

*Regional Cooperation for
Limited Area Modeling in Central Europe*



LACE – DEVELOPMENT IN DYNAMICS

Petra Smolíková (CHMI), Jozef Vivoda (SHMI), Radmila Brožková,
Ján Mašek (CHMI), Alexandra Craciun(NMA) and others
Hirlam coop.: Juan Simarro, Alvaro Subias (AEMET)



Summary

- 1. Finite element method in vertical discretization of NH model (J.Vivoda, P.Smolíková)**
 - based on hydrostatic version of VFE (being developed by A.Untch, M.Hortal)
 - **cooperation with HIRLAM colleagues** (J.Simarro, A.Subias)
- 2. ENO technique in SL interpolations (J.Mašek, A.Craciun)**
- 3. Scale adaptive horizontal diffusion - SLHD tuning for high resolutions (P.Smolíková, R.Brožková)**

Finite elements in vertical for NH

What we want to have:

Pure FE vertical discretisation
in NH model with

- an **arbitrary** choice of the order of basis functions
- **stability** – similar as for FD
- **accuracy** – improved compared to FD



What we have:

FE vertical discretisation in NH
model with

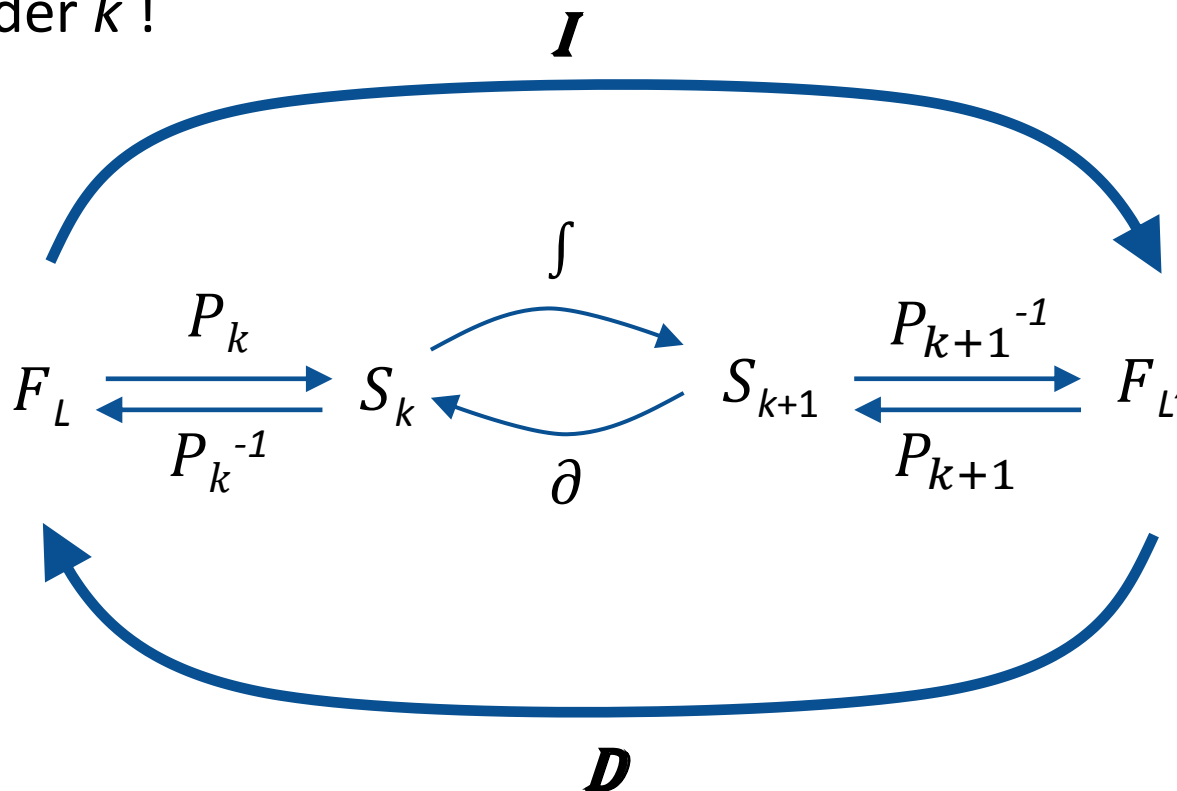
- several **FD features**
(transformations $w \leftrightarrow d$,
BC of vertical Laplacian)
- an **almost arbitrary** choice of the order of basis functions
- **stability** – similar as for FD in 2D and 3D (2.2km) tests
- **accuracy** – improved according to theoretical study for distinct FE operators, not confirmed in 2D and 3D tests

Eliminating FD features

- Stability reasons \Rightarrow **vertical divergence d** in spectral calculations
- Accuracy reasons \Rightarrow **vertical velocity w** in GP calculations
- **Transformations** (ones per time step) by vertical operators **I, D**
Derivative **D** : $w \rightarrow d$ after GP calculations
Integral **I** : $d \rightarrow w$ after SP calculations
- To keep the steady state $\frac{\partial w}{\partial t} = 0$ we need **invertibility**
$$**$I.D$** $f = \mathbf{D.I}$ $f = f$$$
- Looking for operators in the space of B-splines of a given order was unsuccessful \Rightarrow FD version of **I, D** was kept limiting the accuracy of the whole FE scheme

Eliminating FD features

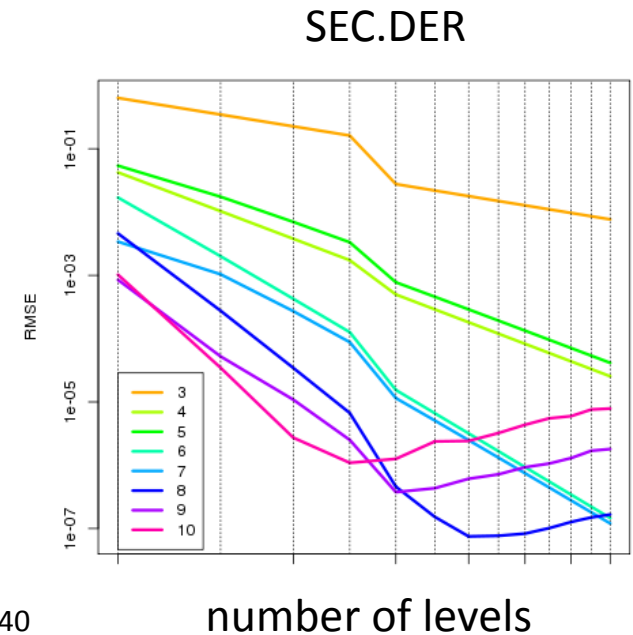
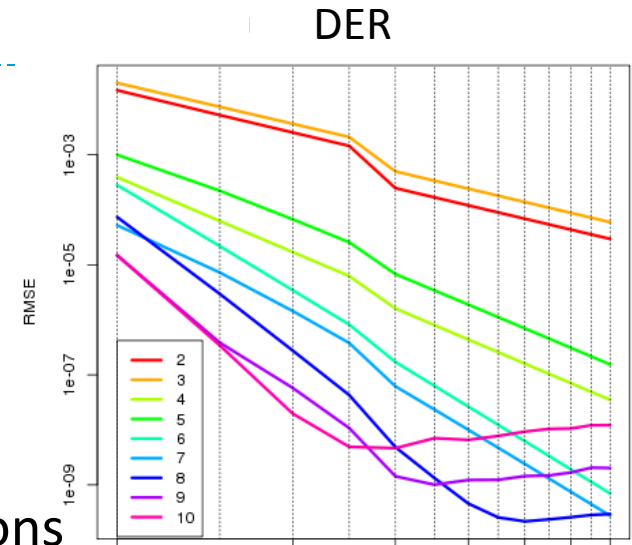
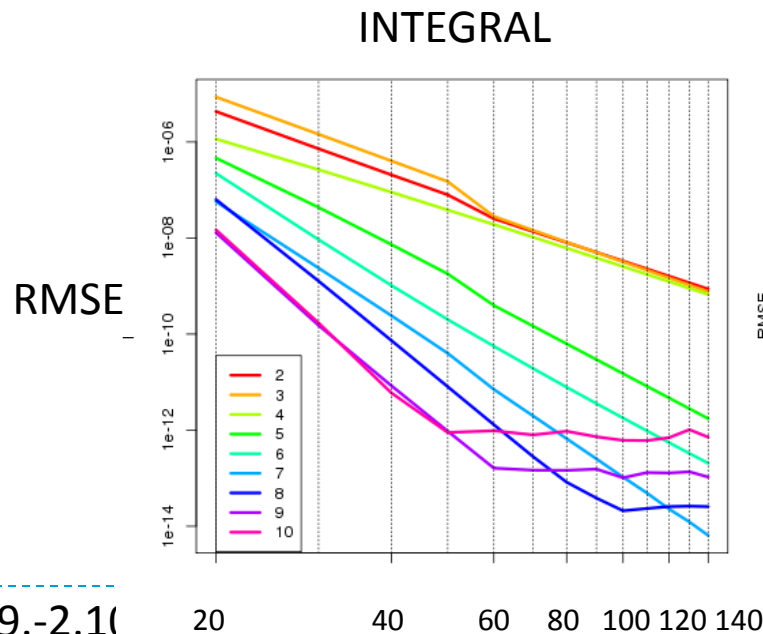
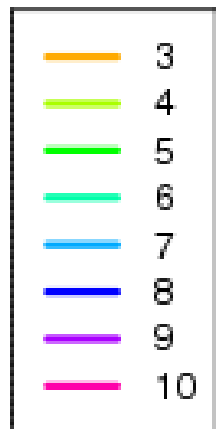
We do not have to restrict ourselves on the space of B -splines of a given order k !



- not yet fully implemented

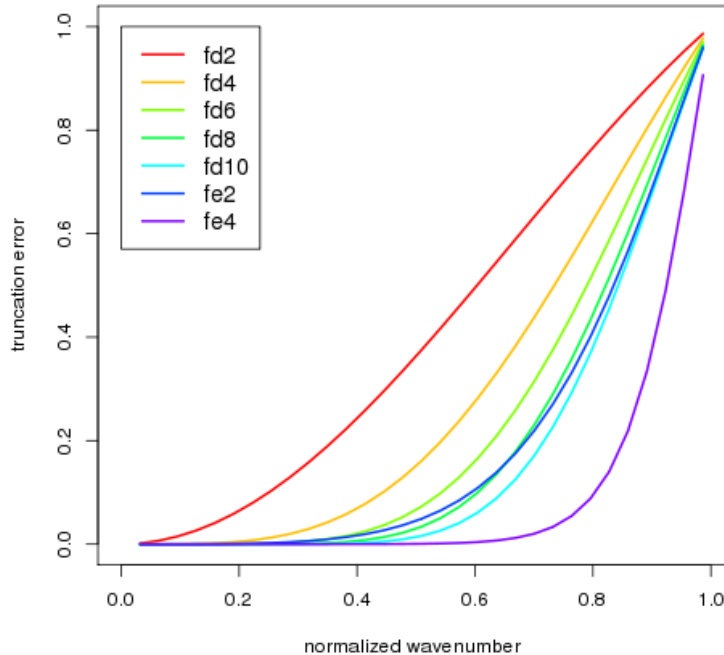
Order of splines

- vertical operators applied on a smooth function $\sin(\pi\eta)^3 \cos(\pi\eta)$
 - regular eta levels
 - saturation for higher orders
 - saturation for higher vertical resolutions
- <= rounding error and high number of operations



Theoretical accuracy of vertical operators – truncation error

First derivative operator



Taylor series expansion
(Staniforth, Wood):

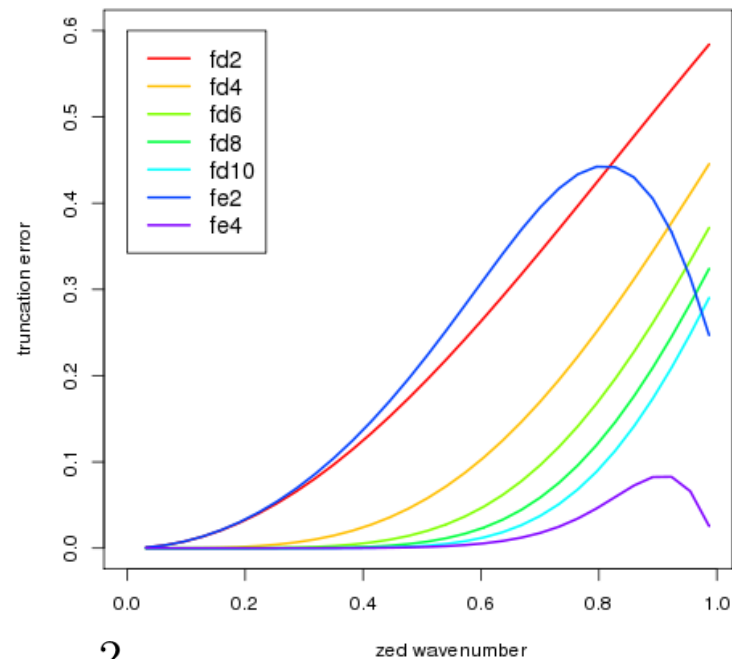
$$\text{FD 4}^{\text{th}} \text{ order} \approx 1 - \frac{x^4}{30} + \mathcal{O}(x^6)$$

$$\text{FE linear spline} \approx 1 - \frac{x^4}{180} + \mathcal{O}(x^6)$$

$$\text{FE cubic spline} \approx 1 - \frac{x^8}{151200} + \mathcal{O}(x^{10})$$

Theoretical accuracy of vertical operators – truncation error

Second derivative operator



FE linear spline $\approx -1 - \frac{x^2}{12} + \mathcal{O}(x^4)$

FE cubic spline $\approx -1 - \frac{x^6}{30240} + \mathcal{O}(x^8)$

Theoretical accuracy of vertical operators – truncation error

Confirmed by application on a smooth function $\sin(\pi\eta)^3 \cos(\pi\eta)$ satisfying operator's boundary conditions

- 140 regular levels
- cubic splines (spline order=4)
- MAE for central part (without boundary effects)

Operator	MAE	Order	
First derivative	8.6228e-12	8.0202	≈ 8
First derivative h->f	1.0362e-05	4.0008	≈ 4
Second derivative	4.4721e-08	6.0648	≈ 6
Integral	4.2197e-16	9.1289	≈ 8

ENO technique in SL interp.

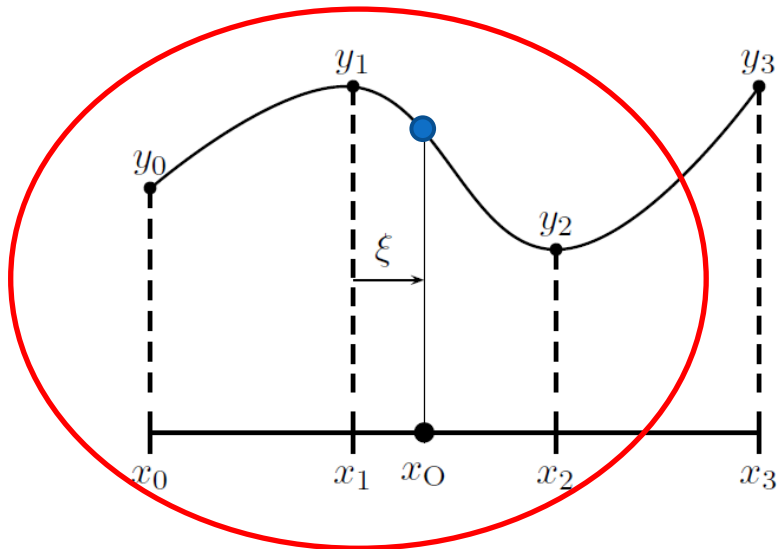
ENO (Essentially Non-Oscillatory)/WENO (Weighted ENO) techniques

Idea of Ján Mašek (inspired by literature):

- to explore alternative interpolators which are
- less overshooting than Lagrange polynomials
- more accurate than their quasi-monotonic versions

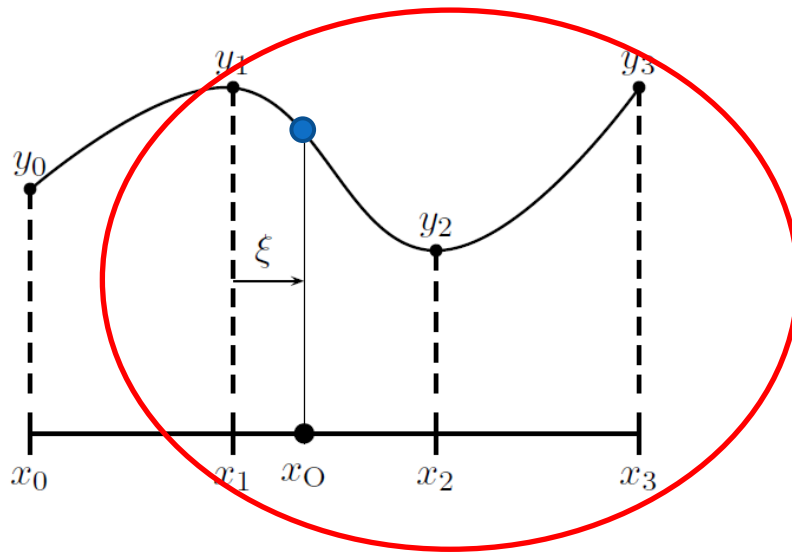
⇒ interpolation depends on the smoothness of the interpolated field

ENO technique in SL interp.



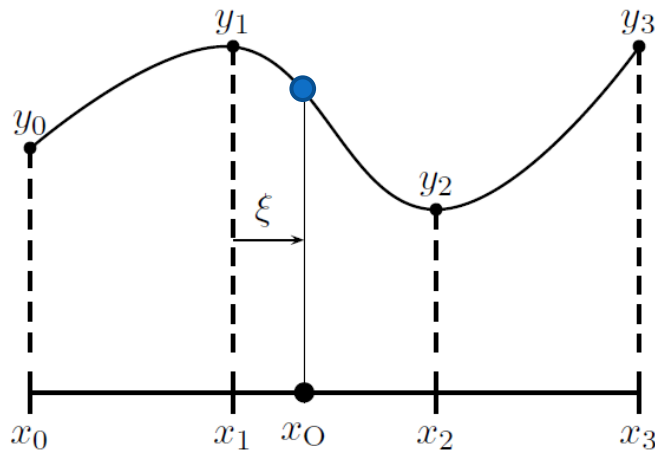
Second order interpolation scheme (quadratic) needs 3 points to find ●: we may choose the first stencil

ENO technique in SL interp.



Second order interpolation scheme (quadratic) needs 3 points to find \bullet :
or the second stencil

ENO technique in SL interp.



p_1, p_2 interpolated values on the first and second stencil

$$y = p_1 \cdot w_1 + p_2 \cdot w_2, \quad w_1 + w_2 = 1$$

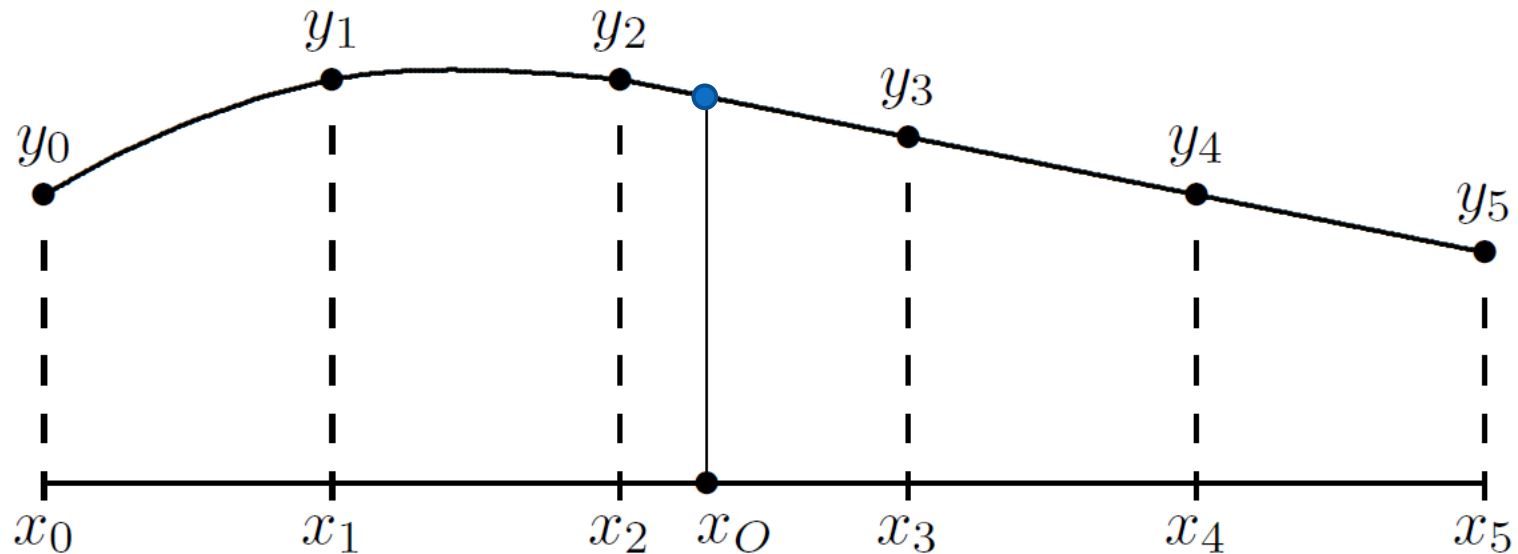
ENO chooses the smoothest solution ($w_1=0$ or $w_2=0$)

WENO weighted combination based on smoothness

Linear/cubic p_1, p_2 interpolated with linear/cubic Lagrange polynomial, weights depend on smoothness

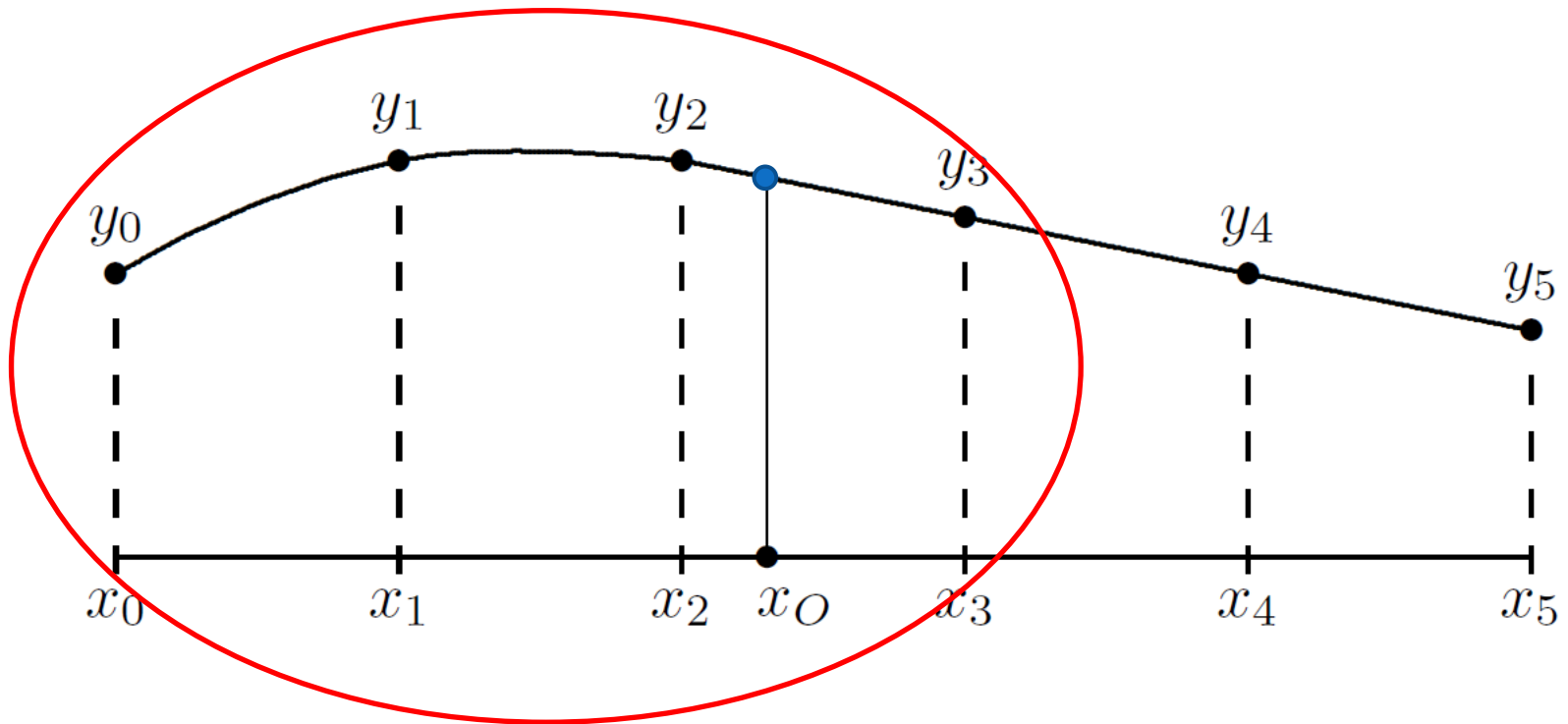
ENO technique in SL interp.

Third order interpolation scheme (cubic) needs 4 points to find ●:



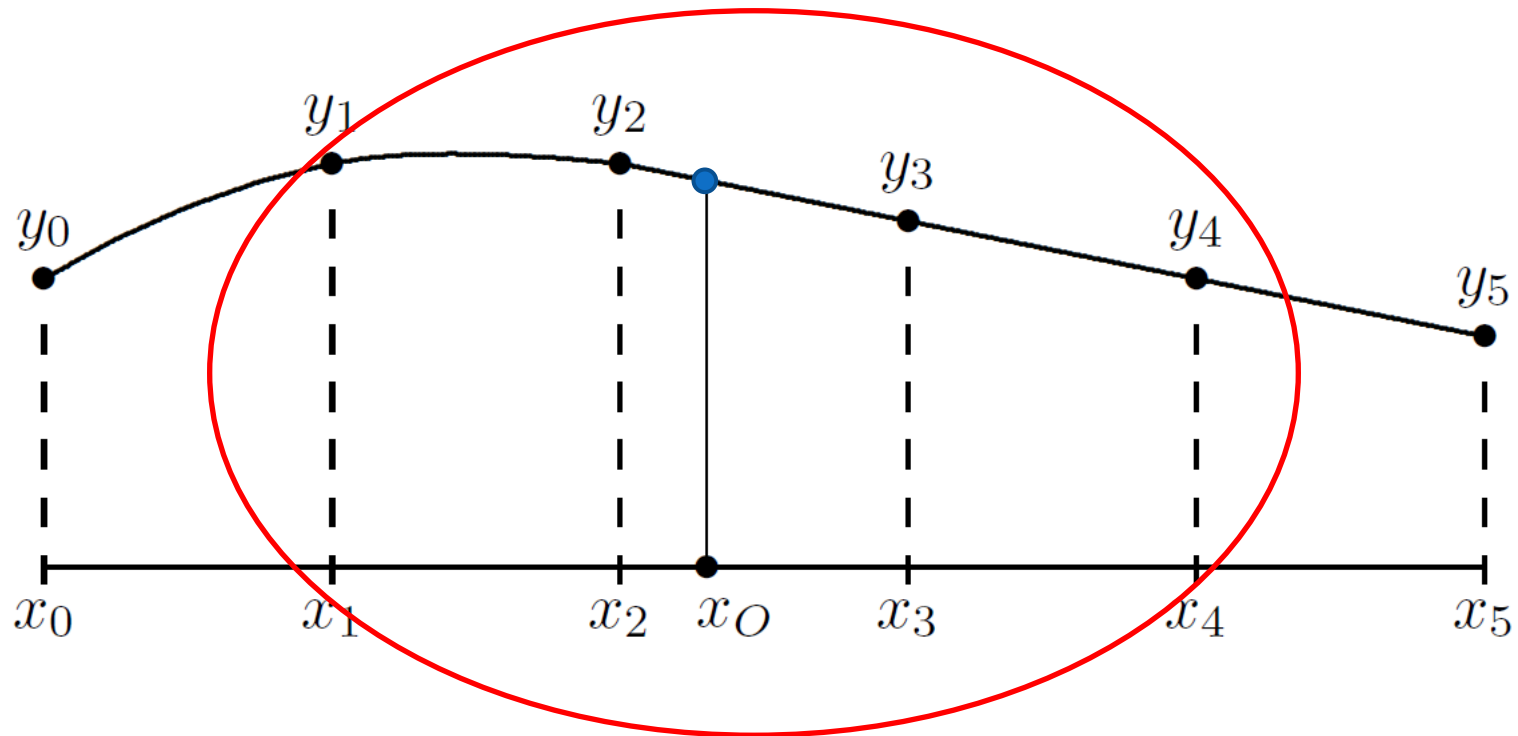
ENO technique in SL interp.

Third order interpolation scheme (cubic) needs 4 points to find ●:



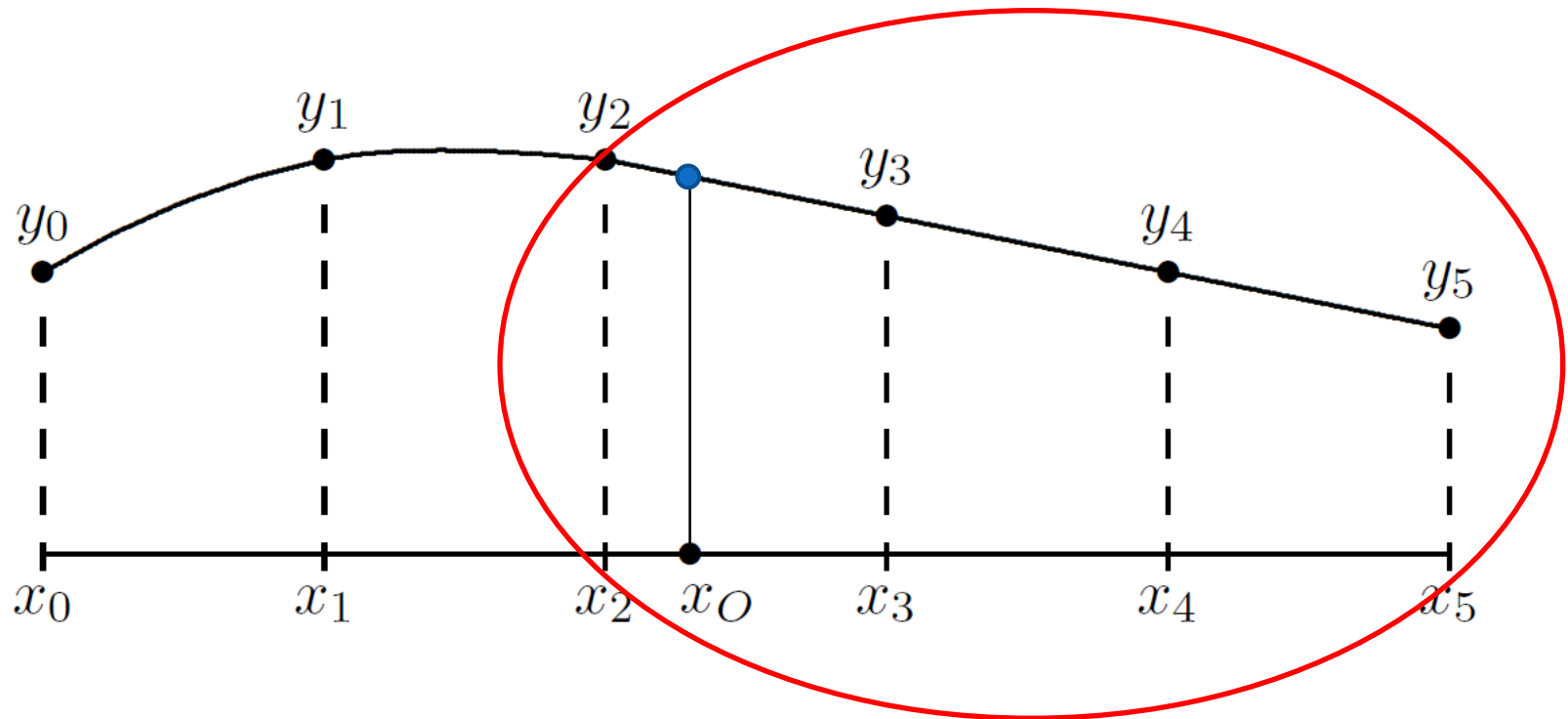
ENO technique in SL interp.

Third order interpolation scheme (cubic) needs 4 points to find ●:



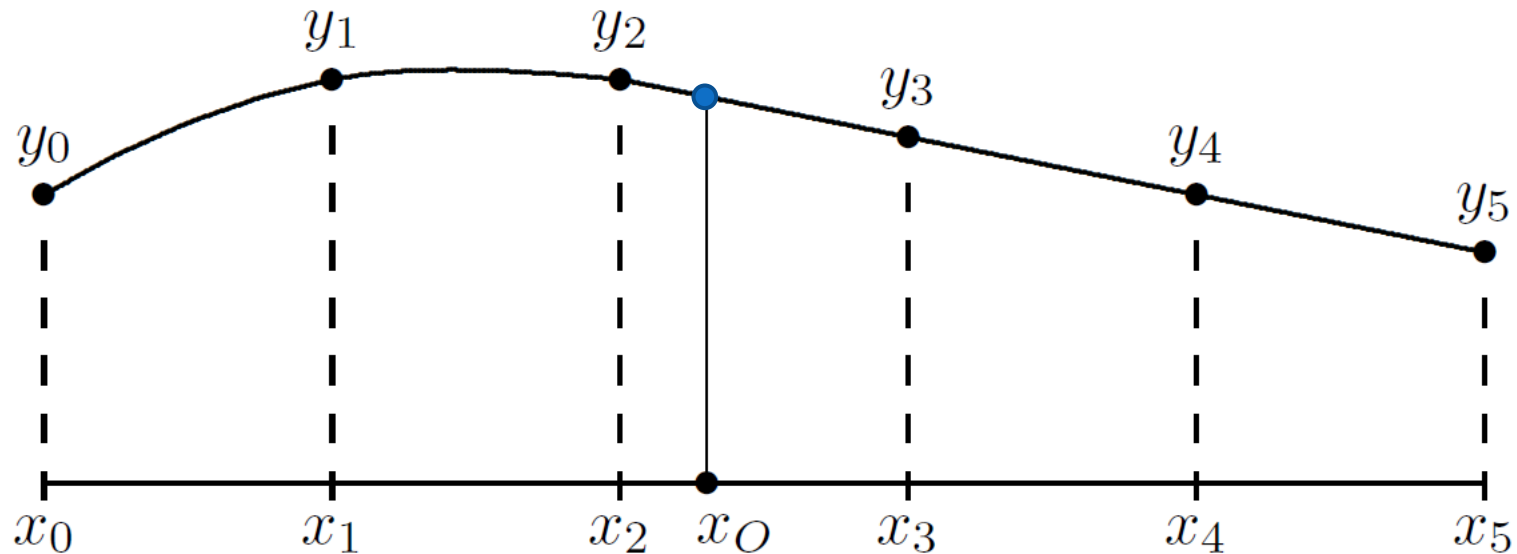
ENO technique in SL interp.

Third order interpolation scheme (cubic) needs 4 points to find ●:



ENO technique in SL interp.

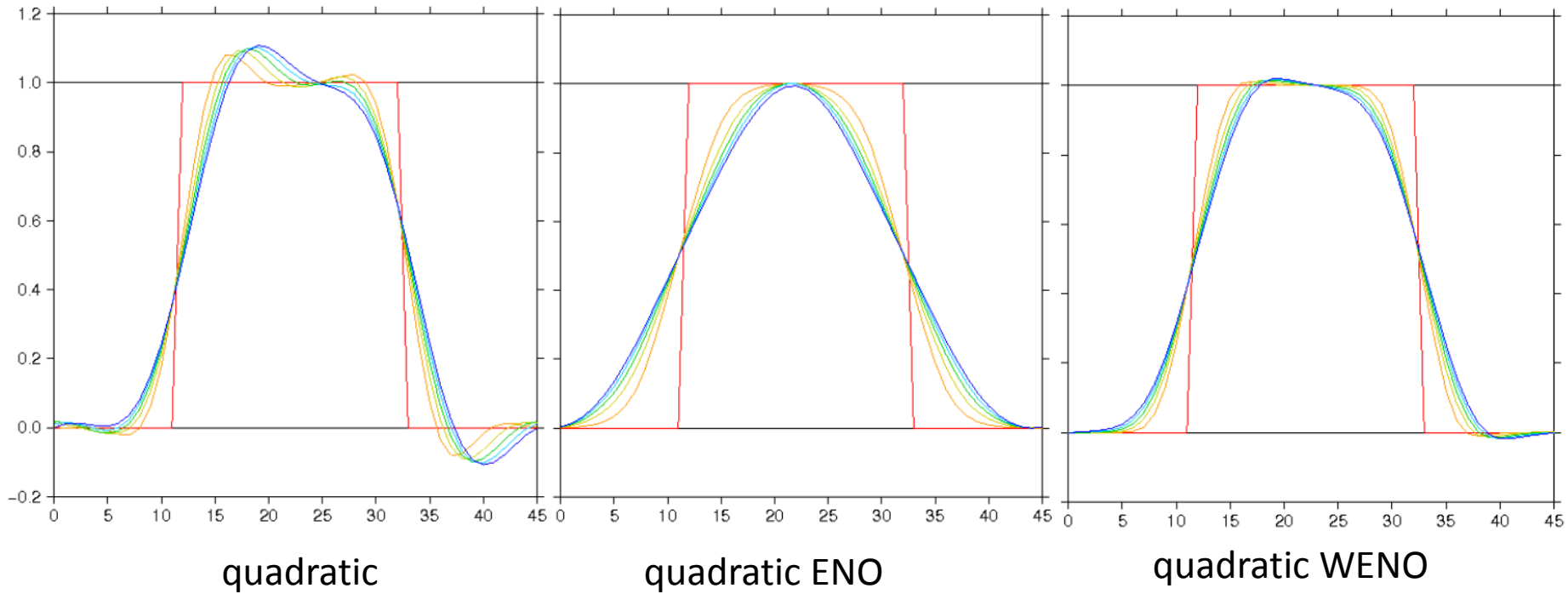
Third order interpolation scheme (cubic) needs 4 points to find ●:



=> **6 points stencil** needed for ENO/WENO interpolations !!!

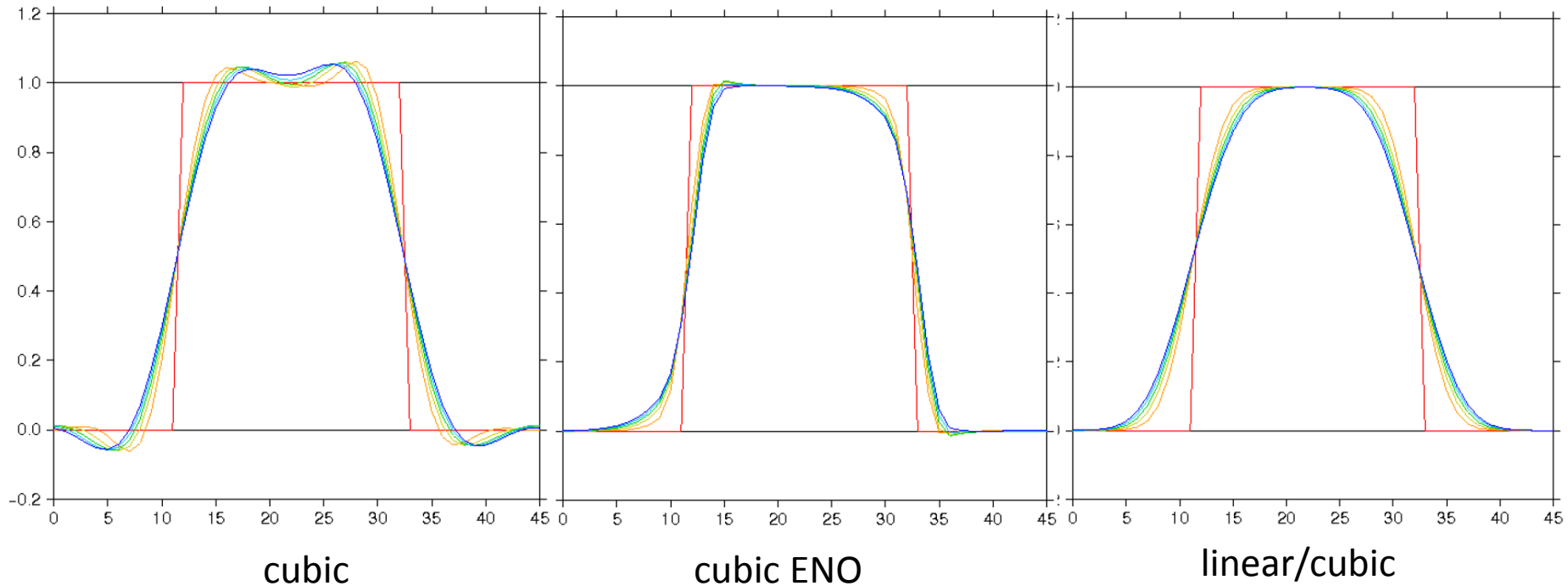
ENO technique in SL interp.

Toy model – 1D linear advection of rectangular pulse in a periodic domain (courtesy of Ján Mašek)



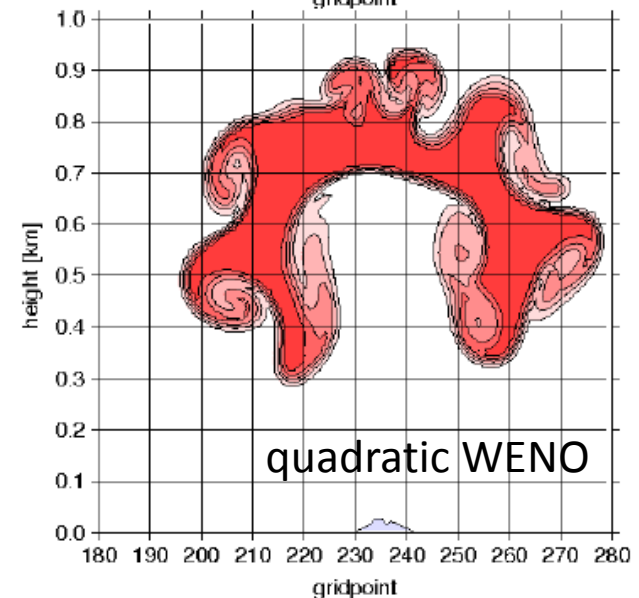
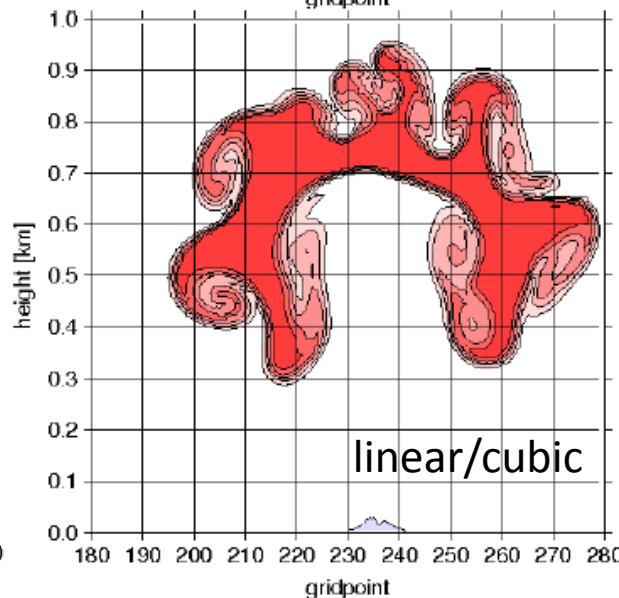
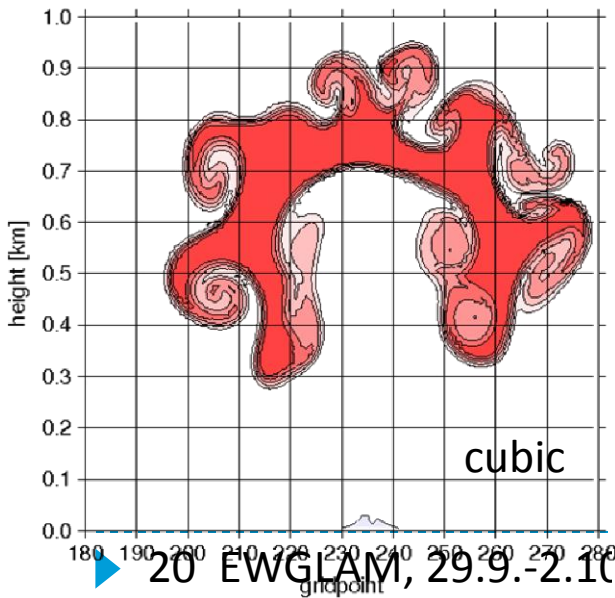
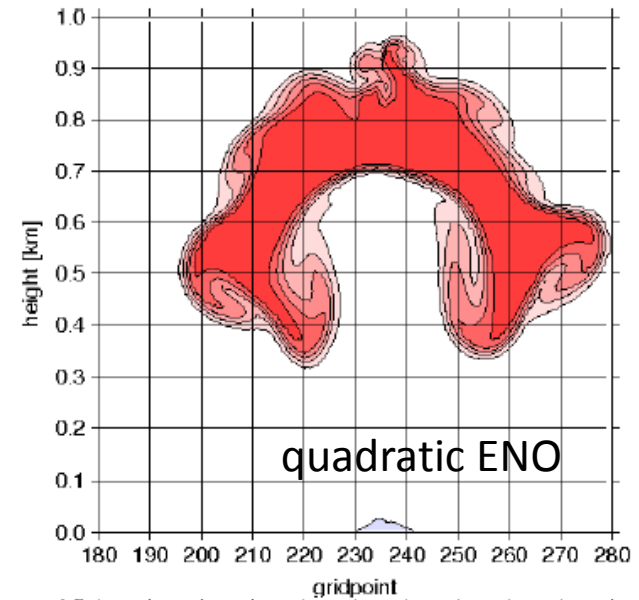
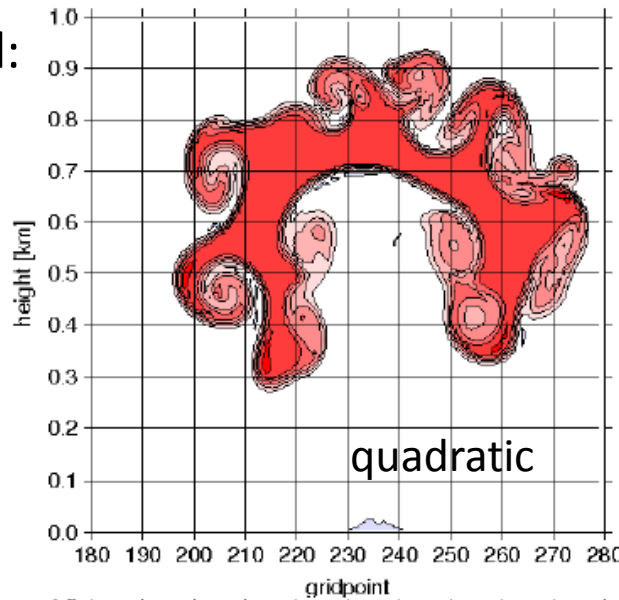
ENO technique in SL interp.

Toy model – 1D linear advection of rectangular pulse in a periodic domain (courtesy of Ján Mašek)



ENO technique in SL interp.

Robert's test in 2D model:
warm bubble (+0.5K)
in the field of potential
temperature (300K)
advected with the
wind speed 2m/s
(courtesy of A.Craciu)



Conclusions:

- Quadratic interpolator too smoothing to work well
 - Cubic ENO/WENO technique promising, but technically and computationally demanding (number of cubic interpolations increased from 7 to 21 !!!)
 - Combined linear/cubic interpolation may be easily tested and gives promising results – controlled damping depending on the interpolated field
- => 2 last points worth to be tested in 3D – planned for future work

SLHD tuning in ALARO 1km

Semi-Lagrangian horizontal diffusion (implemented to ALADIN by Filip Váňa)

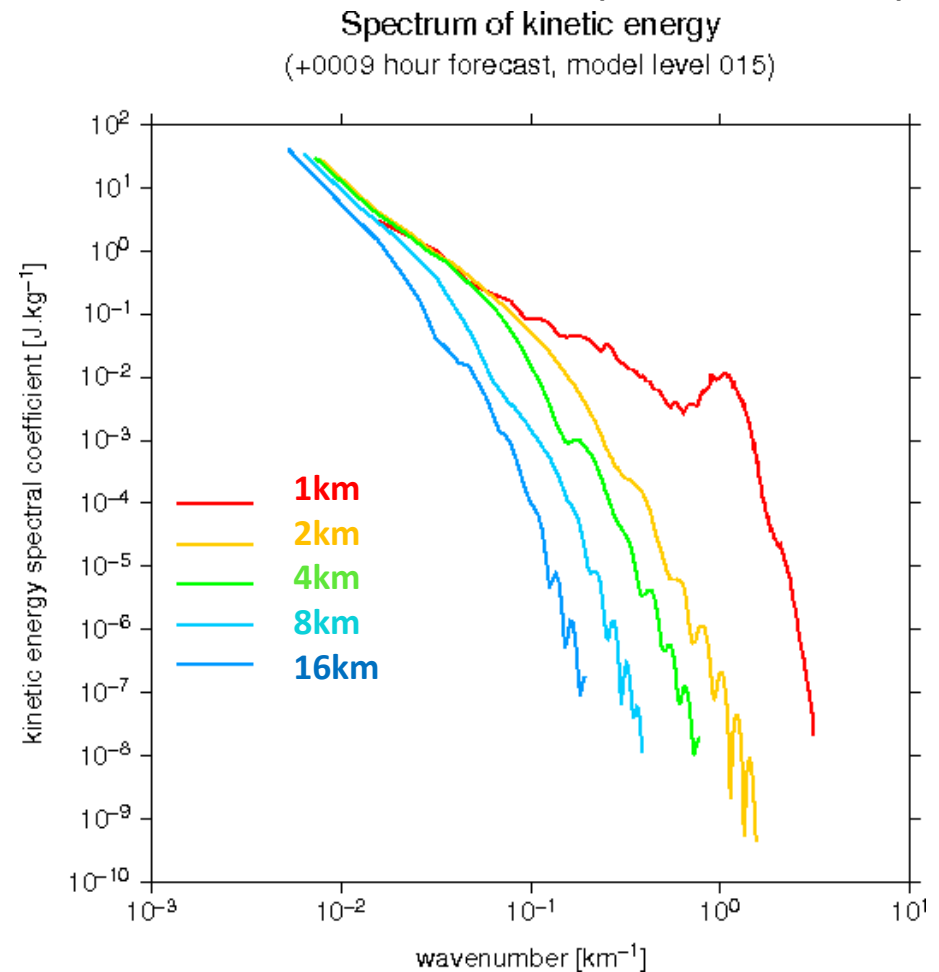
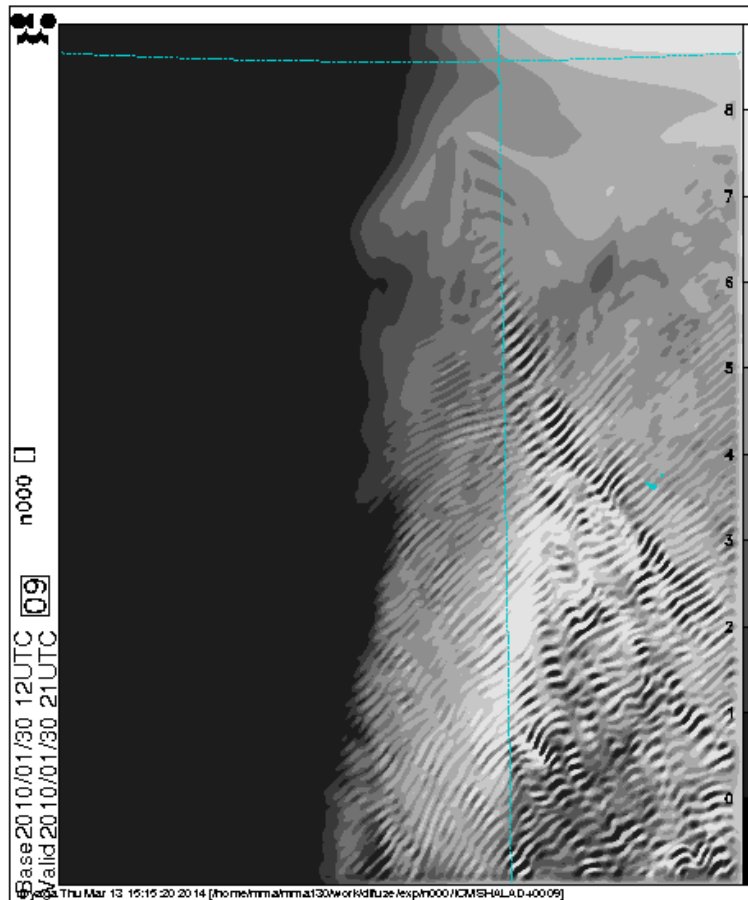
For the following purposes:

- 1) To represent the subgrid horizontal effect of **turbulence** and molecular dissipation
- 2) To **damp** the waves without predictive skills (to improve model scores)
- 3) To avoid the accumulation of energy at the end of the **model spectrum**

The diffusion coefficient for any diffused field is a function of **deformation** with several tunable parameters.

SLHD tuning in ALARO 1km

Grey zone experiment in the cascade of resolutions (R. Brožková)



SLHD tuning in ALARO 1km

Gridpoint part of SLHD

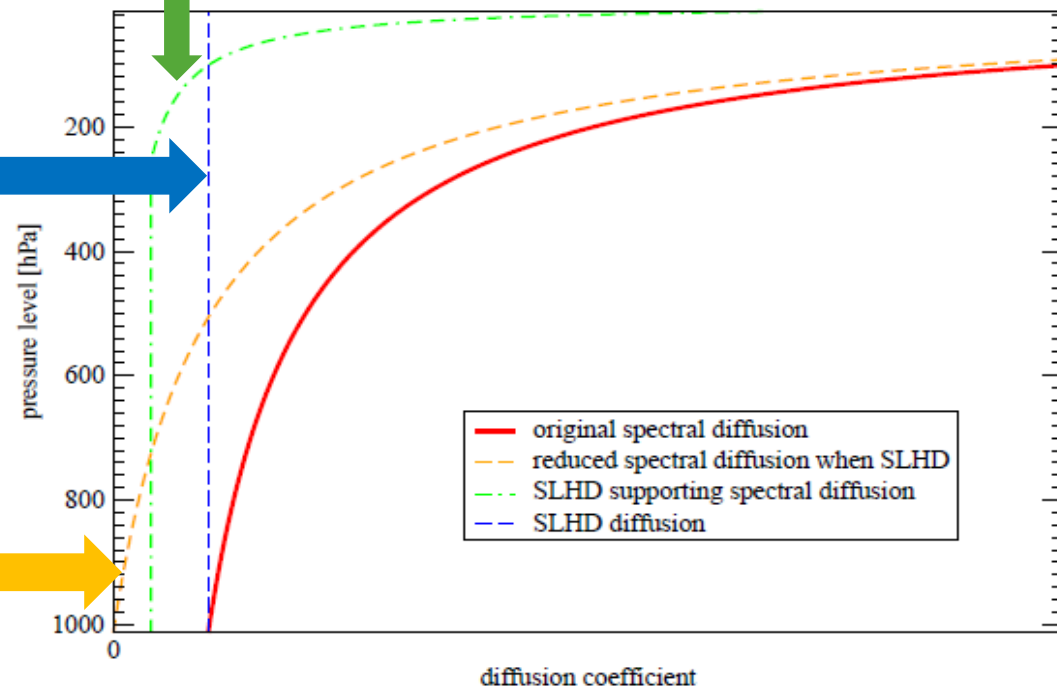
LSLHD_X = .T.
SLHDA0 = 0.25
SLHDB = 4.
SLHDD00 = 6.5E-05
ZSLHDP1 = 1.7 **adaptation on**
ZSLHDP3 = 0.6 **resolution**
YX_NL%LSLHD = .T.
SLHDEPSH = 0.016
SLHDEPSV = 0.016
SLHDKMAX = 6.
SLHDKMIN = -0.6

Reduced spectral diffusion - enhanced damping with height

RRDXTAU=123.
REXPDH = 2.
SDRED = 1.
RDAMPX = 0.,...,1. for X=T,Q,VOR,DIV,VD,PD
SLEVDPH = 0.1

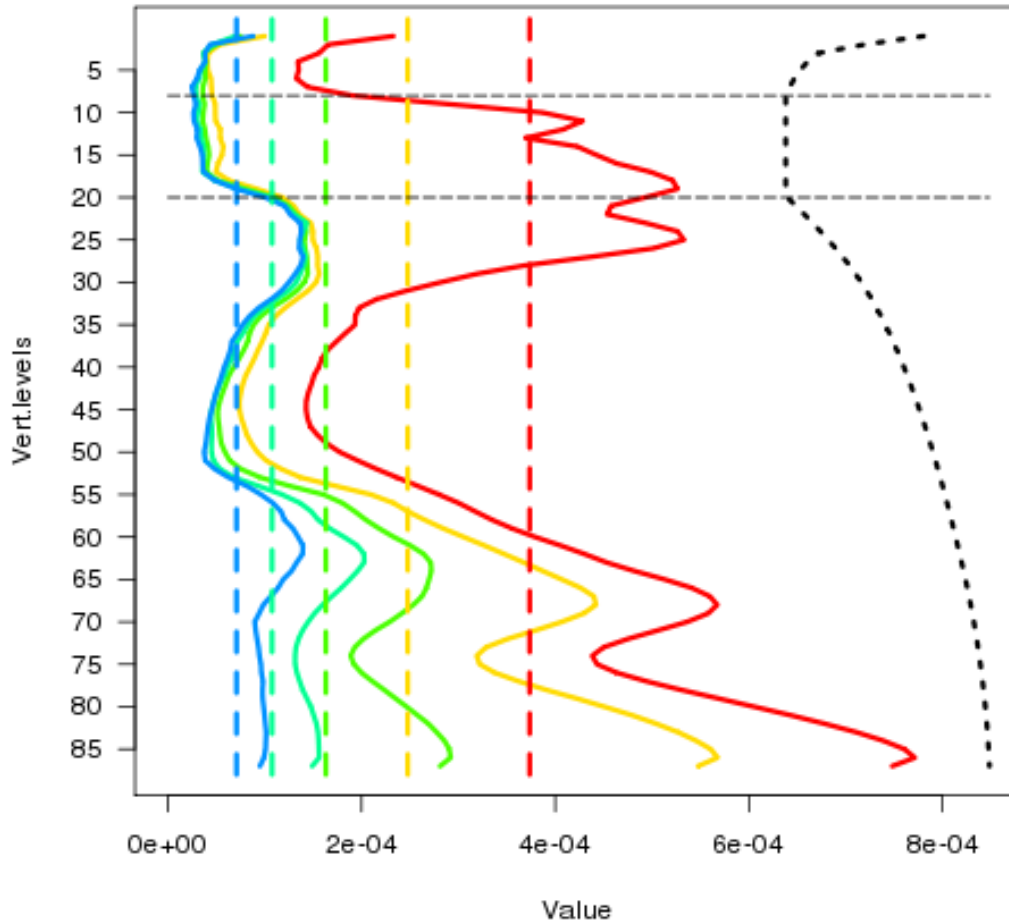
Supporting spectral diffusion – to control impact of orography

REXPDHS = 6.
RDAMPXS = 10.
SLEVDPHS = 1.



SLHD tuning in ALARO 1km

75th percentile, 25% points have bigger deformation



--- temperature profile

Horizontal resolution:

1km

2km

4km

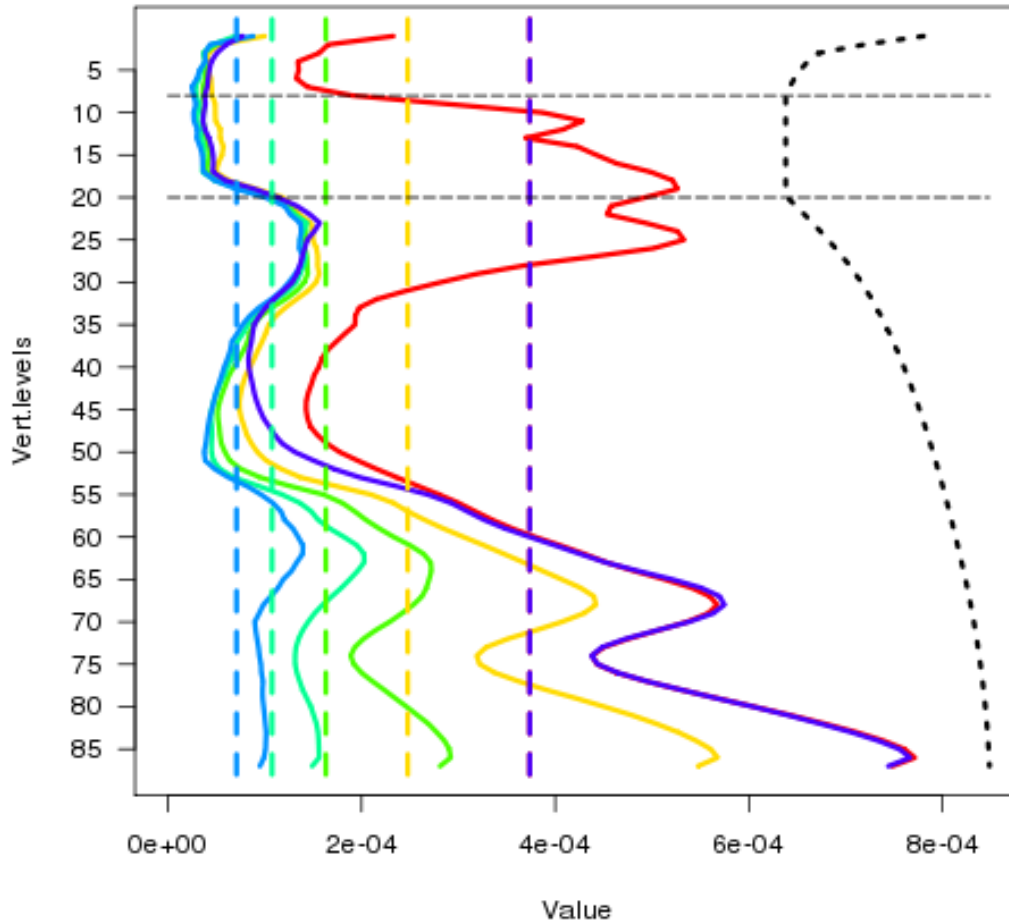
8km

16km

--- characteristic deformation
based on resolution

SLHD tuning in ALARO 1km

75th percentile, 25% points have bigger deformation



--- temperature profile

Horizontal resolution:

1km tuned

1km

2km

4km

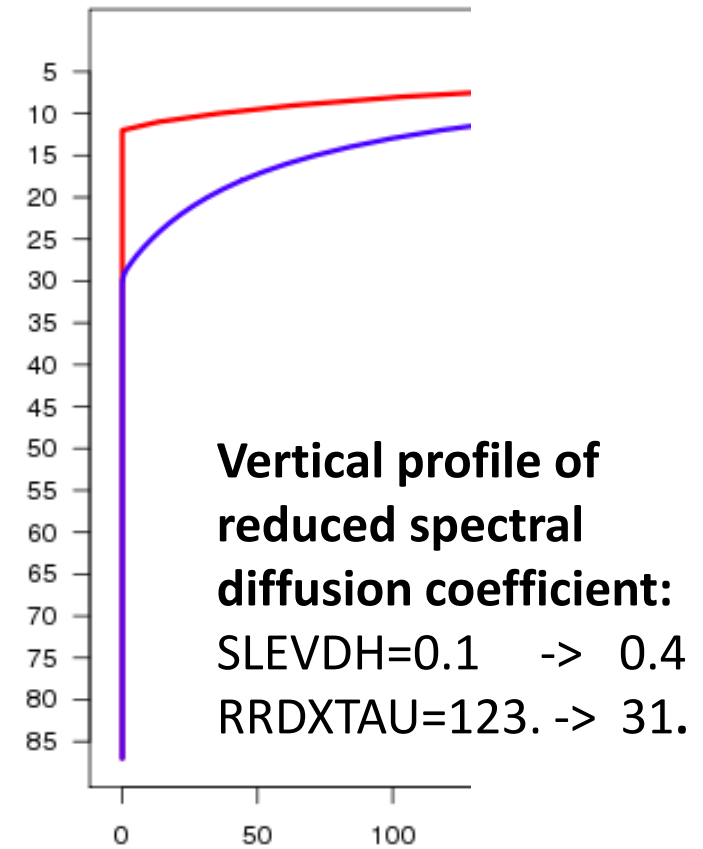
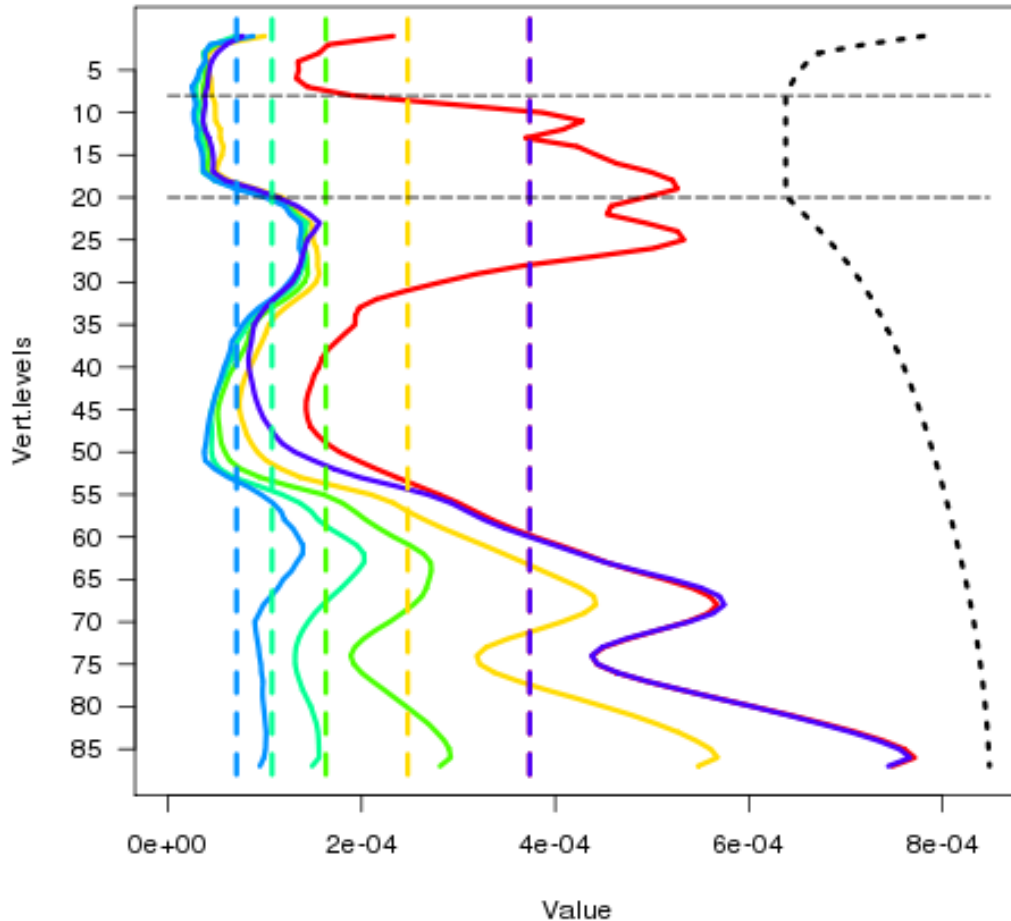
8km

16km

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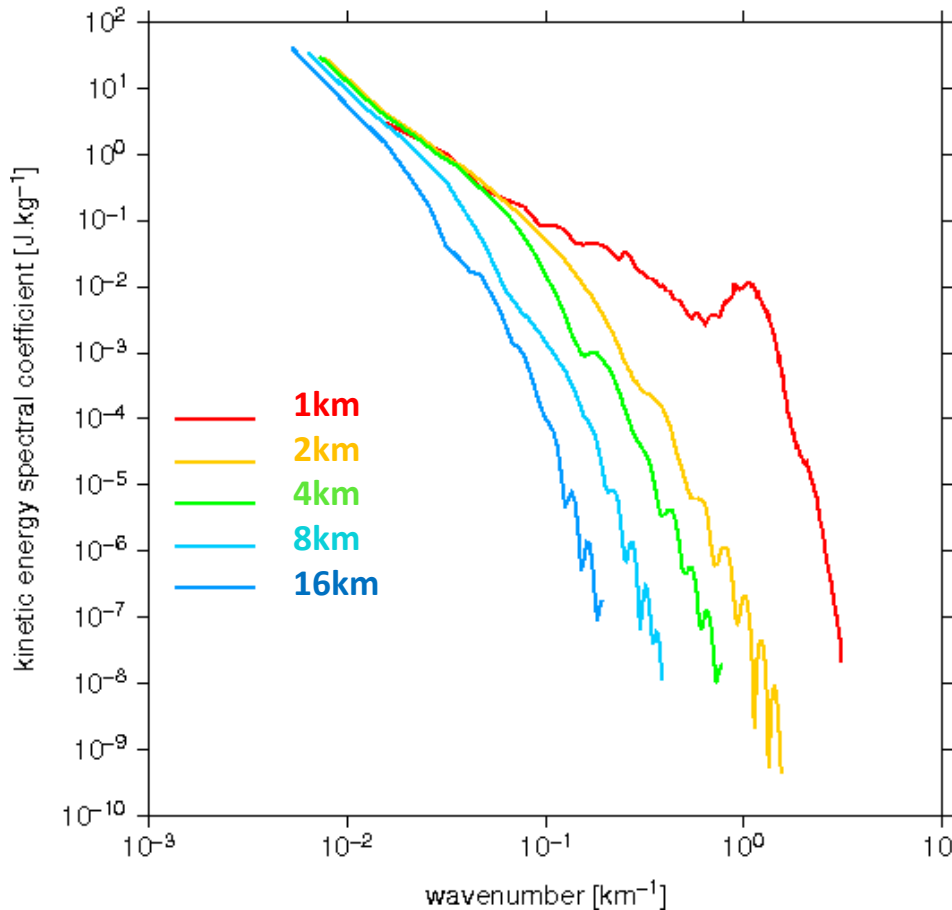
SLHD tuning in ALARO 1km

75th percentile, 25% points have bigger deformation

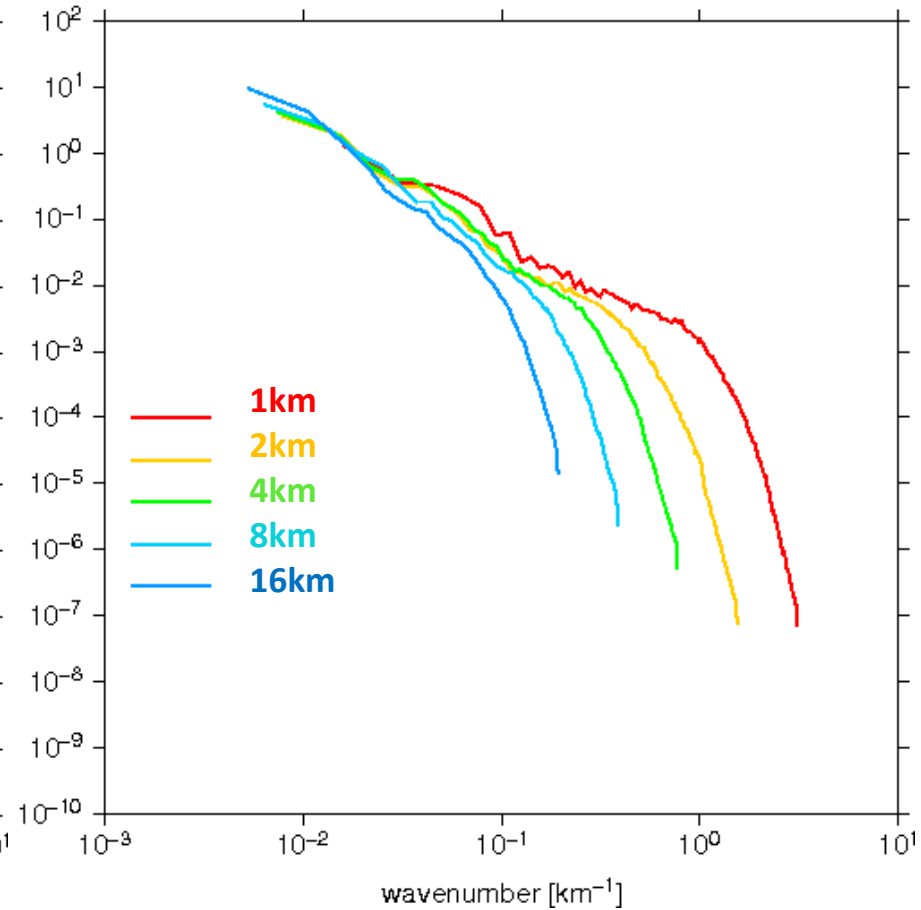


SLHD tuning in ALARO 1km

Spectrum of kinetic energy
(+0009 hour forecast, model level 015)

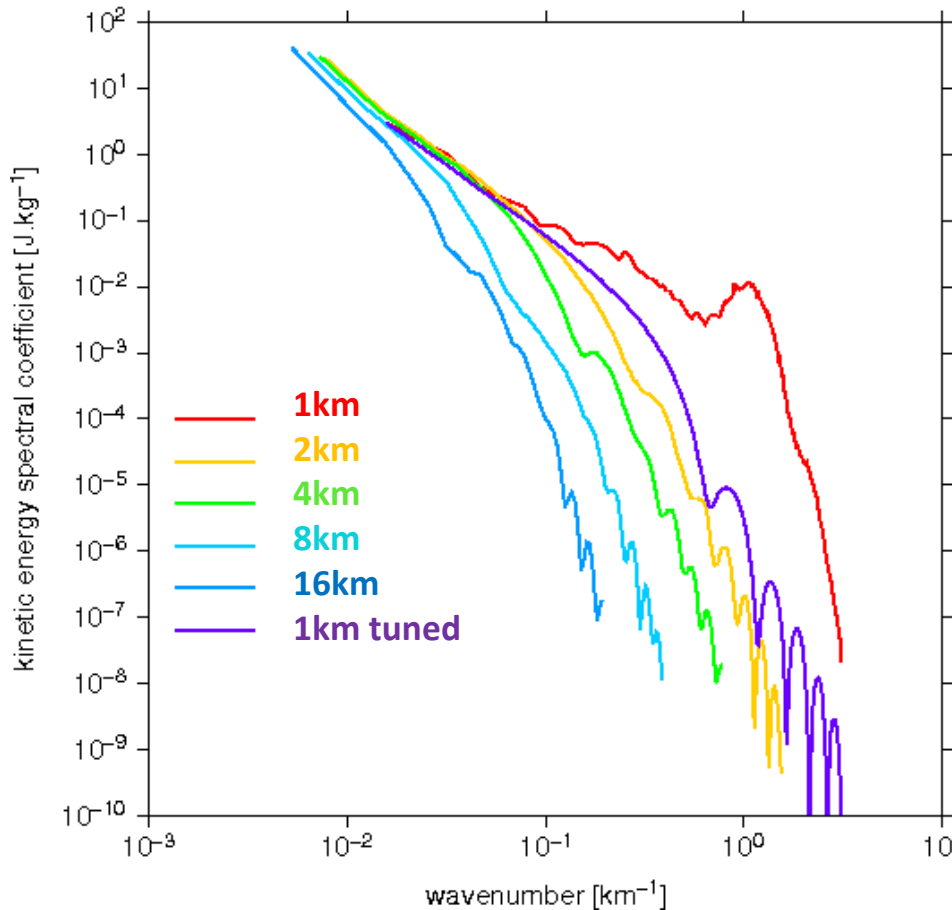


Spectrum of kinetic energy
(+0009 hour forecast, model level 065)

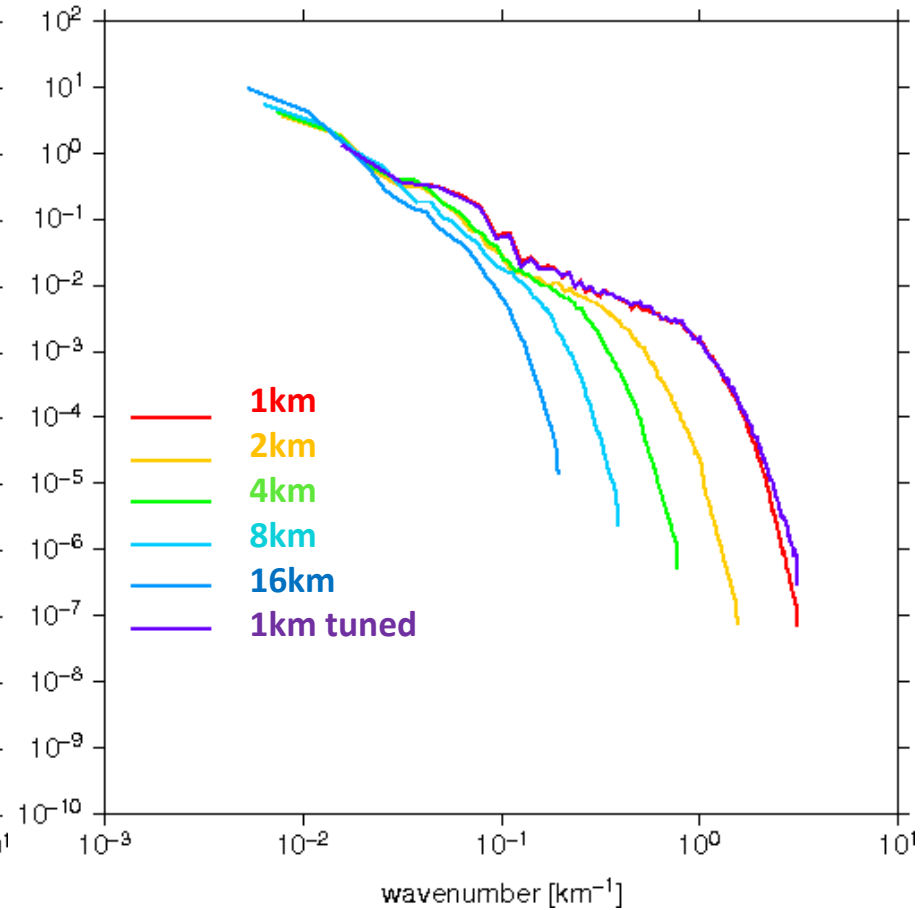


SLHD tuning in ALARO 1km

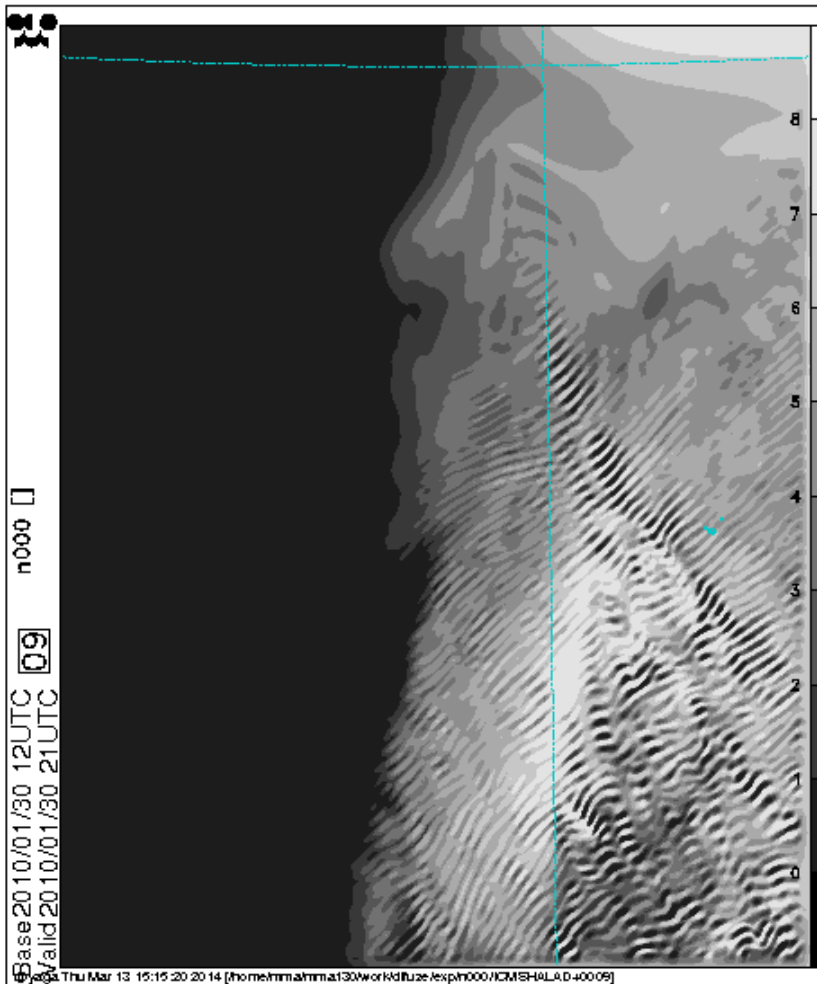
Spectrum of kinetic energy
(+0009 hour forecast, model level 015)



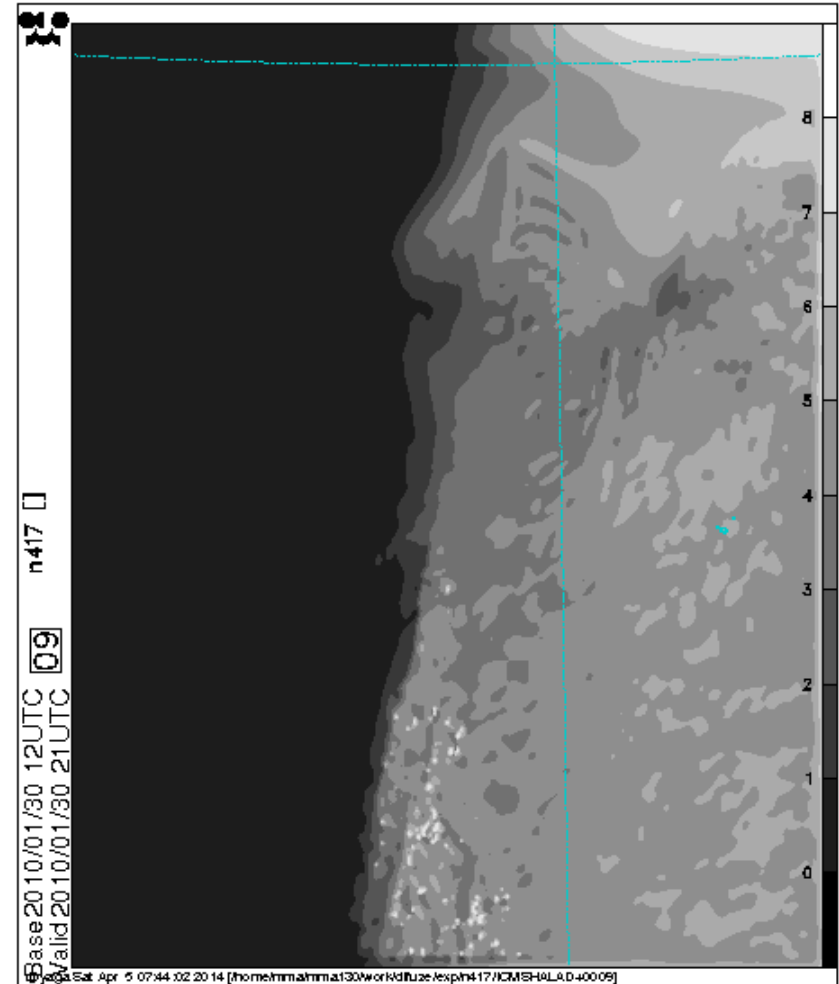
Spectrum of kinetic energy
(+0009 hour forecast, model level 065)



SLHD tuning in ALARO 1km



High level cloudiness: reference



tuned with SLEVDH=0.4,RRDXTAU=0.31

**Thank you for
your attention !**

**Ich danke Ihnen
für Ihre
Aufmerksamkeit!**