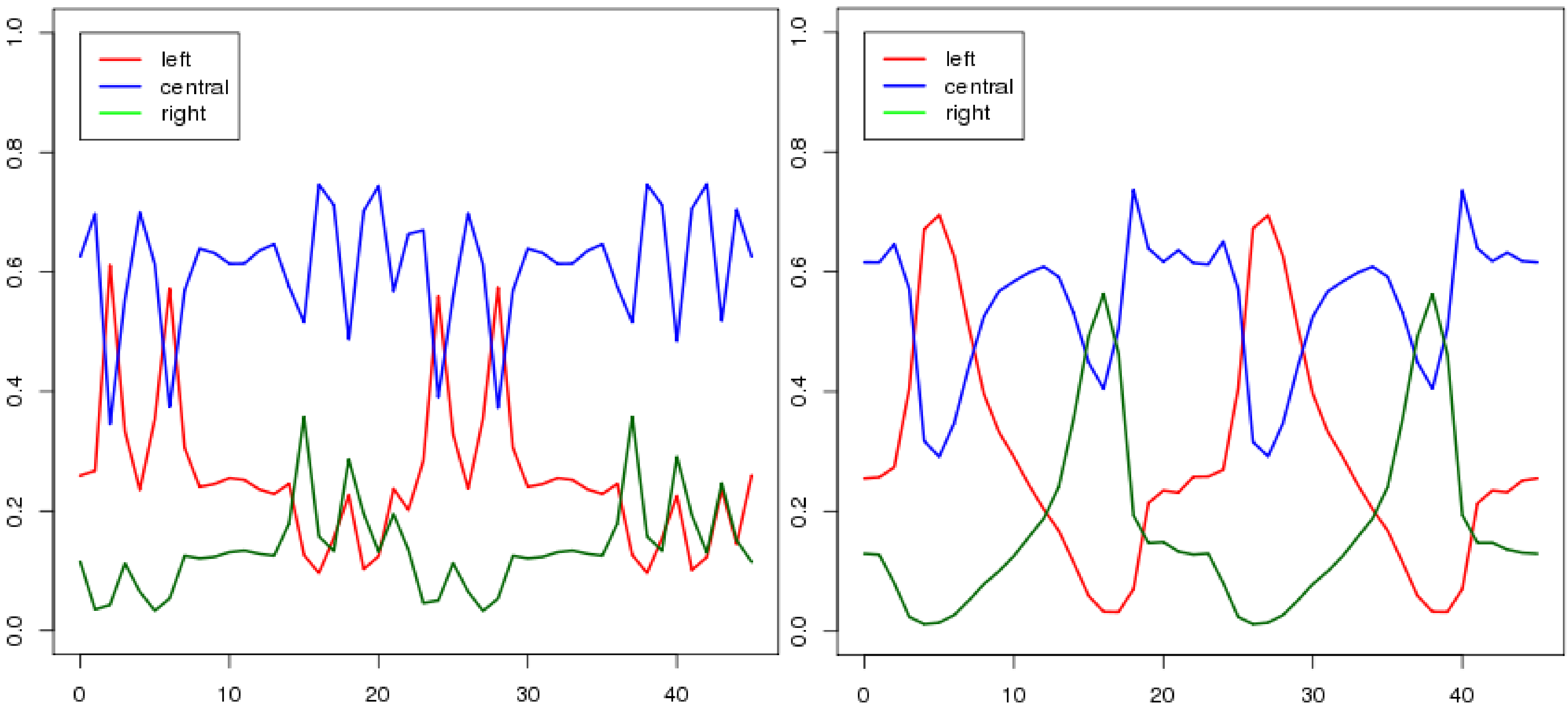

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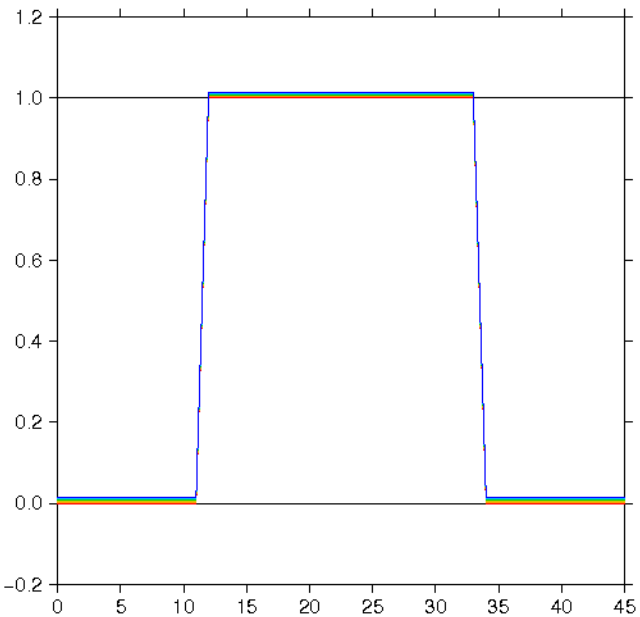
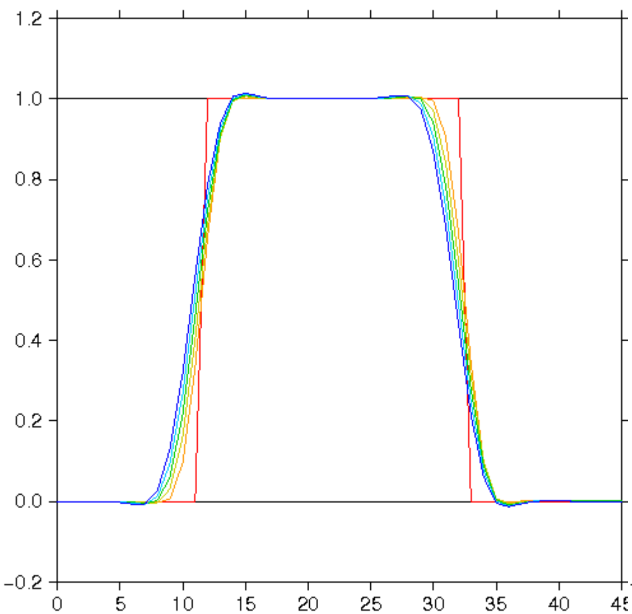
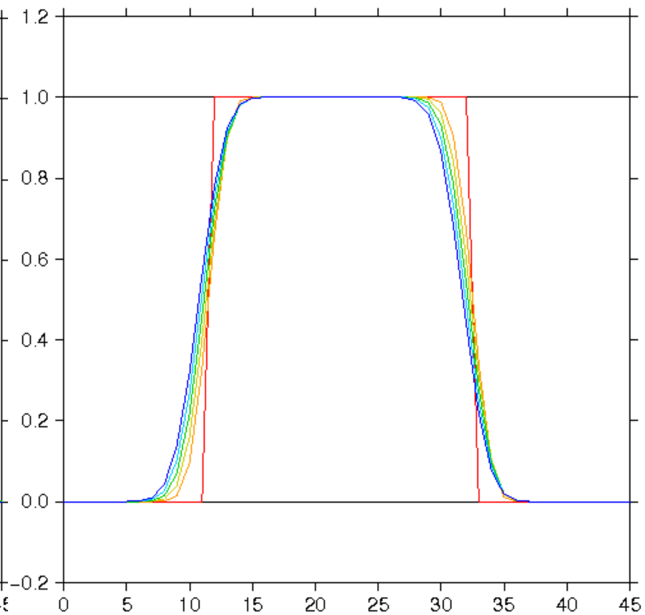


ideal advection



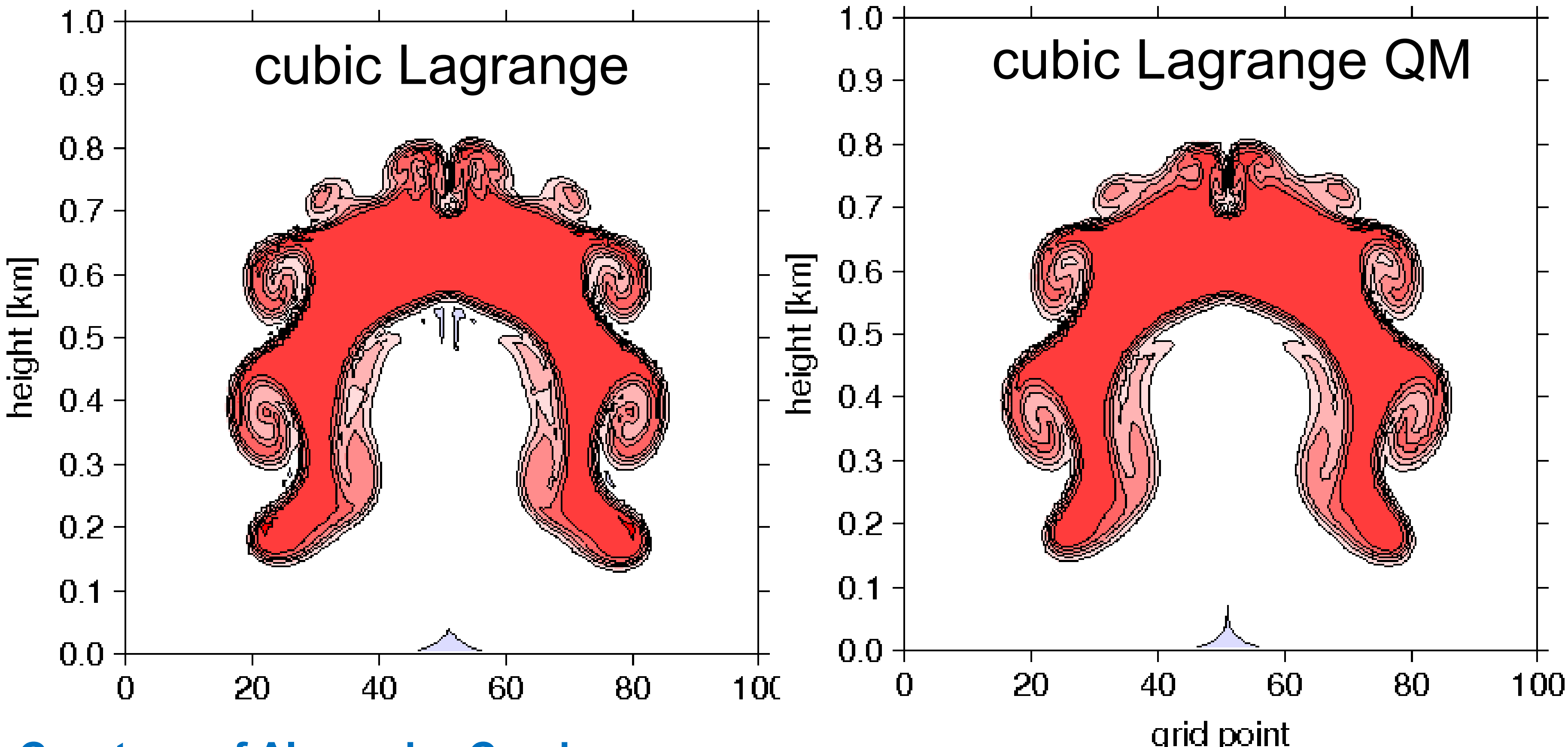
WENO 1



WENO 2

ENO technique in SL interpolations

Robert's test in 2D model: warm bubble (+0.5K) in the field of potential temperature (300K) without advection



Courtesy of Alexandra Craciun

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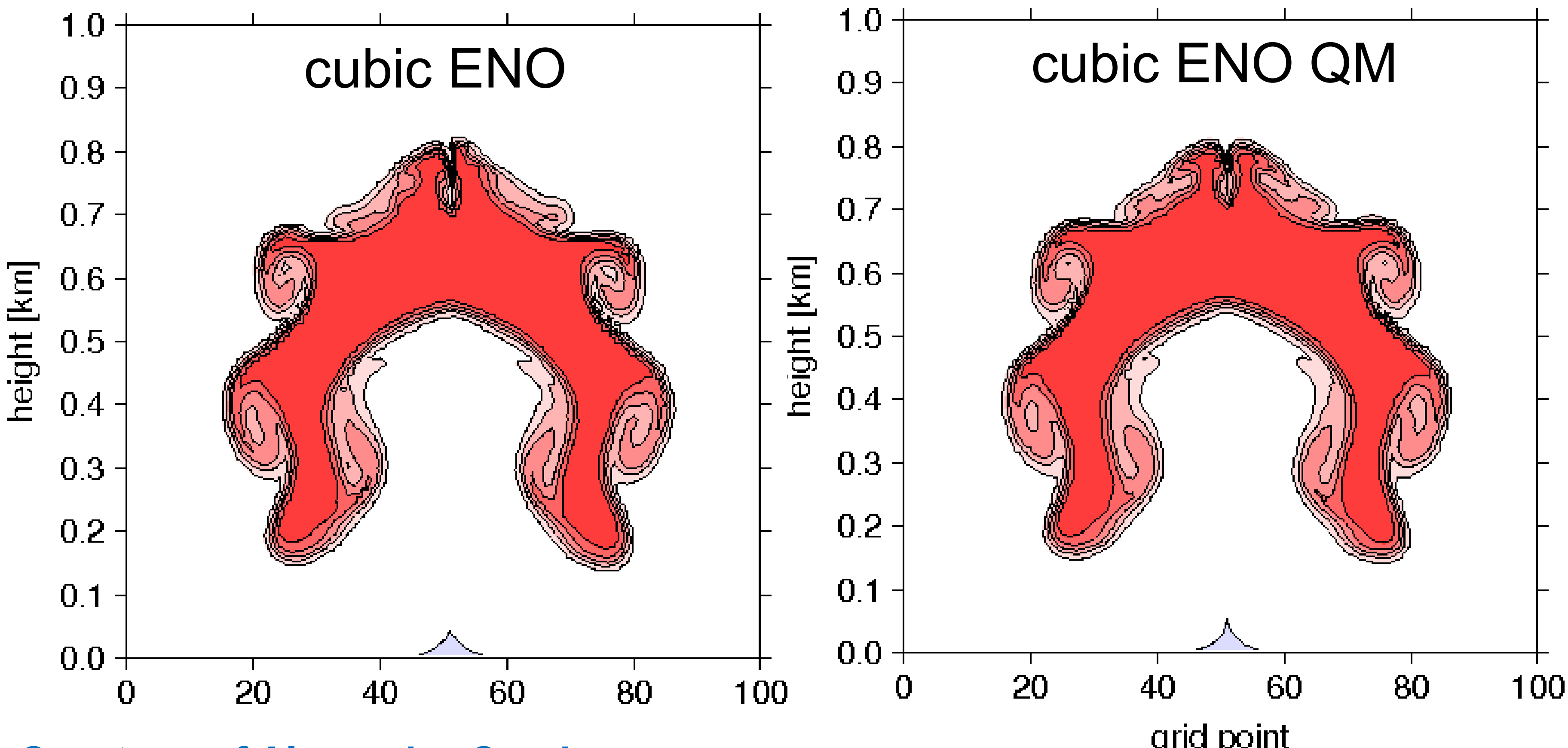

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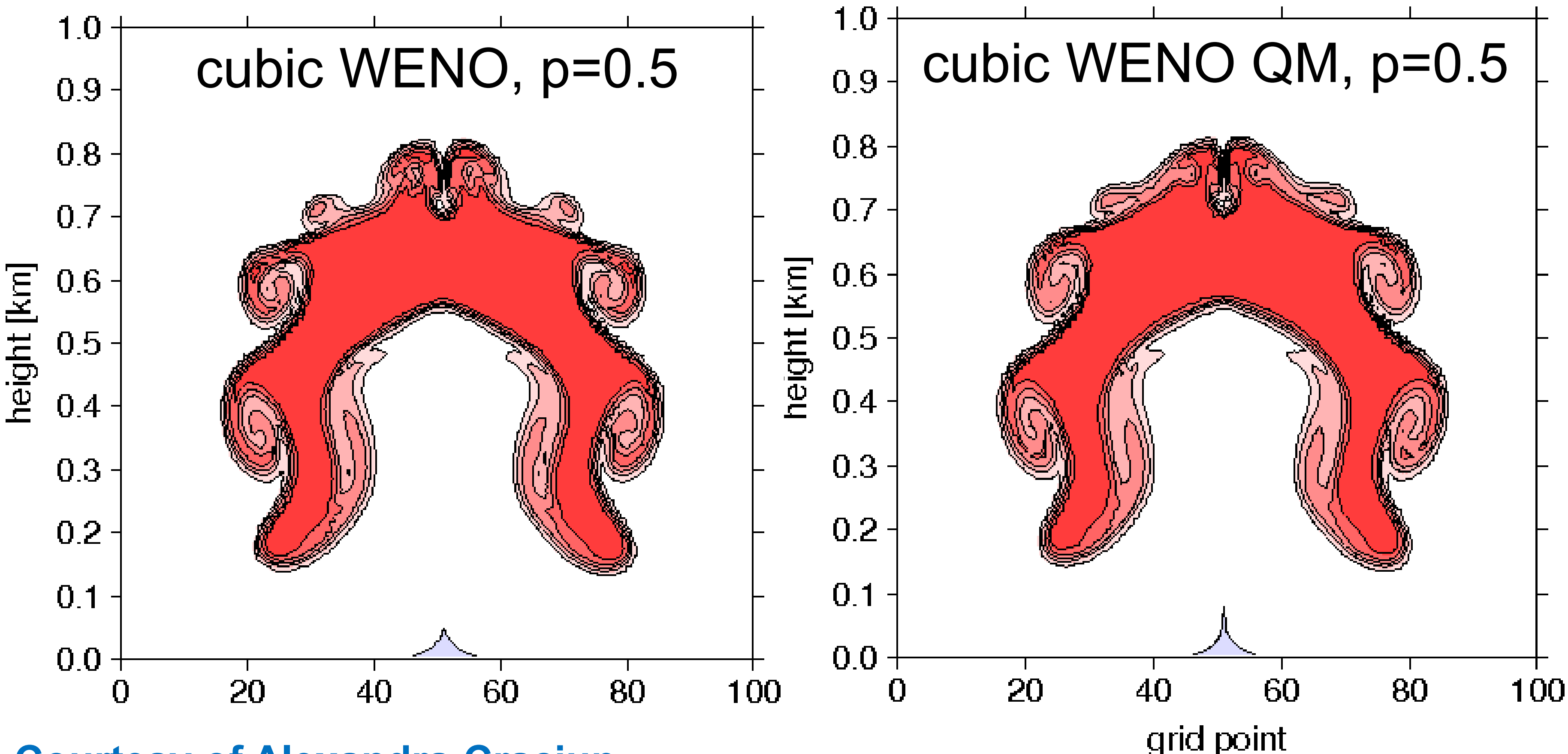

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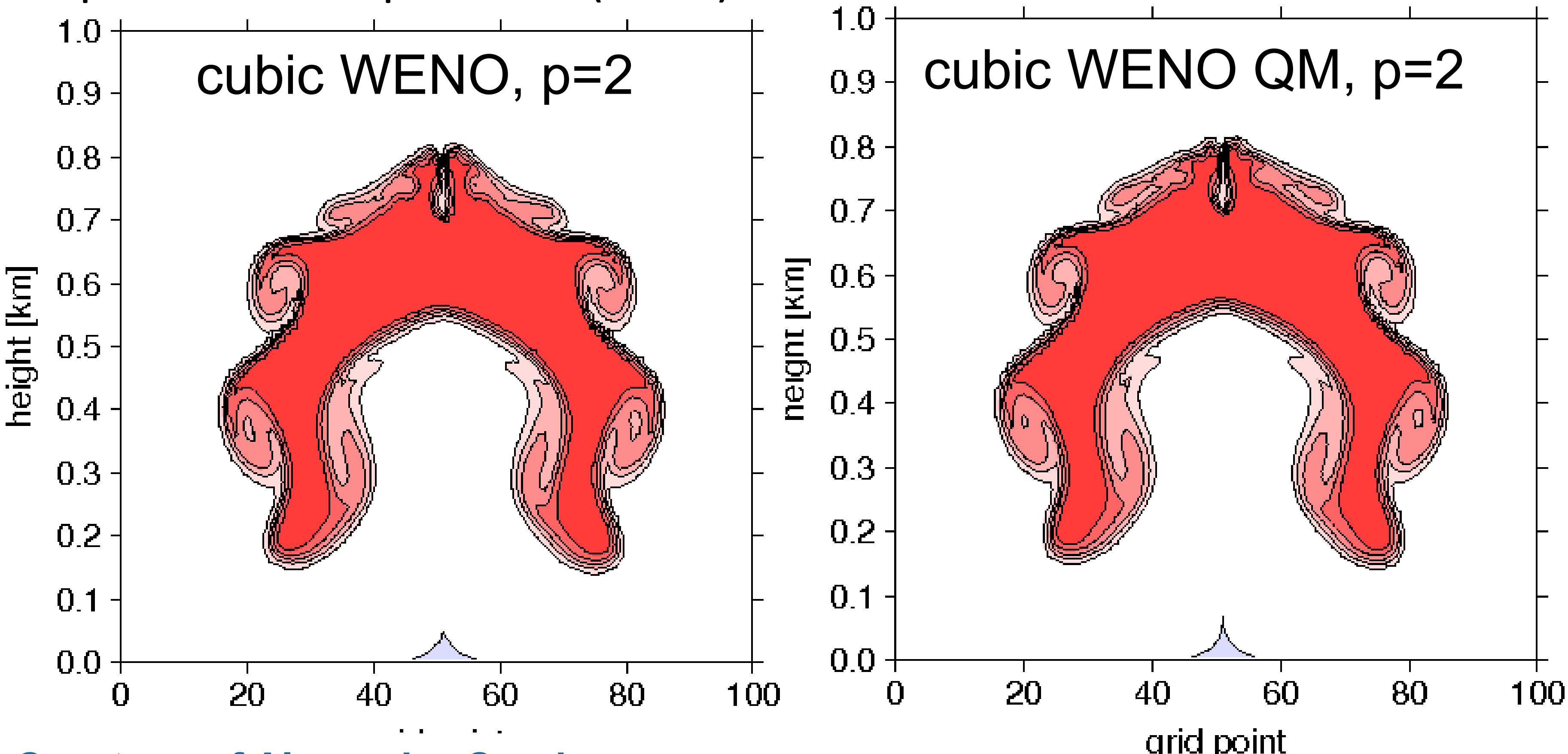

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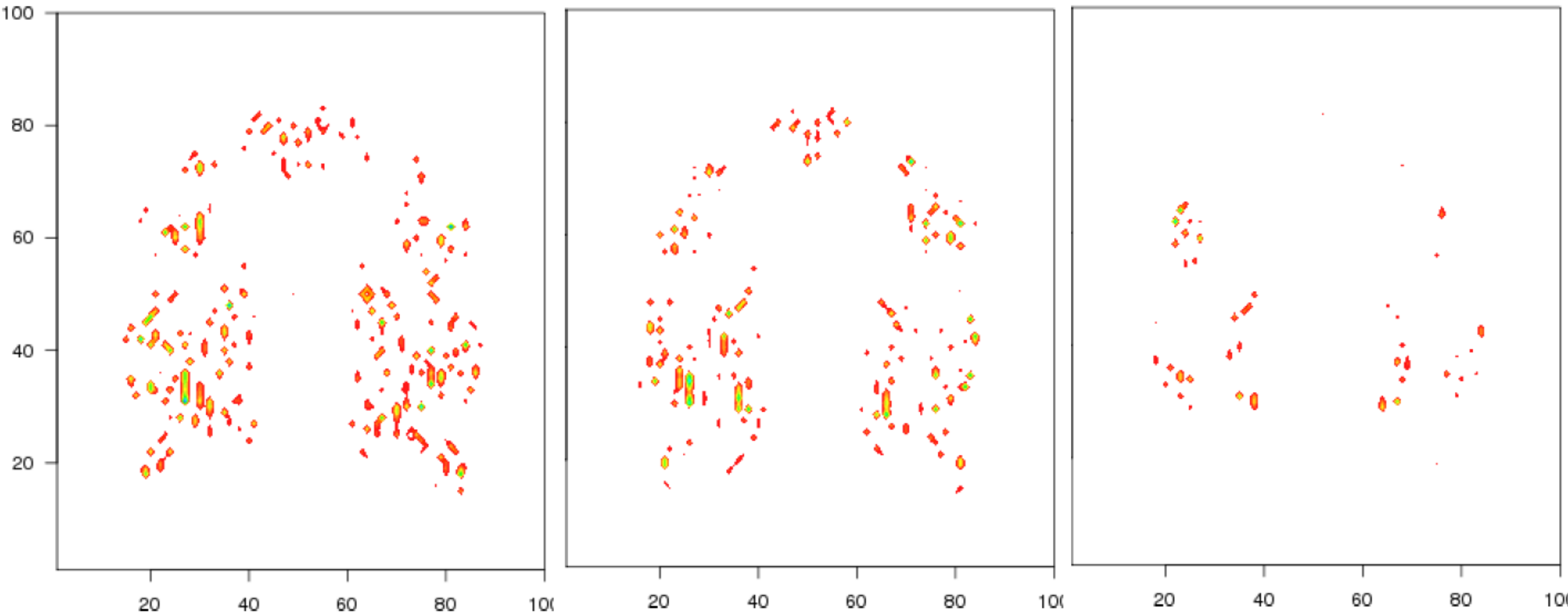
ENO technique in SL interpolations

Over/undershooting in one vertical layer

Cubic Lagrange | WENO, p=0.5 | WENO, p=2



ENO technique in SL interpolations

In 3D : 6 vertical levels

3 linear interpolations in top and bottom level

\+

5 eno and 2 linear interpolations in 4 middle levels

\+

1 vertical eno interpolation

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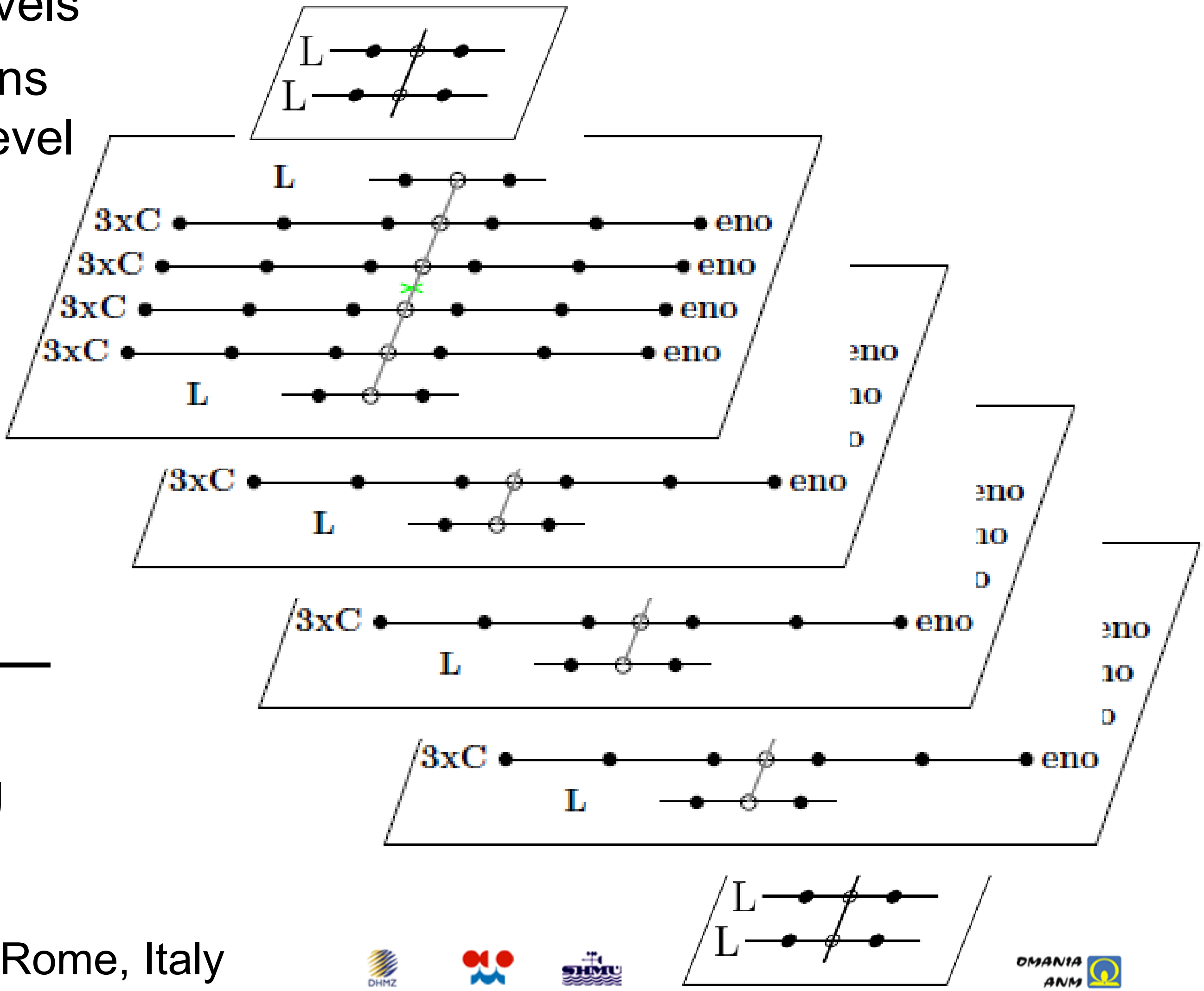
14 linear + 21 eno interpolations using 120 grid points



ENO technique in SL interpolations

Conclusions:

- Interpolations are subject of a trade off between accuracy and noise production near discontinuities; more smoothing schemes give less over/undershoots while more accurate results suffer from noise created near sharp gradients or discontinuities in the interpolated field.
- Slight improvement in the production of over/undershoots observed for the best behaving choice of the k and p parameters does not compensate the increase in the computational cost of the new WENO scheme compared to the classical cubic Lagrange solution.

Thank you for your attention!
Grazie per la vostra attenzione!