

*Regional Cooperation for
Limited Area Modeling in Central Europe*



LACE – DEVELOPMENT IN DYNAMICS

Petra Smolíková (CHMI)

ALADIN team: Jozef Vivoda (SHMI), Radmila Brožková, Ján Mašek (CHMI), David Lancz (OMSZ), Alexandra Craciun (ANM)

Hirlam coop.: Juan Simarro, Alvaro Subias (AEMET)



1. Finite element method in vertical discretization of NH model (J.Vivoda, P.Smolíková)

- based on hydrostatic version of VFE (being developed by A.Untch, M.Hortal)
- **cooperation with HIRLAM colleagues** (J.Simarro, A.Subias)

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- 3. ENO technique for SL interpolations (J.Mašek, A.Craciun)**

Current status: there is a working implementation of the VFE method in the NH model in the cycle CY40T1

How well does it work?

- 1. Idealized tests** with known analytical solution in 2D vertical plane
- 2. Real case study in high resolution** to show that the proposed method is stable and robust
- 3. Accuracy study** of the vertical FE operators to show the advantage of FE discretization over FD one
- 4. Problems**

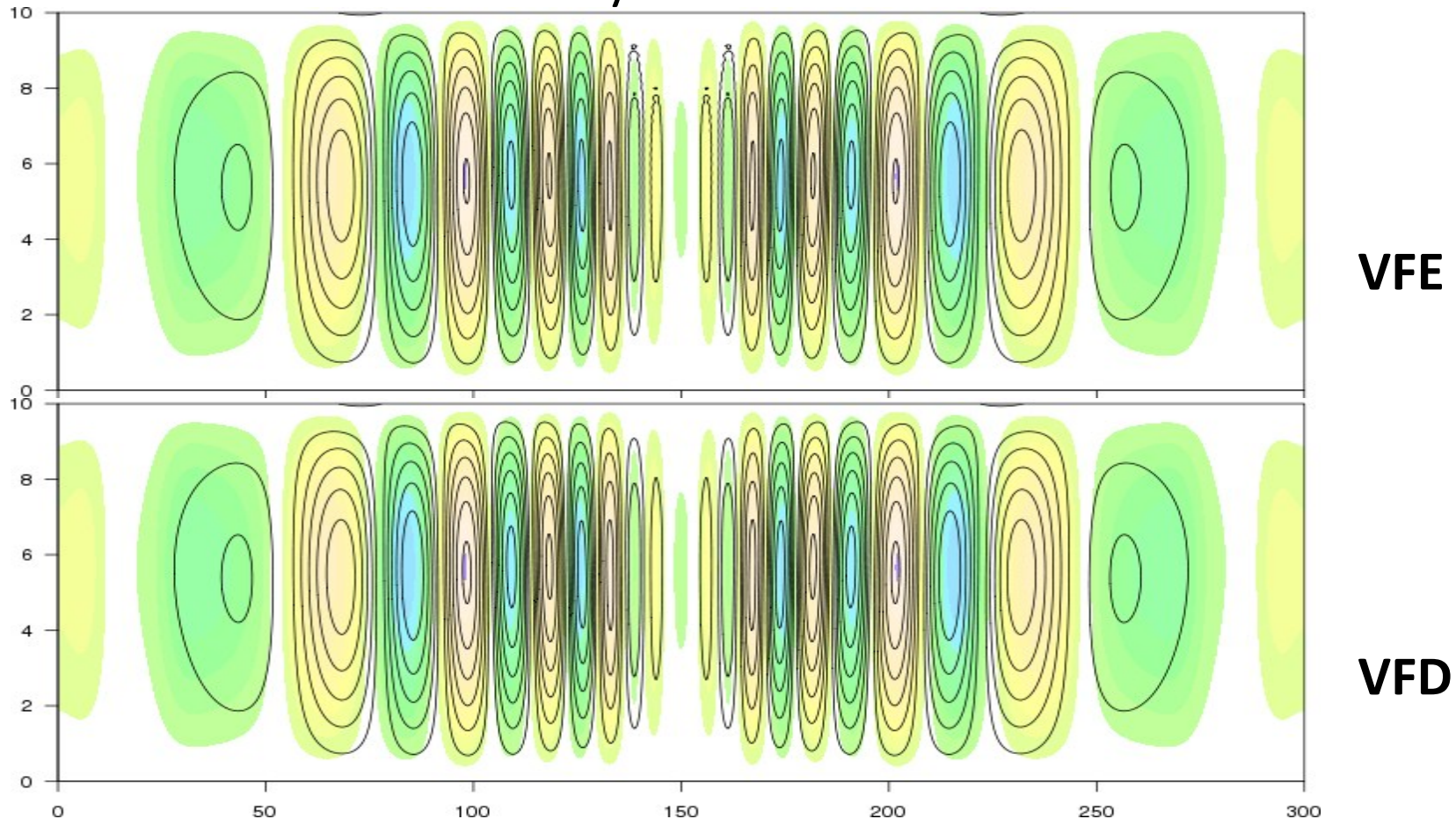
Idealized tests:

- 1. Baldauf-Brdar** : the linear expansion of sound and gravity waves in a channel induced by a weak warm bubble
 - originally in height based coordinate
 - with the boundary conditions: $w=0$ at top and bottom boundary
 - the solution: a set of waves that propagate horizontally
 - **in mass based vertical coordinate**: no fixed top boundary => atmosphere can evolve freely and move up and down at the top, we may impose strict sponge conditions, but not to avoid the vertical propagation of waves
 - vertical resolution 125m, regular levels in height up to 10km

VFE in NH

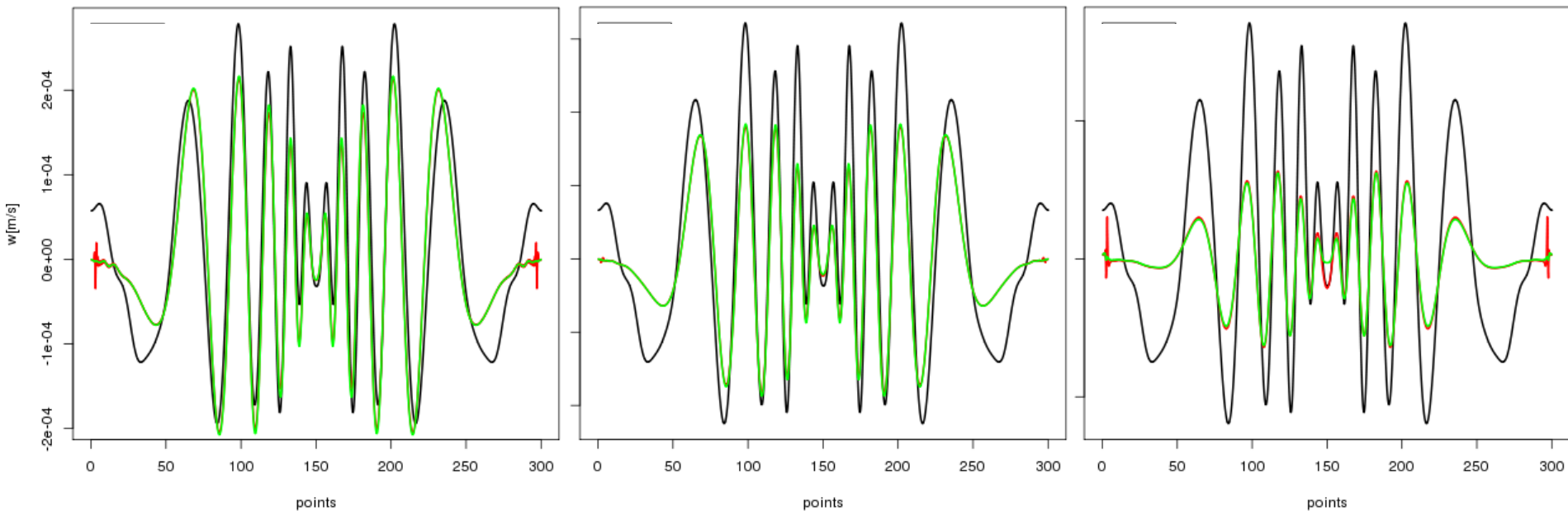
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Idealized tests:

1. **Baldauf-Brdar** : the linear expansion of sound and gravity waves in a channel induced by a weak warm bubble



Potential temperature:

4th level in 0.5km, 40th level in 5km, 76th level in 9.5km

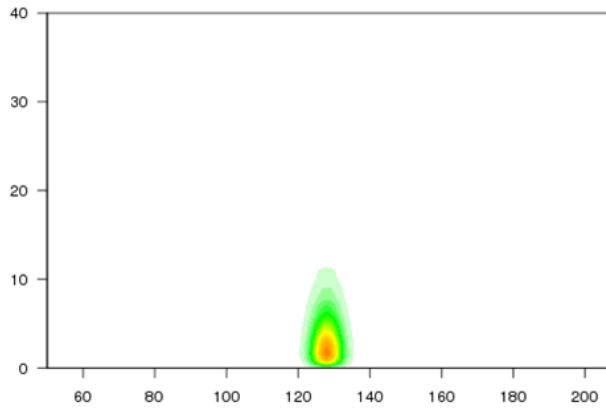
— ANALYTICAL
— VFD
— VFE

2. Simarro's test: the linear 3-dim expansion of sound and gravity waves induced by a weak warm bubble

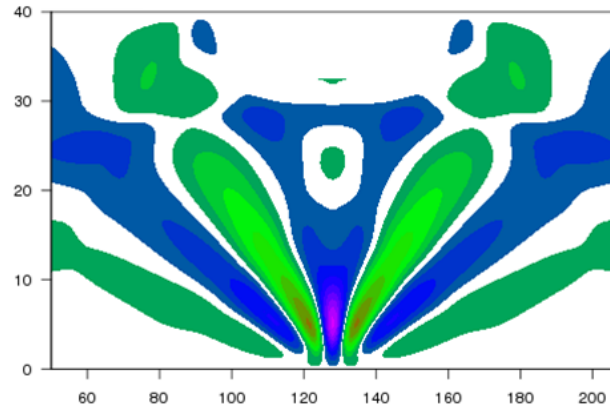
- initial perturbation localized in the lower atmosphere
- propagation of a set of waves horizontally and vertically
- it will take some time to reach the upper atmosphere
- the vertical velocity at the top is zero in the analytical solution and there is no need to impose artificially the condition of the vertical velocity being zero at the top

2. Simarro's test: the linear 3-dim expansion of sound and gravity waves induced by a weak warm bubble

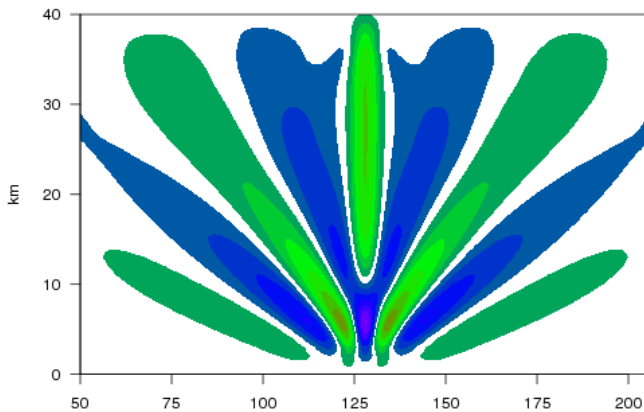
initial field of Θ



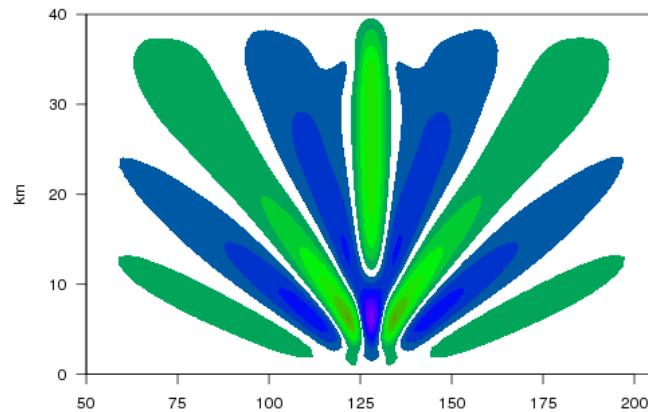
w after 600s ~ 1200 time steps



**ANALYTICAL
SOLUTION**



VFD



VFE

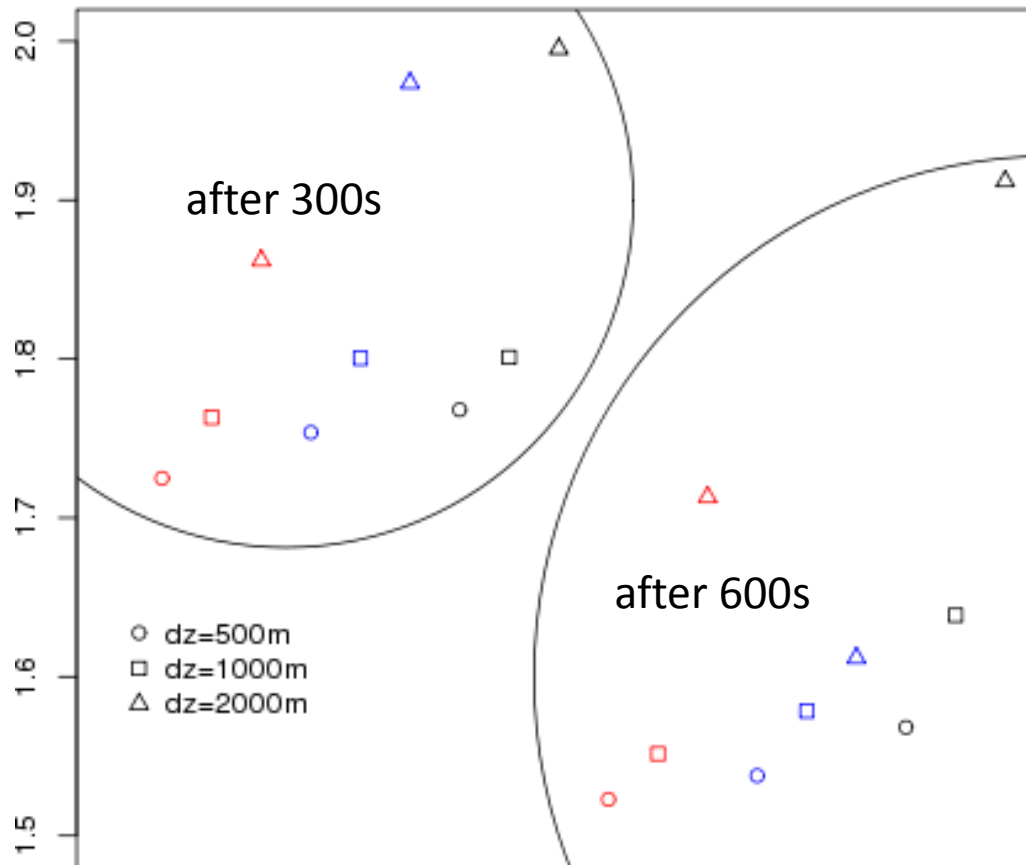
numerical solution in time 600s

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▶ 9 April 2015, Helsingør, Denmark



2. Simarro's test: the linear 3-dim expansion of sound and gravity waves induced by a weak warm bubble



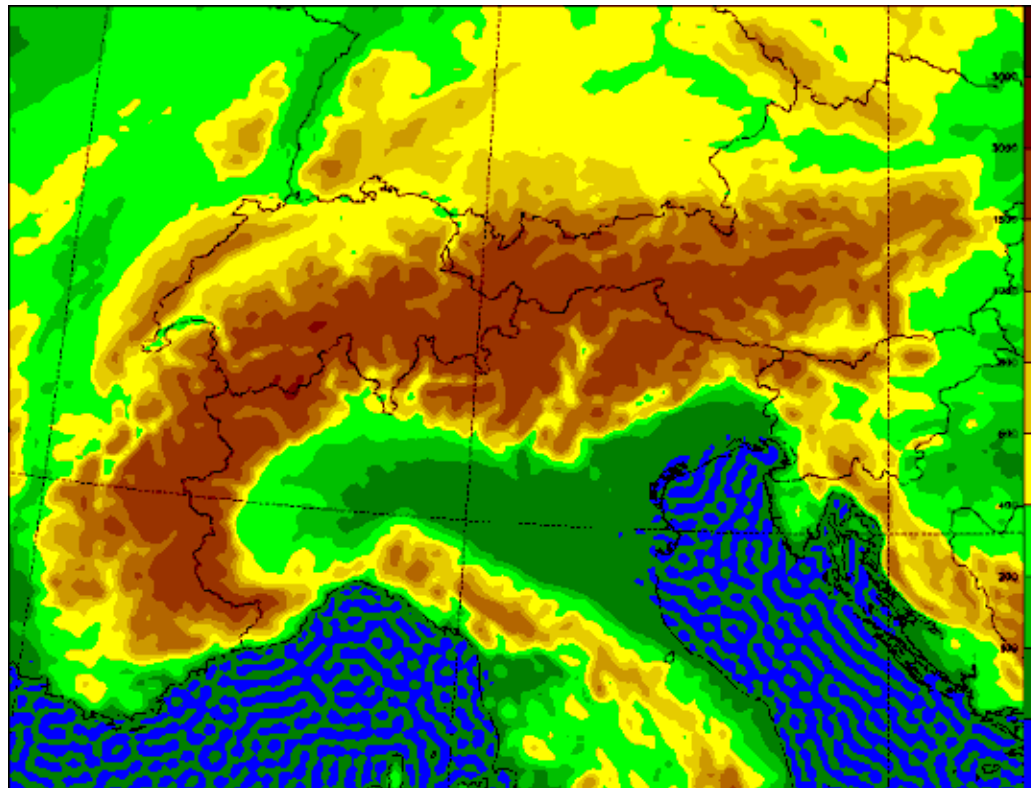
Relative l_2 -error for w .

black FD

red FE

blue FE approx

Real simulation: domain over Alpine region, 1.25km resolution, 87 vertical levels, 2tl PC + LGWADV, 2 series – January 2014 and July 2014, once per day from 00UTC + 24hours, timestep = 50s

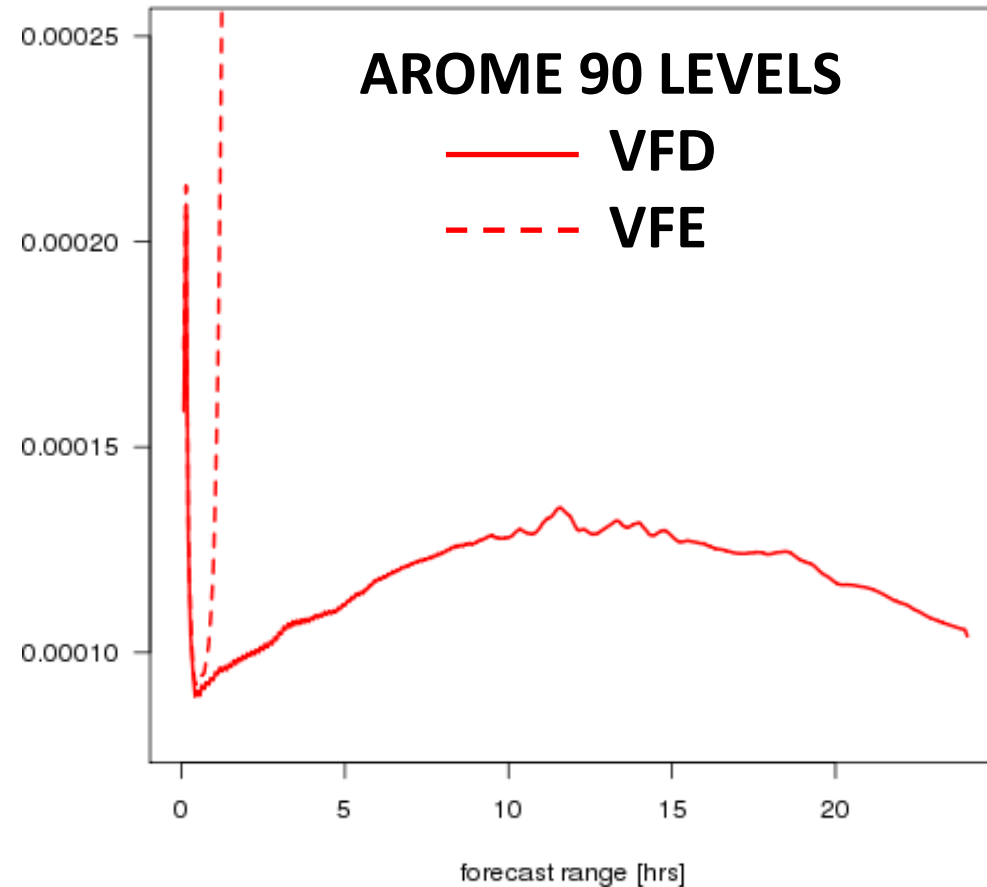


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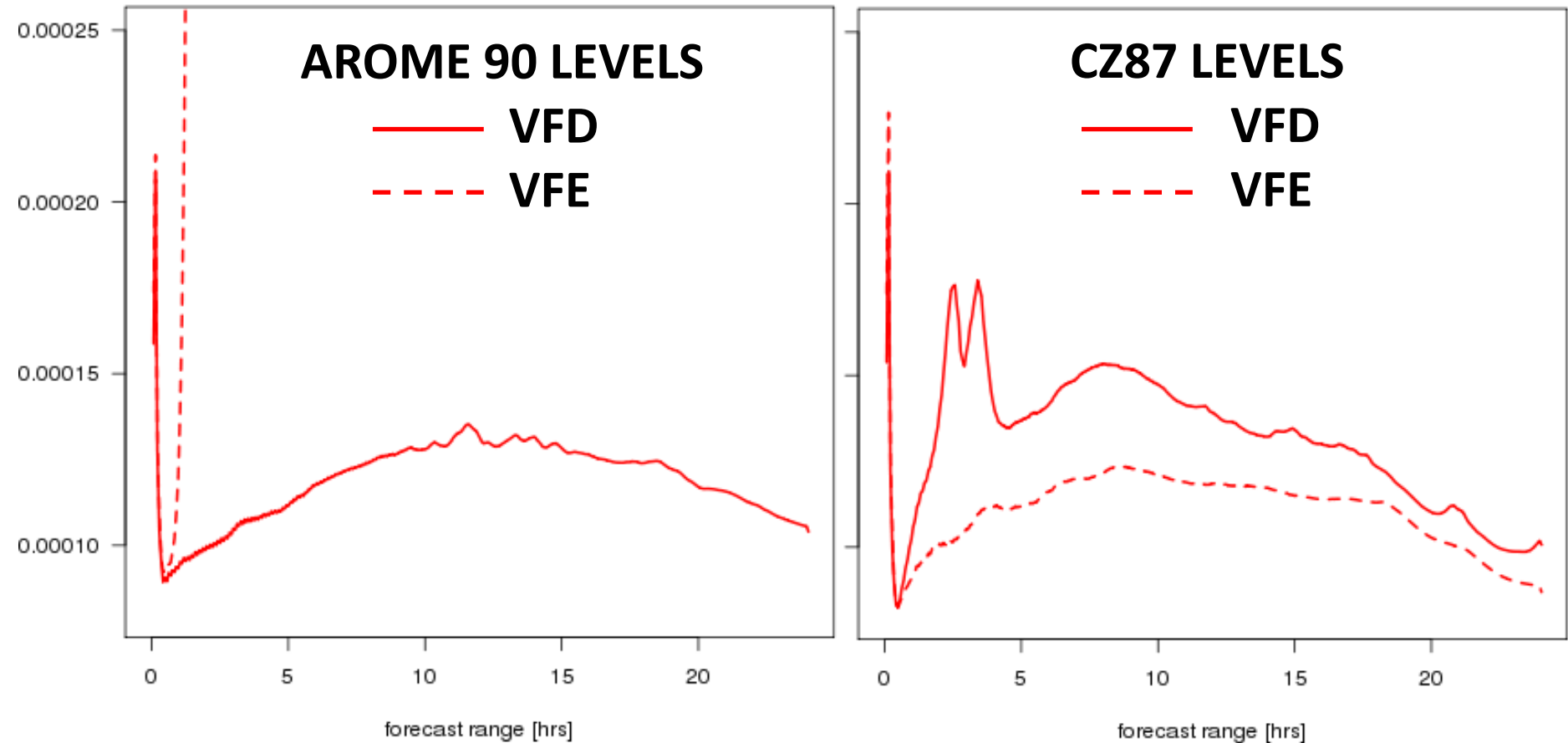
Results:

- VFE scheme used in NH with proper setting of FE parameters and “well defined” vertical levels placement may be as stable as FD scheme
 - no clear impact of vertical discretisation method on objective scores
 - there is an impact of the vertical discretisation method on the precipitation field
- confirms results obtained for coarser horizontal resolution

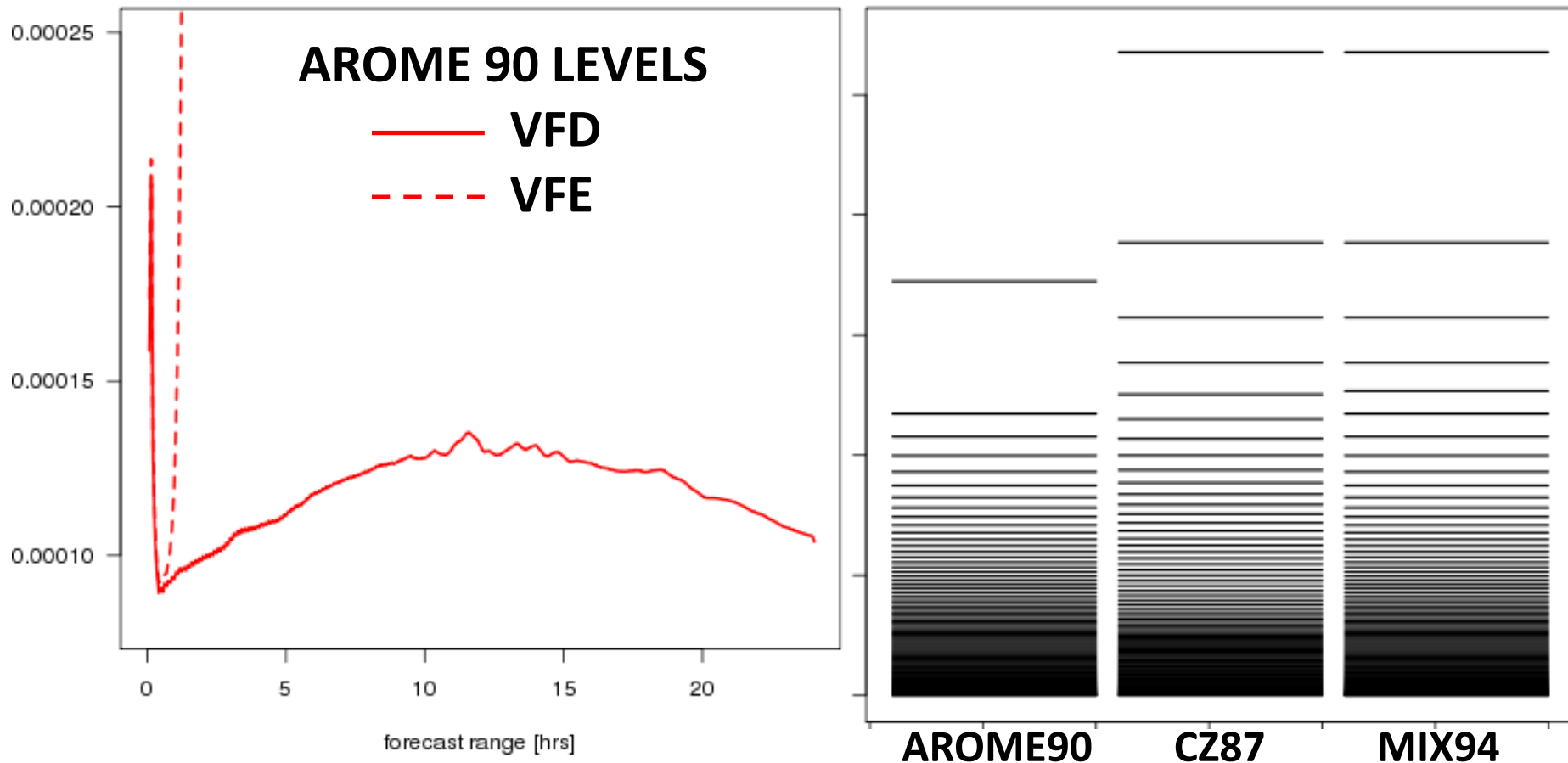
Real simulation: stability measured by the average spectral norm of pressure departure variable



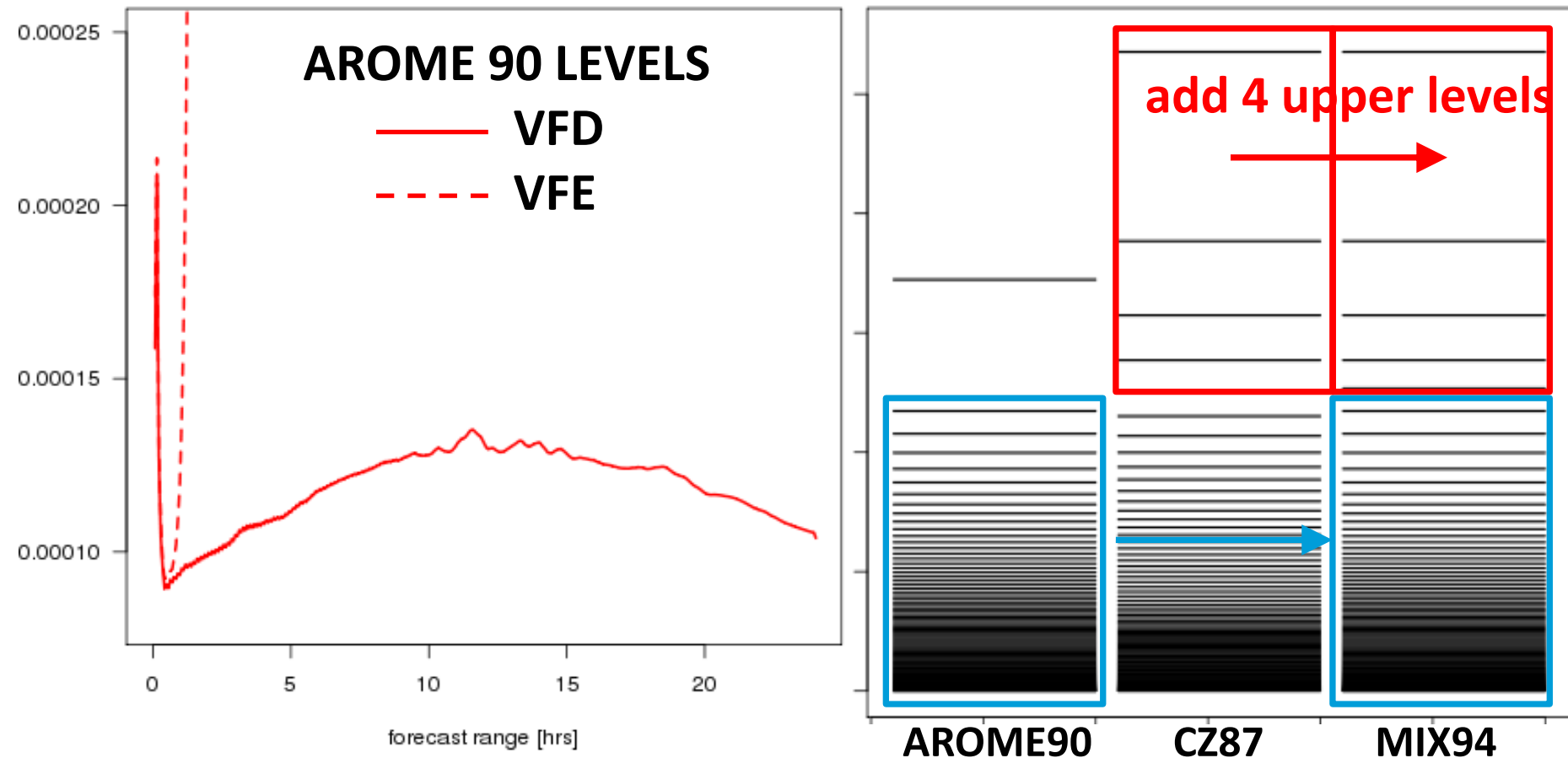
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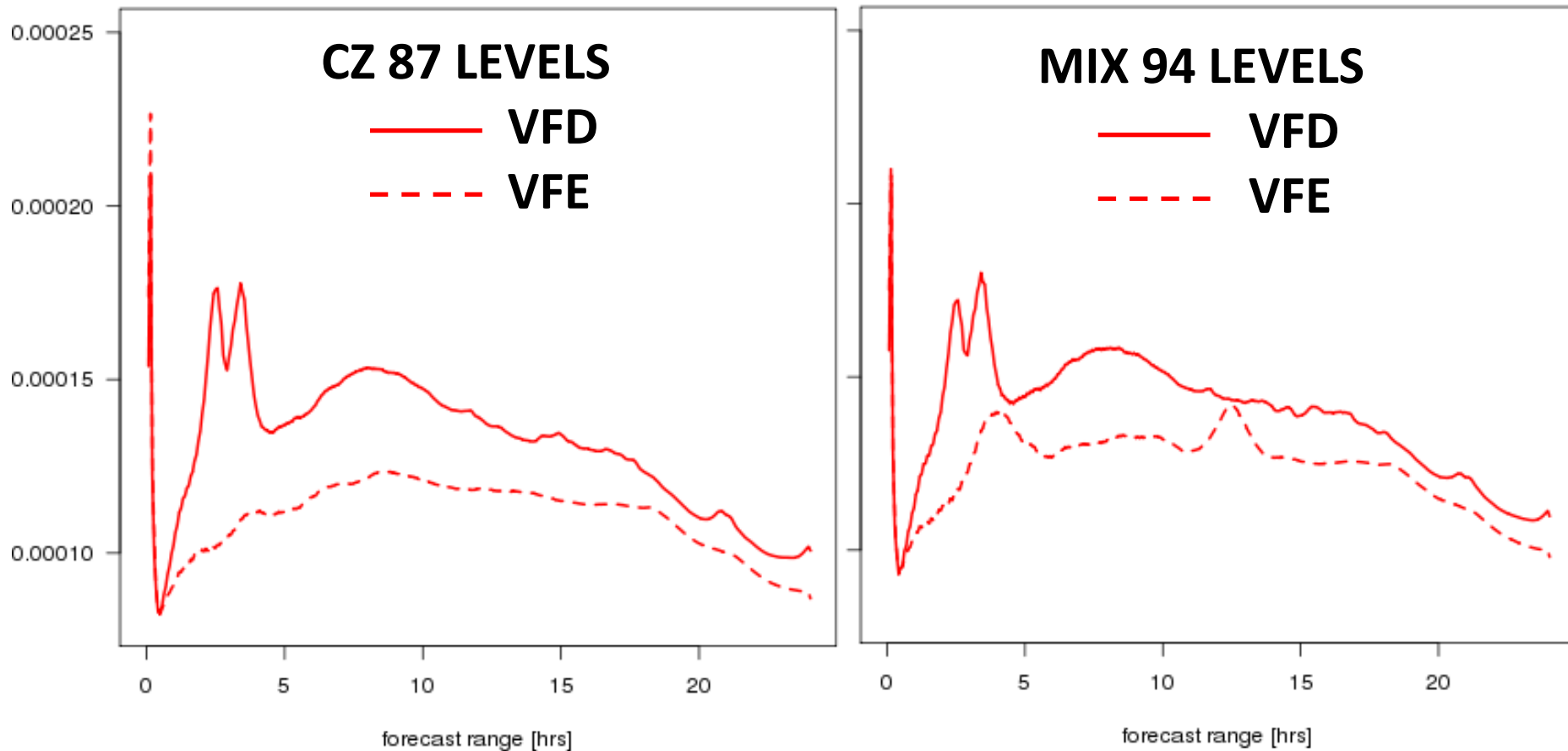
Real simulation: stability measured by the average spectral norm of pressure departure variable



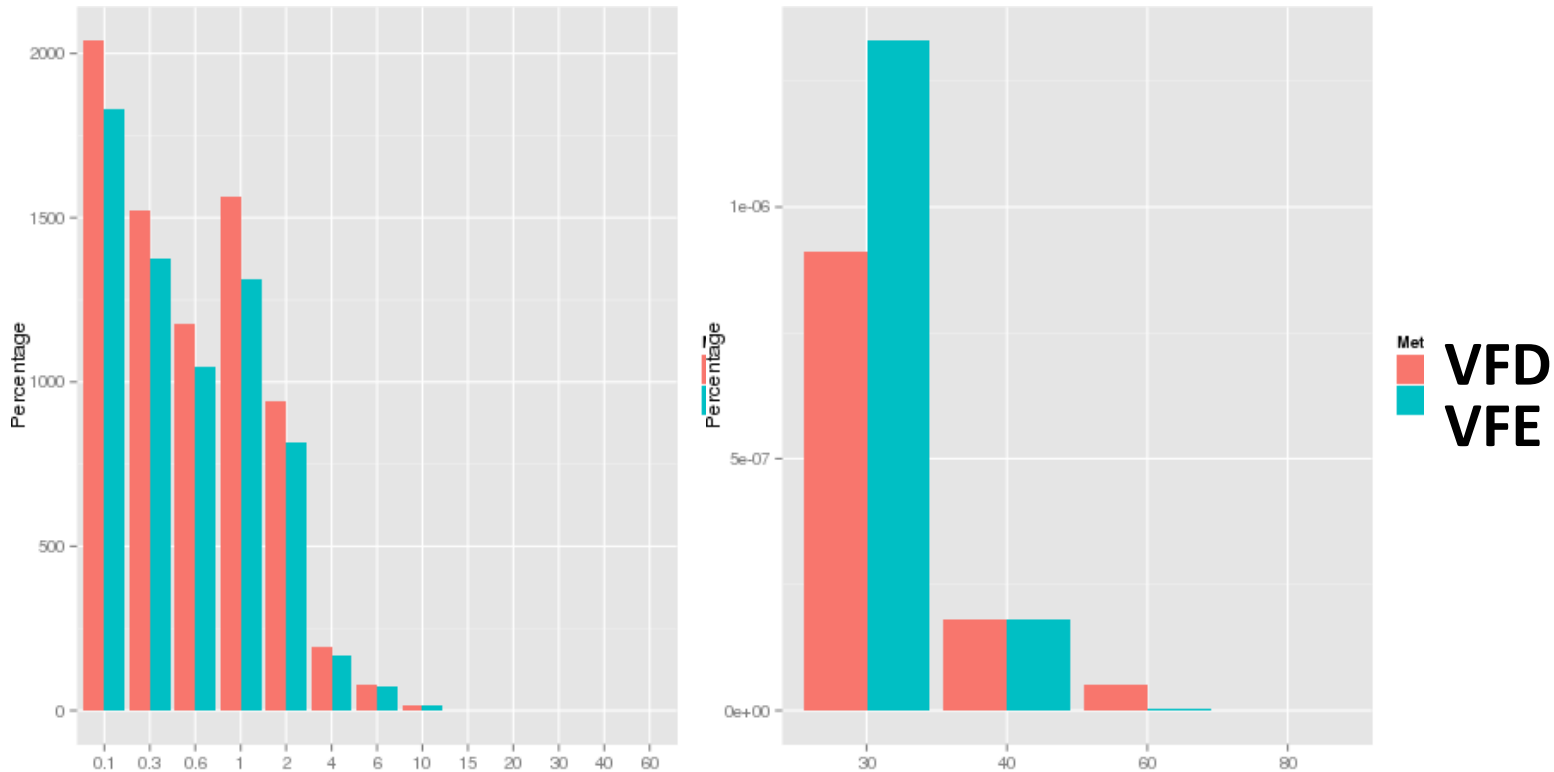
Real simulation: stability measured by the average spectral norm of pressure departure variable



Real simulation: stability measured by the average spectral norm of pressure departure variable

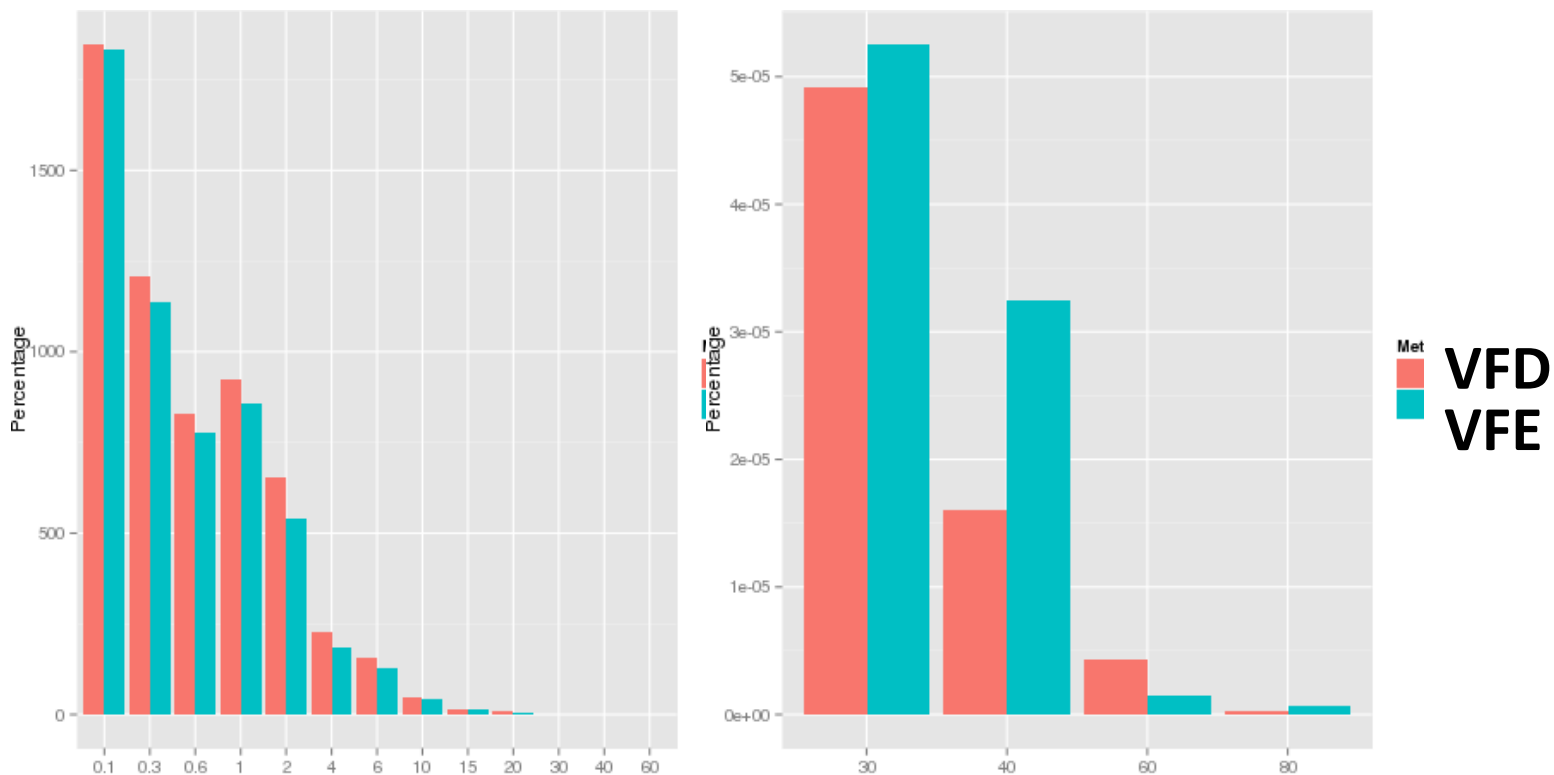


Real simulation: statistical analysis of 1hour cumulated precipitation – winter period



Rescaled results - only 10% of common events are taken with the remaining number of events in which two series differ

Real simulation: statistical analysis of 1hour cumulated precipitation – summer period



Rescaled results - only 10% of common events are taken with the remaining number of events in which two series differ

Summary for real simulations:

1. VFE scheme with proper setting of parameters and vertical levels is as **stable** as VFD scheme in 1.25km resolution.
2. It is difficult to find any benefit from FE used in vertical discretization concerning **objective scores**.
3. The **precipitation field** is modified by FE in such a way that there is bigger number of grid points without rain (cumulated precipitations for 1 hour $< 0.1\text{mm}$) and bigger number of grid points with highest values of cumulated precipitations ($>30\text{mm}/\text{hour}$). Consequently, there is smaller number of grid points with modest rain between 0.1 and 30mm.

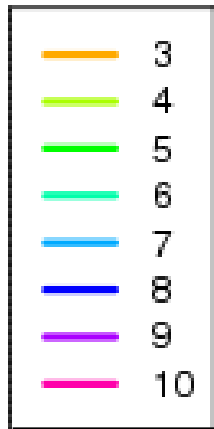
Problems:

1. saturation with higher orders and vertical resolution
2. remaining FD features
3. ...

VFE in NH

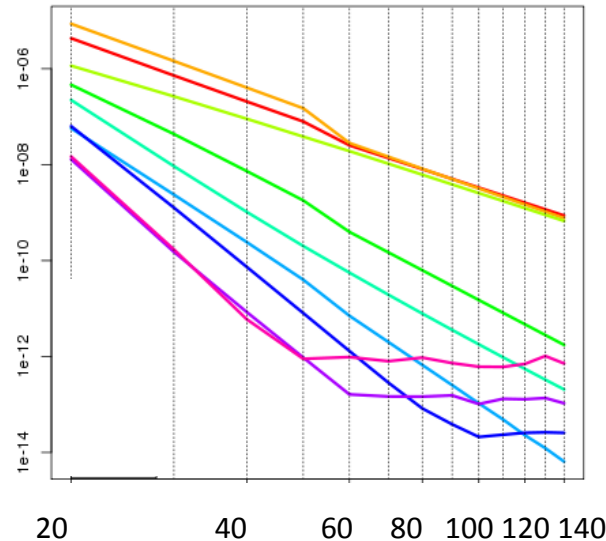
- vertical operators applied on a smooth function $\sin(\pi\eta)^3 \cos(\pi\eta)$
- regular eta levels
- saturation for higher orders
- saturation for higher vertical resolutions
 \leq rounding error and high number of operations

Order of splines

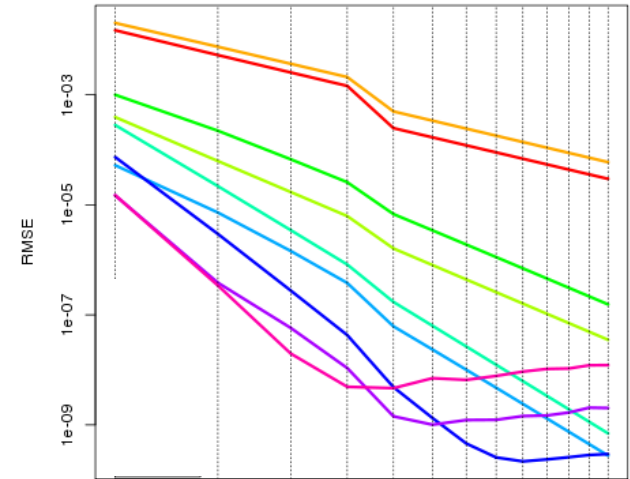


RMSE

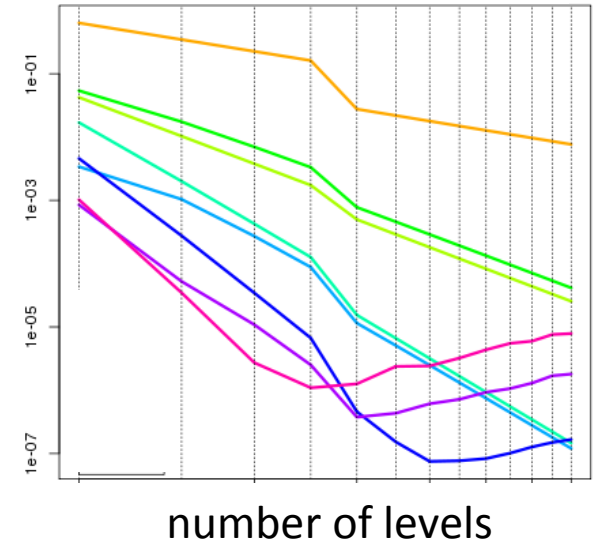
INTEGRAL



DER



SEC.DER



Work in progress:

- remaining FD features
- transformations $w \leftrightarrow d$ (Alvaro Subias invertible operators)
- BC in vertical Laplacian - **study for σ coordinate and 5 vertical levels** (J.Vivoda):

the choice of BBC and TBC between $f=0$ and $\frac{\partial f}{\partial \sigma}=0$ for 2 vertical Laplacian definitions:

$$\mathcal{L}_v^* X = \sigma \frac{\partial}{\partial \sigma} \left(\sigma \frac{\partial}{\partial \sigma} + 1 \right) X$$
$$\mathcal{L}_v^* X = \sigma \left(2 \frac{\partial}{\partial \sigma} + \sigma \frac{\partial^2}{\partial \sigma^2} \right) X$$

=> conclusion: form 2 with BBC imposed gives real and negative eigenvalues, other choices are wrong

ENO (Essentially Non-Oscillatory)/WENO (Weighted ENO) techniques (Alexandra Craciun and Ján Mašek)

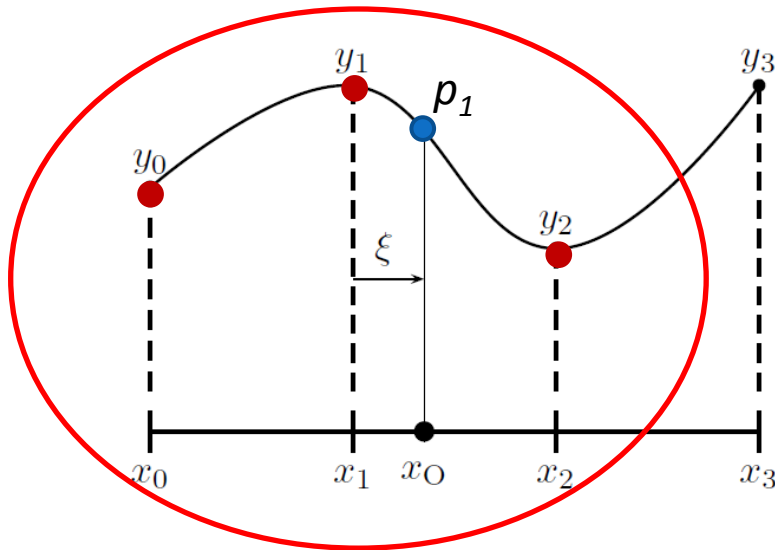
Idea of Ján Mašek (inspired by literature):

to explore alternative interpolators which are

- less overshooting than Lagrange polynomials close to discontinuities
- more accurate than their quasi-monotonic versions

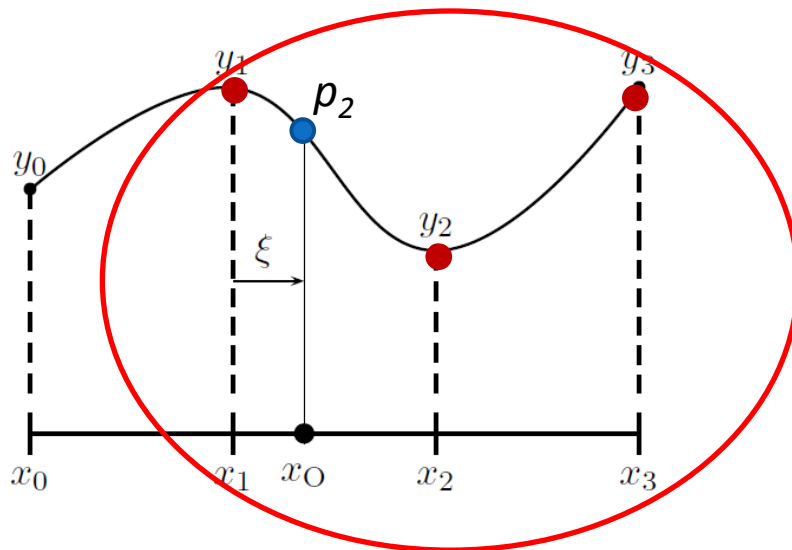
⇒ interpolation depending on the smoothness of the interpolated field

ENO technique in SL interp.



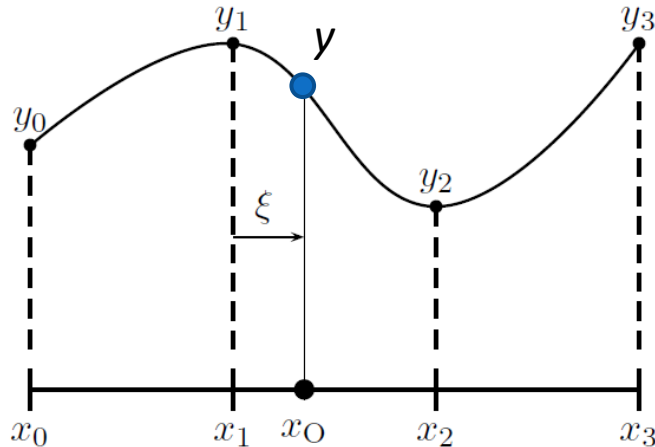
Second order interpolation scheme (quadratic) needs 3 points to find \bullet , we have 4 points stencil:
we may choose the first triplet
 \Rightarrow interpolated value p_1

ENO technique in SL interp.



Second order interpolation scheme (quadratic) needs 3 points to find \bullet , we have 4 points stencil:
or the second triplet
 \Rightarrow interpolated value p_2

ENO technique in SL interp.



p_1, p_2 interpolated values on the first and second triplet

$$y = p_1 \cdot w_1 + p_2 \cdot w_2, \quad w_1 + w_2 = 1$$

ENO chooses the smoothest solution ($w_1=0$ or $w_2=0$)

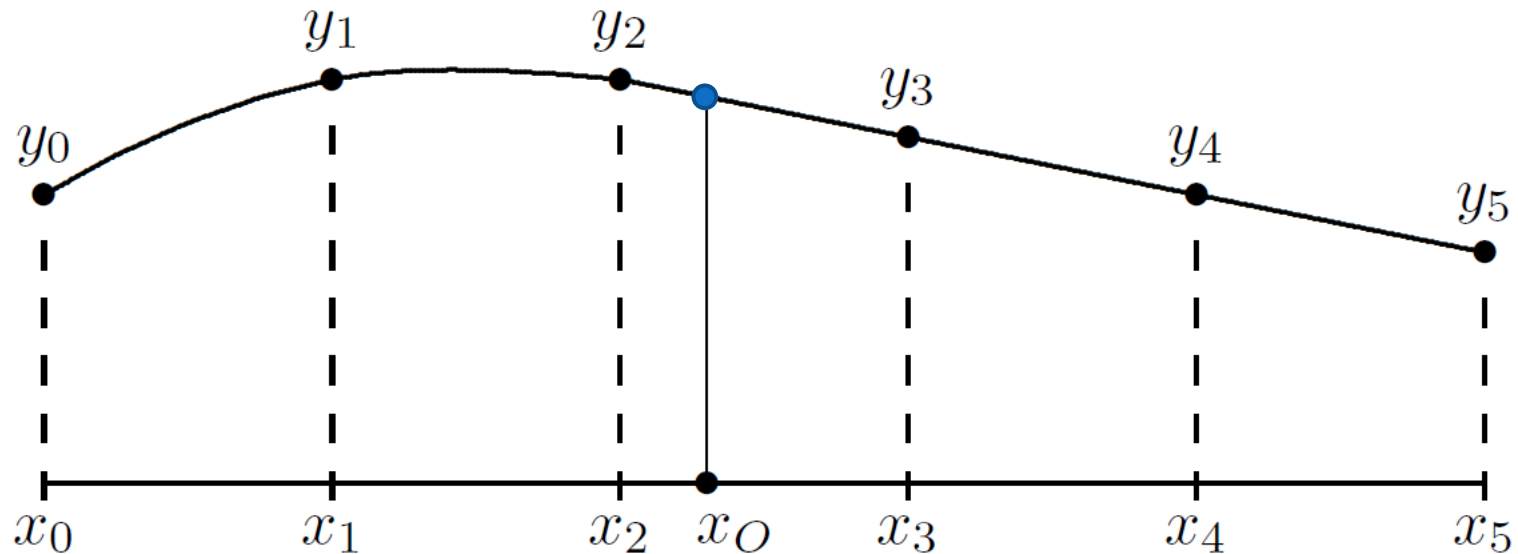
WENO weighted combination based on smoothness

$$S_i = |y_{i+1} - 2y_i + y_{i-1}|$$

Linear/cubic p_1, p_2 interpolated with linear/cubic Lagrange polynomial, weights depend on smoothness

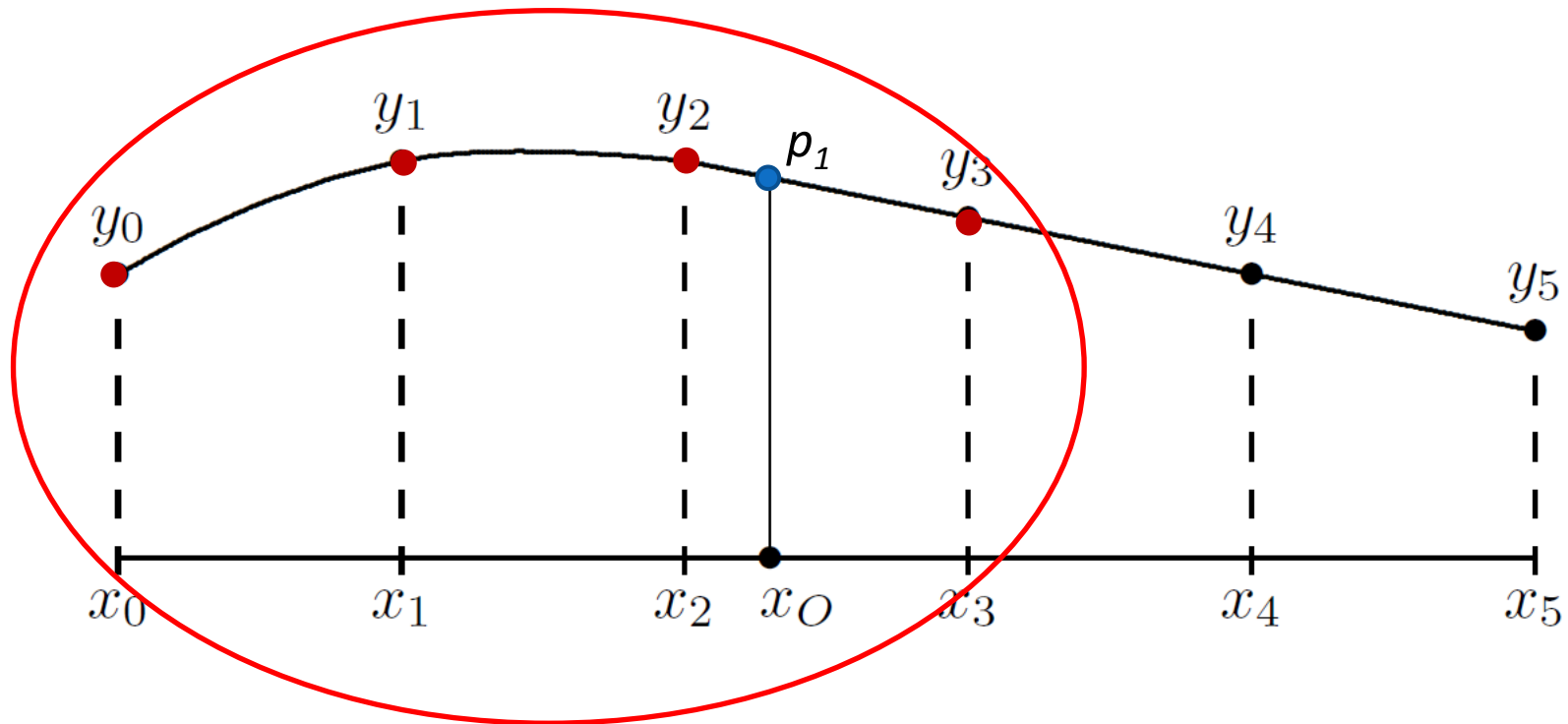
ENO technique in SL interp.

Third order interpolation scheme (cubic) needs 4 points to find ●:



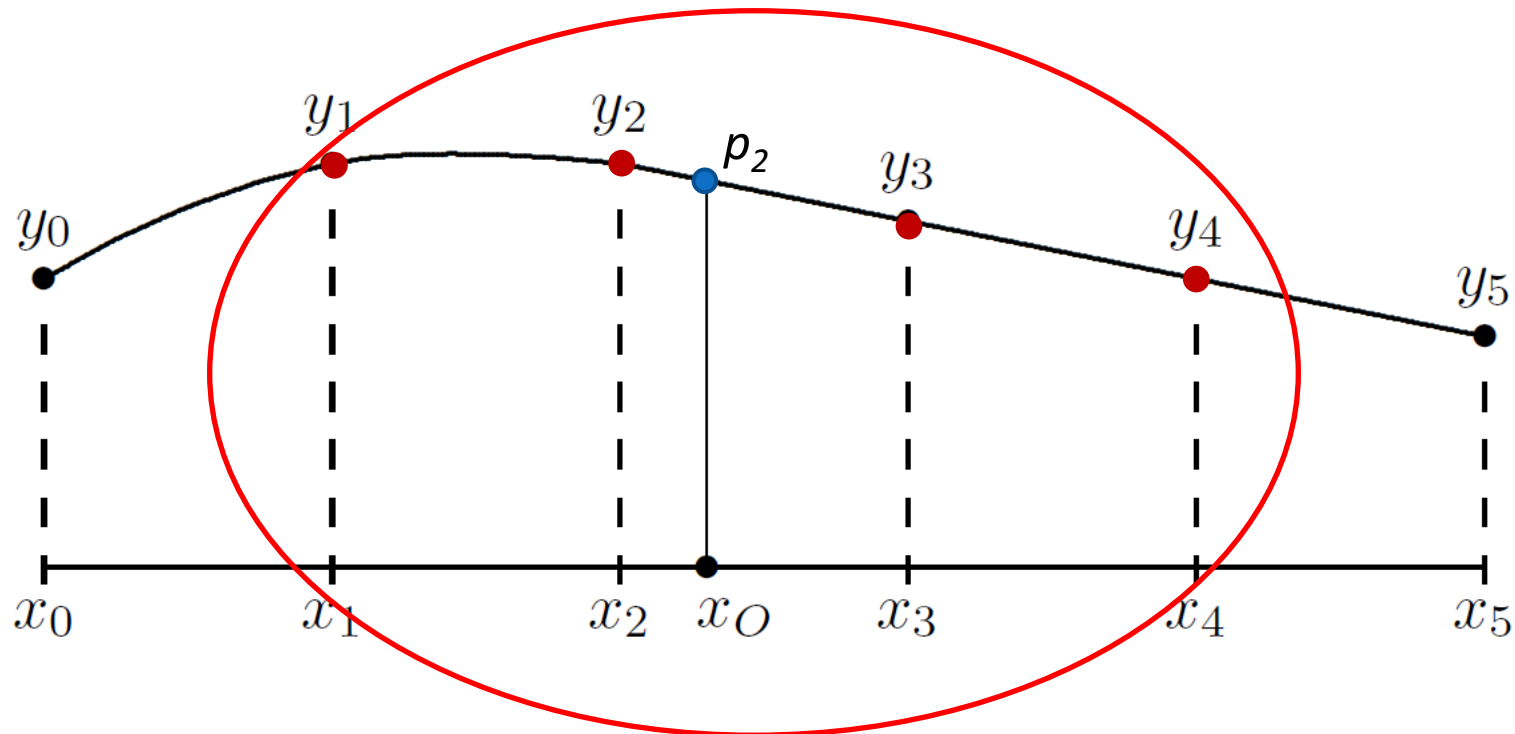
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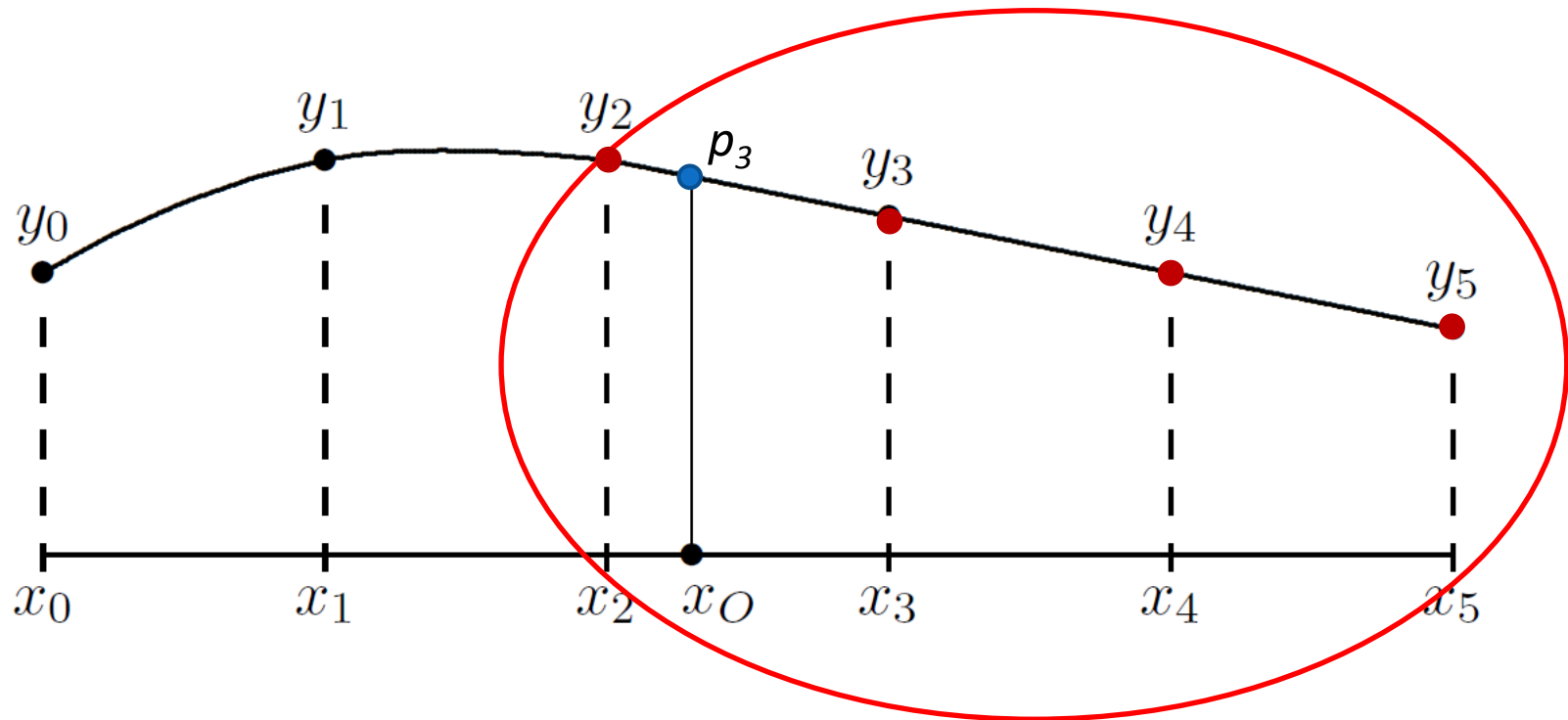
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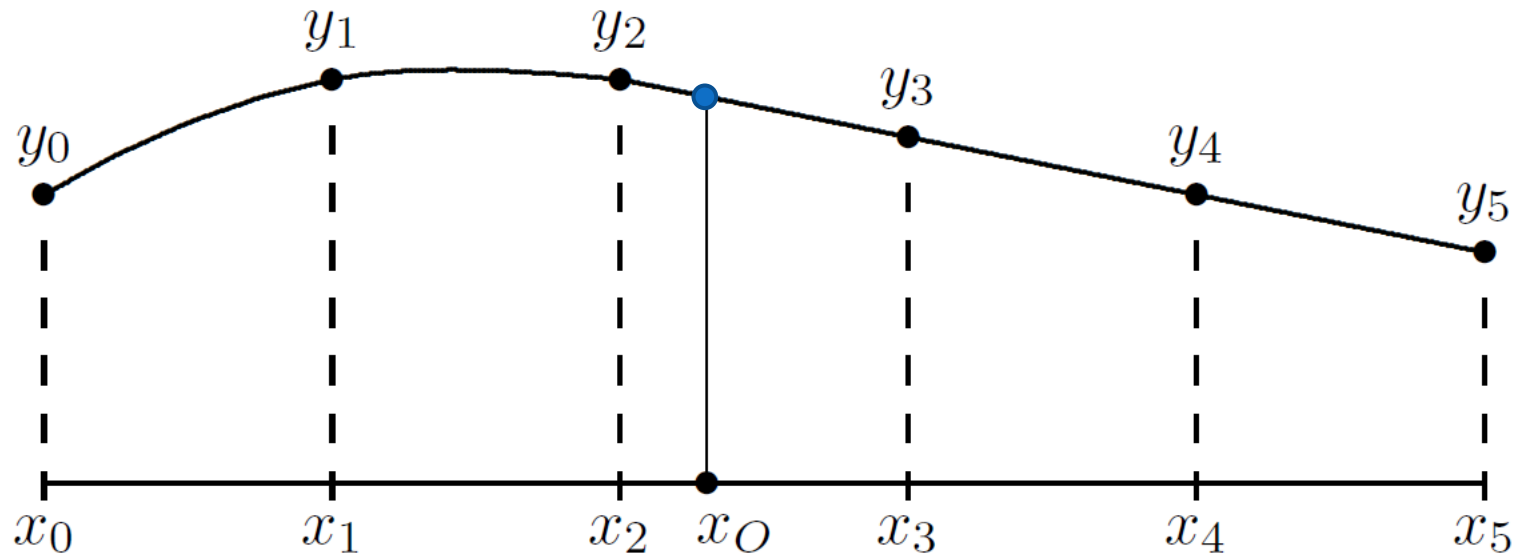
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final interpolated value $y = w_1 p_1 + w_2 p_2 + w_3 p_3$

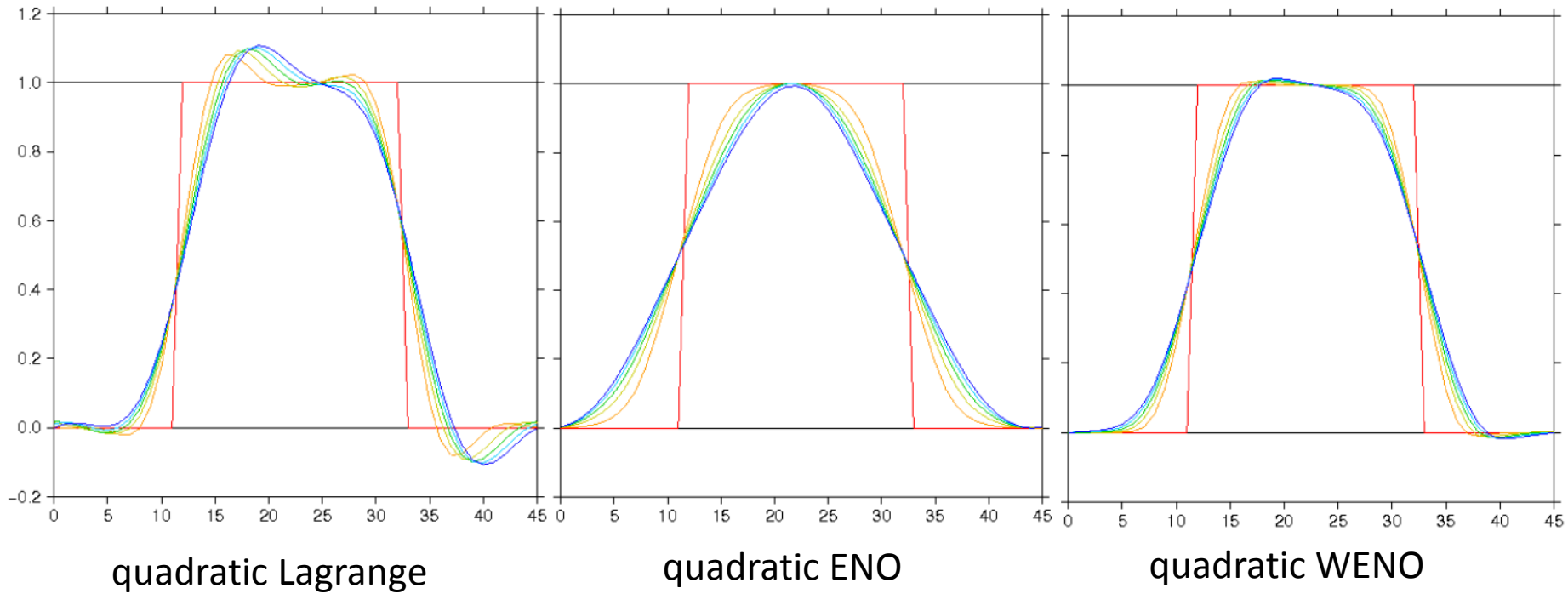
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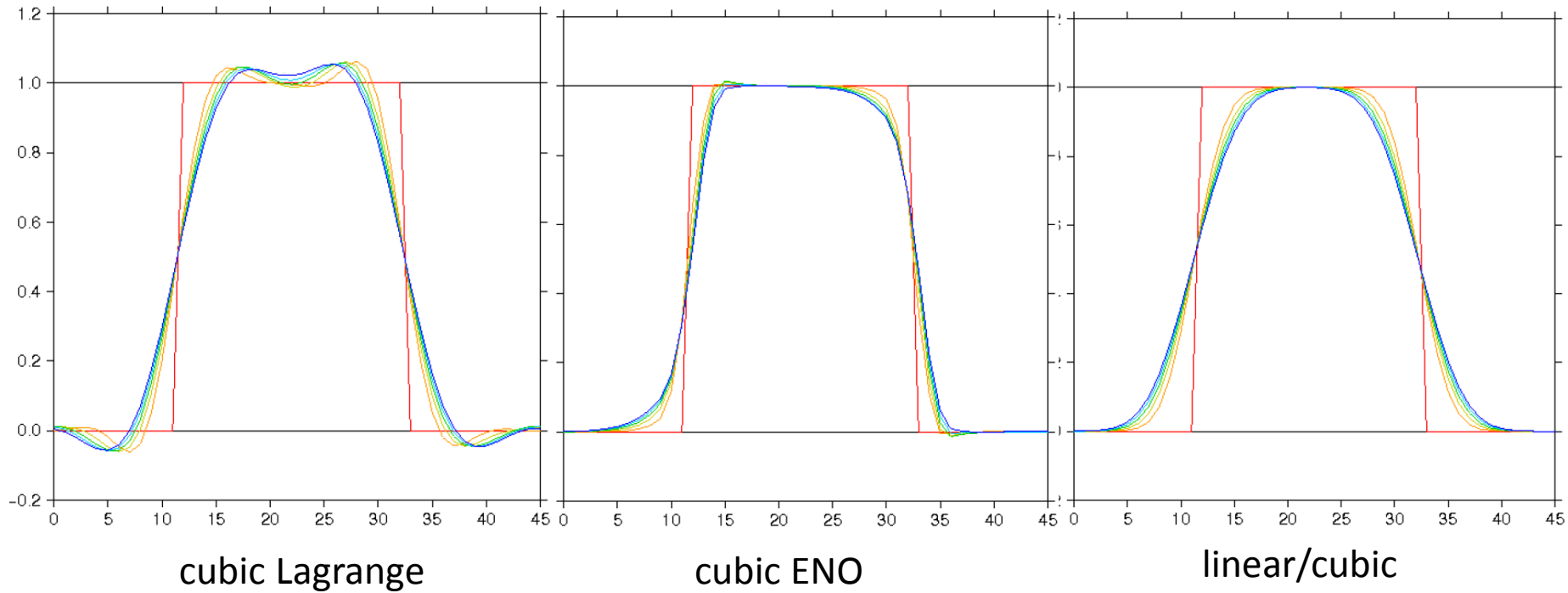


=> **6 points stencil** needed for ENO/WENO interpolations !!!

Toy model – 1D linear advection of rectangular pulse in a periodic domain (courtesy of Ján Mašek)

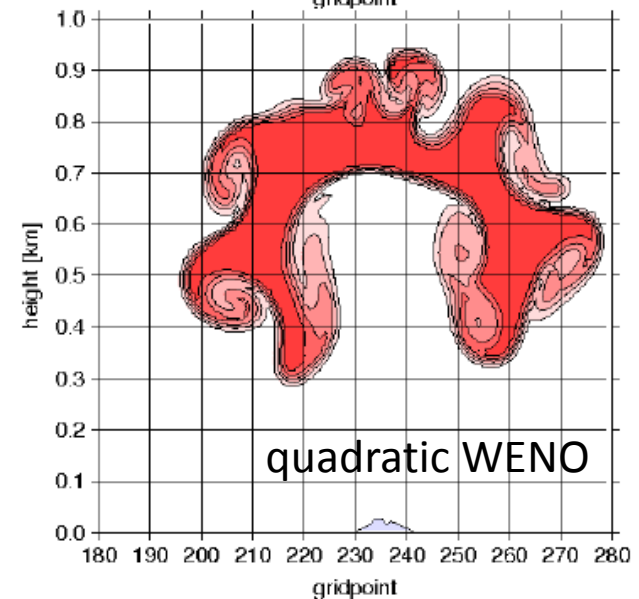
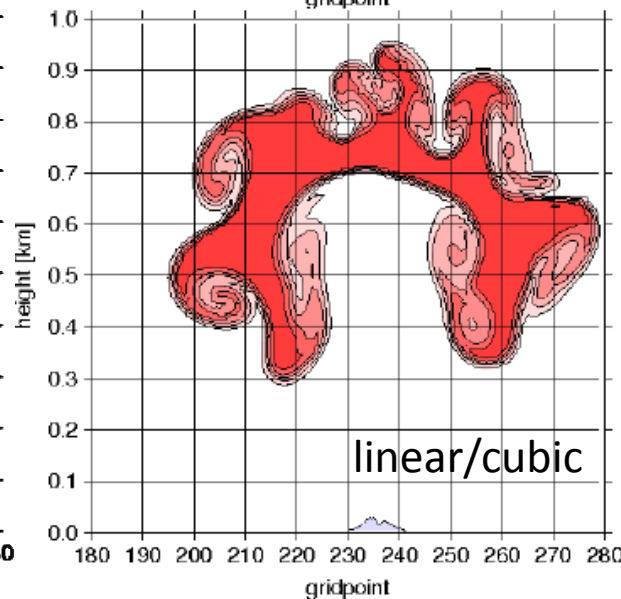
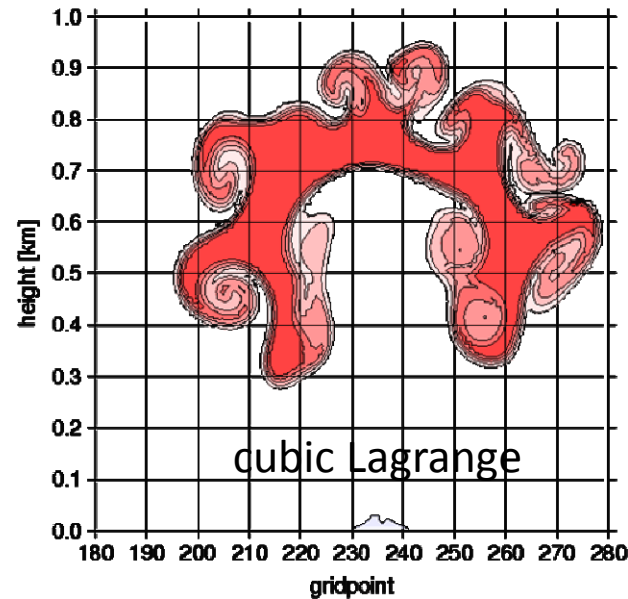
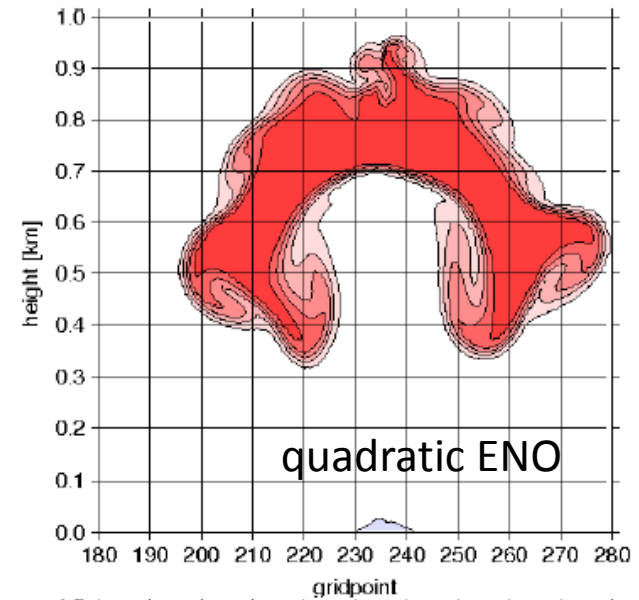
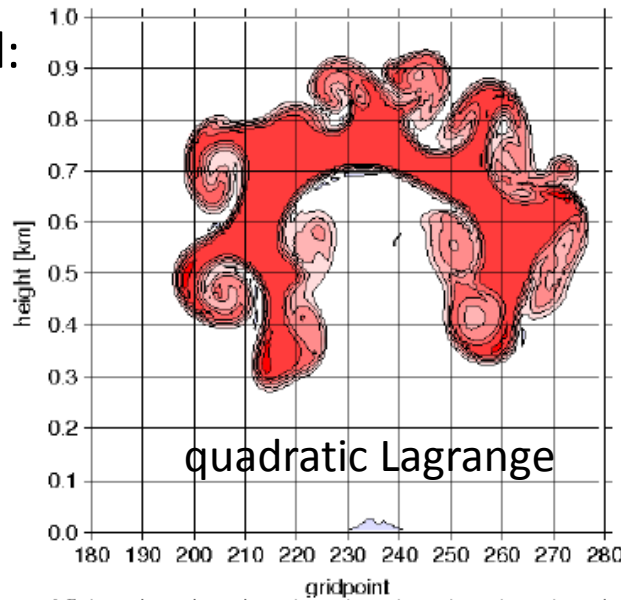


Toy model – 1D linear advection of rectangular pulse in a periodic domain (courtesy of Ján Mašek)



ENO technique in SL interp.

Robert's test in 2D model:
warm bubble (+0.5K)
in the field of potential
temperature (300K)
advected with the
wind speed 2m/s
(courtesy of A.Craciu)



Conclusions:

- Quadratic interpolator too smoothing to work well
 - Cubic ENO/WENO technique promising, but technically and computationally demanding (number of cubic interpolations increased from 7 to 21 !!!, usage of NSTENCILWIDE=3 ???)
 - Combined linear/cubic interpolation may be easily tested and gives promising results – controlled damping depending on the interpolated field
- ⇒ 2 last points worth to be tested in 3D – planned for future work

