Regional Cooperation for
Limited Area Modeling in Central Europe

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LACE news in dynamics - finite elements in vertical discretization of ALADIN NH

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Finite elements in vertical in ALADIN-NH

- Being solved since 2006 (ECMWF, ALADIN, LACE, HIRLAM)
- As an enhancement of FE used in hydrostatic model (Untch, Hortal, 2003, for global model IFS, adapted to LAM ALADIN) => keep all choices: SI time scheme, SL advection, mass based vertical coordinate & get similar stability
- Hydrostatic model – only integral vertical operators appear
- NH in height based vertical coordinate (Juan Simarro, Mariano Hortal)

  – only derivative vert.operators
- NH in mass based vertical coordinate => both, **integral and derivative** vertical operators appear
- In continuous case: vertical operators satisfy **2 conditions** (C1,C2)

  In discretized case: NOT SATISFIED
- VFD: designed to SATISFY C1 & approximation to ALMOST SATISFY C2
- VFE: iterative stationary method to solve the implicit problem – believed non converging, CONVERGES with new vertical discretization in real cases

Finite elements in vertical in ALADIN-NH

2012 – new implementation with several improvements:

- general order of the B-splines
- variation diminishing approach to define vertical coordinate eta
- new definition of knots (centripetal method)
- imposed top and bottom boundary conditions on all the vertical operators, various for distinct terms

Testing:

1) Stability
2) Robustness
3) Convergence of the SI solver
4) Speed of the convergence of the SI solver
5) Accuracy

Prognostic variables

Different in GP and SP space for stability reasons

Grid-point space $\vec{V}, T, q_s = \ln(\pi_s), \hat{q} = \ln\left(\frac{p}{\pi}\right), gw$

on half levels

Spectral space $D, \zeta, T, q_s, \hat{q}, d = \frac{p}{mRT}\frac{\partial gw}{\partial \eta} + \frac{p}{mRT}\nabla\phi\frac{\partial \vec{V}}{\partial \eta}$

$\Rightarrow$ transformations gw $\leftrightarrow$ d needed

Vertical coordinate – mass based one $\pi(\eta) = A(\eta) + B(\eta)\pi_s$

Interpolation with B-spline curve

PROBLEM: to interpolate the data points $(\pi_i, f(\pi_i))$ known on full levels and material boundaries with parametric B-spline curve

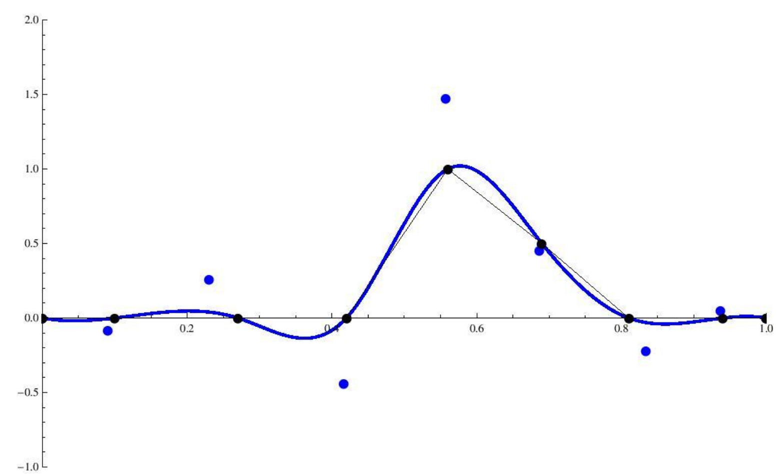
$$S(\eta, f(\eta)) = \sum_{i=0}^{L+1} (\hat{\eta}_i, \hat{f}_i) \cdot \mathbf{a_i}(\eta)$$

STEPS:

1) Define knots to construct B-spline basis $\mathbf{a}_i$, use deBoor's algorithm

2) Determine value of parameter eta in data points from known π_i

3) Determine spline curve control points $\hat{f} = A^{-1} f$



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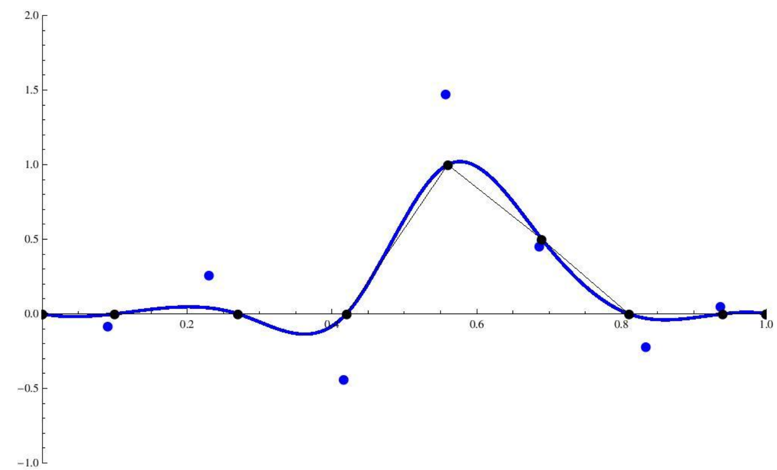

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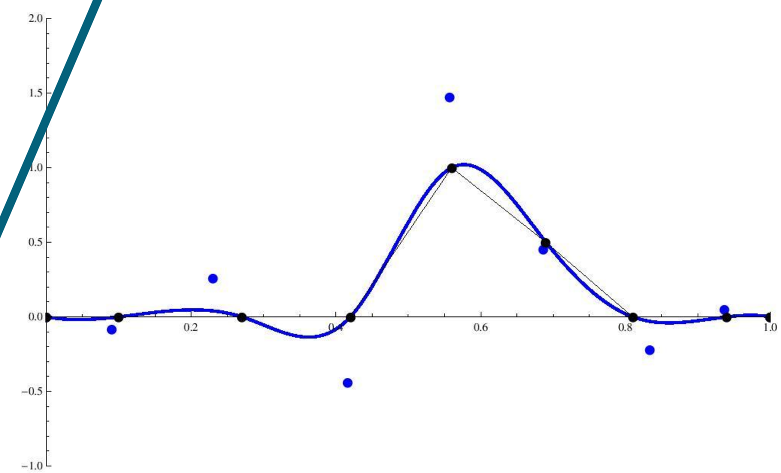


Finite element process to define vertical operator $\Psi(f(\eta)) = g(\eta)$

To interpolate with

$$\sum_{i=0}^{L+1} \langle \hat{\eta}_i, \hat{f}_i \rangle \cdot \Psi(\mathbf{a}_i(\eta)) = \sum_{i=0}^{L+1} \langle \hat{\eta}_i, \hat{g}_i \rangle \cdot \mathbf{b}_i(\eta),$$

Use **mean weighted residual approach** with weighting functions $\mathbf{a}_i$

$$\sum_{i=0}^{L+1} \left(\int_0^1 \Psi(\mathbf{a}_i(\eta)) \mathbf{a}_j(\eta) d\eta \right) \langle \hat{\eta}_i, \hat{f}_i \rangle = \sum_{i=0}^{L+1} \left(\int_0^1 \mathbf{b}_i(\eta) \mathbf{a}_j(\eta) d\eta \right) \langle \hat{\eta}_i, \hat{g}_i \rangle, \text{ i.e. } S\,.\,\hat{f} = M.\,\hat{g}$$

Evaluate the value of vertical operator at locations η_k

$$g(\eta_k) = \sum_{i=0}^{L+1} \mathbf{b}_i(\eta_k) \langle \hat{\eta}_i, \hat{g}_i \rangle, \text{ i.e. } g = B\hat{g}$$

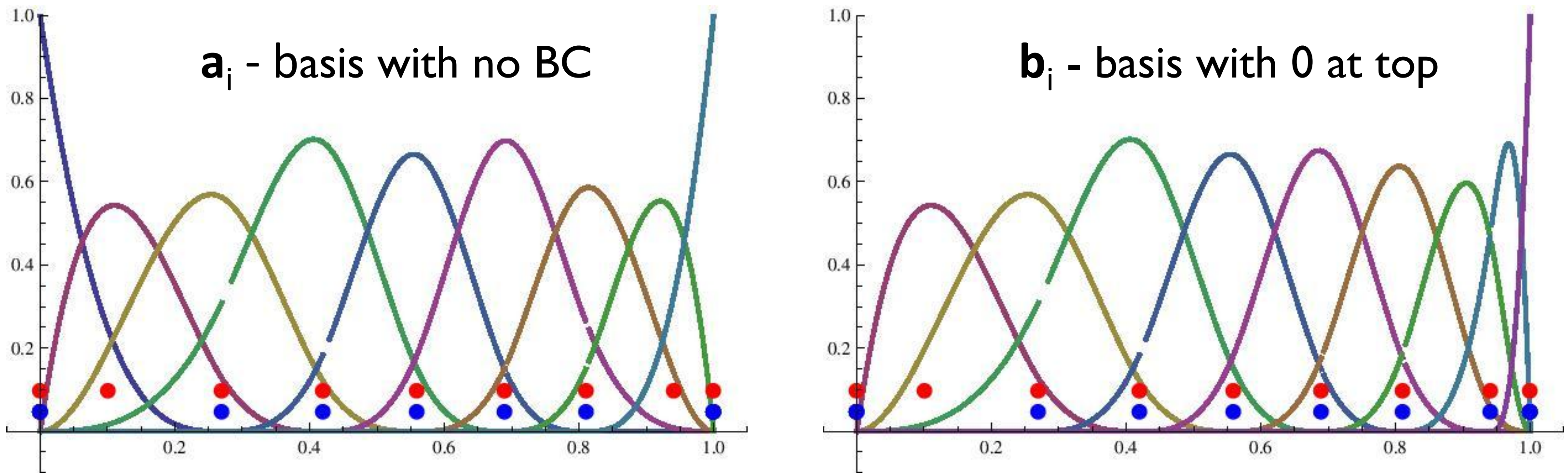
We represent vertical operator with one single matrix $B\ M^{-1} S A^{-1}$.

Newton or Dirichlet boundary conditions

are imposed on material boundaries:

1) On input quantity directly (prescribed values of f or $\frac{\partial f}{\partial \eta}$ at model top and model bottom)

2) On output quantity by adjusting the basis functions $\mathbf{b}_i$



Linear system

$$\frac{\partial D}{\partial t} = R\mathcal{G}^* \triangle T + RT^*(\mathcal{G}^* - 1)\triangle \hat{q} - RT^* \triangle q_s - \triangle \phi_s,$$

$$\frac{\partial d}{\partial t} = -\frac{g^2}{RT_e^*}\mathcal{L}^* \hat{q},$$

$$\frac{\partial T}{\partial t} = -\frac{RT^*}{C_v}(D + d),$$

$$\frac{\partial \hat{q}}{\partial t} = S^* D - \frac{C_p}{C_v}(D + d),$$

$$\frac{\partial q_s}{\partial t} = -\mathcal{N}^* D,$$

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Integral VFE operators

From model top $(\mathbf{K}\psi)_\eta = \int_0^\eta \psi d\eta$

From model surface $(\mathbf{P}\psi)_\eta = (\mathbf{K}\psi)_1 - (\mathbf{K}\psi)_\eta = \int_\eta^1 \psi d\eta$

with boundary conditions

$$\text{Input: } \left(\frac{\partial\psi}{\partial\eta}\right)_0 = 0,\ \left(\frac{\partial\psi}{\partial\eta}\right)_{L+1} = 0$$

$$\text{Output: } (\mathbf{K}\psi)_0 = 0$$

Linear operators

$$\begin{aligned}
\mathcal{S}^*\psi(\eta_l) &\approx \frac{1}{\pi_l^*}(\mathbf{K}m^*\psi)_l \\
\mathcal{G}^*\psi(\eta_l) &\approx (\mathbf{P}\frac{m^*}{\pi^*}\psi)_l \\
\mathcal{N}^*\psi(\eta_l) &\approx (\mathcal{S}^*\psi)_{L+1}
\end{aligned}$$

Derivative VFE operators

Laplacian term
$$\begin{aligned}\mathcal{L}^*\psi &= \frac{1}{m^*}\frac{\partial}{\partial\eta}\Big(\frac{\pi^{*2}}{m^*}\Big)\frac{\partial\psi}{\partial\eta} + \Big(\frac{\pi^*}{m^*}\Big)^2\frac{\partial^2\psi}{\partial\eta^2} \\ &= \frac{1}{m^*}\mathbf{D_1}\Big(\frac{\pi^{*2}}{m^*}\Big)\mathbf{D_2}\psi + \Big(\frac{\pi^*}{m^*}^2\Big)\mathbf{DD}\psi\end{aligned}$$

with boundary conditions

Operator:	Input:	Output:
$\mathbf{D_1}\psi$	$\psi_0 = 0, \psi_{L+1} = \psi_L$	-
$\mathbf{D_2}\psi$	$\psi_0 = 0, \left(\frac{\partial\psi}{\partial\eta}\right)_{L+1} = 0$	$(\mathbf{D_2}\psi)_{L+1} = 0$
$\mathbf{DD}\psi$	$\psi_0 = 0, \left(\frac{\partial\psi}{\partial\eta}\right)_{L+1} = 0$	$(\mathbf{DD}\psi)_{L+1} = 0$

Implicit problem

In continuous case vertical operators satisfy 2 conditions:

C_1: $$\mathcal{C} = -\mathcal{G}^*\mathcal{S}^* + \mathcal{G}^* + \mathcal{S}^* - \mathcal{N}^* = 0$$

C_2: $$\mathcal{L}^*\left(\mathcal{S}^*\mathcal{G}^* - \frac{C_p}{C_v}\mathcal{S}^* - \frac{C_p}{C_v}\mathcal{G}^*\right) = \frac{R}{C_v}$$

In VFE discretization the conditions are NOT FULFILLED !

Implicit problem in 2L dimension for C $\neq$ 0: full elimination of variables not possible

$$\begin{pmatrix} \mathbb{H} & \mathbb{F}\mathbb{C} \\ -\mathbb{B} & \mathbb{A}+\mathbb{C} \end{pmatrix} \begin{pmatrix} d \\ D \end{pmatrix} = \begin{pmatrix} \mathbb{A} & \mathbb{F} \\ 0 & \mathbb{I} \end{pmatrix} \begin{pmatrix} d^\bullet \\ D^\bullet \end{pmatrix}$$

Stationary iterative method: Predictor as if C=0 => full elimination

$$\begin{pmatrix} \mathbb{H} & 0 \\ -\mathbb{B} & \mathbb{A} \end{pmatrix} \begin{pmatrix} d \\ D \end{pmatrix}^{(0)} = \begin{pmatrix} \mathbb{A} & \mathbb{F} \\ 0 & \mathbb{I} \end{pmatrix} \begin{pmatrix} d^\bullet \\ D^\bullet \end{pmatrix}$$

Corrector with C on the RHS => full elimination

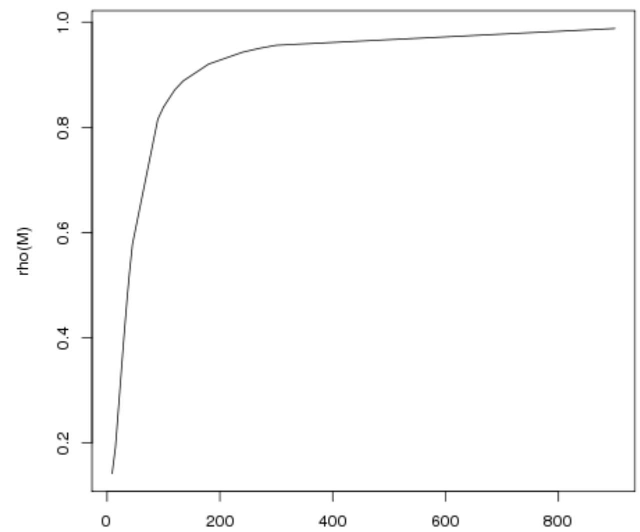
$$\begin{pmatrix} \mathbb{H} & 0 \\ -\mathbb{B} & \mathbb{A} \end{pmatrix} \begin{pmatrix} d \\ D \end{pmatrix}^{(i+1)} = \begin{pmatrix} 0 & -\mathbb{F}\mathbb{C} \\ 0 & -\mathbb{C} \end{pmatrix} \begin{pmatrix} d \\ D \end{pmatrix}^{(i)} + \begin{pmatrix} \mathbb{A} & \mathbb{F} \\ 0 & \mathbb{I} \end{pmatrix} \begin{pmatrix} d^\bullet \\ D^\bullet \end{pmatrix}$$

Convergence of the iterative procedure

- Depends on discretized vertical operators used
- Believed non-converging (shown with old vertical FE operators)
- With the new VFE operators converges in real cases (test realized in the setup through eigenvalues of a given iteration matrix)

< 1 convergence
> 1 non-convergence

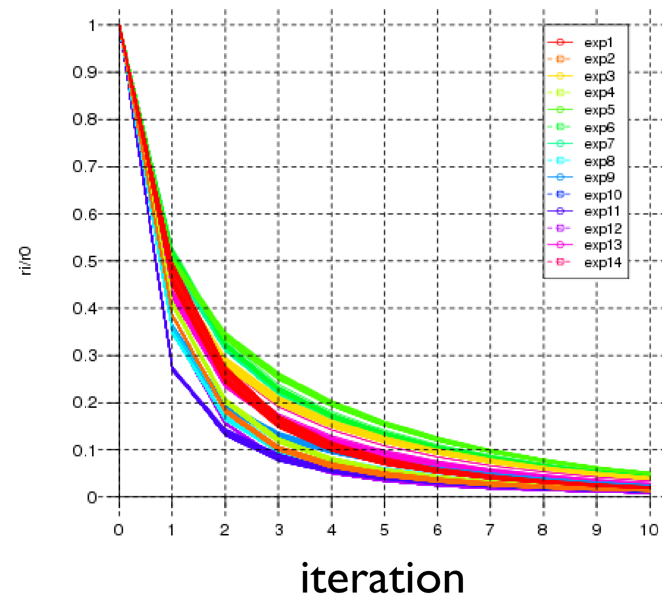
Real case experiment:



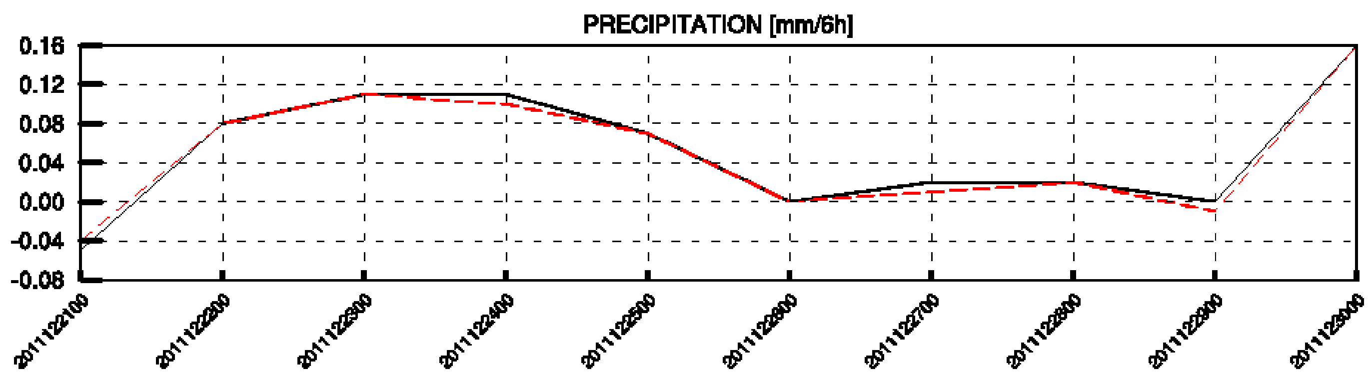
Timestep used

Speed of convergence

- Satisfactory in real cases with stability achieved
- Objective verification scores calculated for 10 days period => one iteration enough, the results are undistinguishable in all parameters except precipitation



1 iteration
8 iterations



Non-linear system

Continuous

$$\frac{d\vec{V}}{dt} = -\frac{RT}{p}\nabla p - \left(\frac{1}{m}\cdot\frac{\partial p}{\partial \eta}\right)\cdot\nabla\phi$$

$$\frac{dgw}{dt} = g^2\cdot\frac{1}{m}\cdot\frac{\partial(p-\pi)}{\partial\eta}$$

$$\frac{dT}{dt} = -\frac{RT}{C_v}D_3$$

$$\frac{d\hat{q}}{dt} = -\frac{C_p}{C_v}D_3 - \frac{\omega}{\pi}$$

$$\frac{\partial q_s}{\partial t} = -\frac{1}{\pi_s}\cdot\int_0^1 (m\vec{V})d\eta$$

Discretized

$$\left(\frac{1}{m}\cdot\frac{\partial p}{\partial\eta}\right)_l = \frac{p_l}{\pi_l} + \left(\frac{p}{m}\mathbf{D}_1\hat{q}\right)_l$$

$$\left(\frac{1}{m}\frac{\partial(p-\pi)}{\partial\eta}\right)_{\tilde{l}} = \left(\frac{1}{m}\mathbf{D}_h(p-\pi)\right)_{\tilde{l}}$$

$$(\nabla\phi)_l = \nabla\phi_s + \left[\mathbf{P}\nabla\left(\frac{mRT}{p}\right)\right]_l$$

$$(D_3)_l = \cdots - \frac{p_l}{m_l R T_l}(\mathbf{D}_1\vec{V})_l\cdot(\nabla\phi)_l$$

$$\omega_l = (\vec{V}\cdot\nabla\pi)_l - (\mathbf{K}\nabla\cdot m\vec{V})_l$$

$$\int_0^1 (m\vec{V})d\eta = \left(\mathbf{K}\nabla\cdot m\vec{V}\right)_L$$

$\mathbf{D}_h$ gives values on half levels when applied on full level variable ψ, with input boundary conditions $\psi_0 = 0, \psi_{L+1} = \psi_L$

Boundary conditions of vertical operators

Summary: we have 1 integral vertical operator and 4 derivative vertical operators with the following boundary conditions

Operator	Input	Output
$\mathbf{K}\psi$	$\left(\frac{\partial\psi}{\partial\eta}\right)_0 = 0, \left(\frac{\partial\psi}{\partial\eta}\right)_{L+1} = 0$	$(\mathbf{K}\psi)_0 = 0$
$\mathbf{D}_1\psi$	$\psi_0 = 0,\ \psi_{L+1} = \psi_L$	$-$
$\mathbf{D}_2\psi$	$\psi_0 = 0,\ \left(\frac{\partial\psi}{\partial\eta}\right)_{L+1} = 0$	$(\mathbf{D}_2\psi)_{L+1} = 0$
$\mathbf{D}_h\psi$	$\psi_0 = 0,\ \psi_{L+1} = \psi_L$	$-$
$\mathbf{DD}\psi$	$\psi_0 = 0,\ \left(\frac{\partial\psi}{\partial\eta}\right)_{L+1} = 0$	$(\mathbf{DD}\psi)_{L+1} = 0$

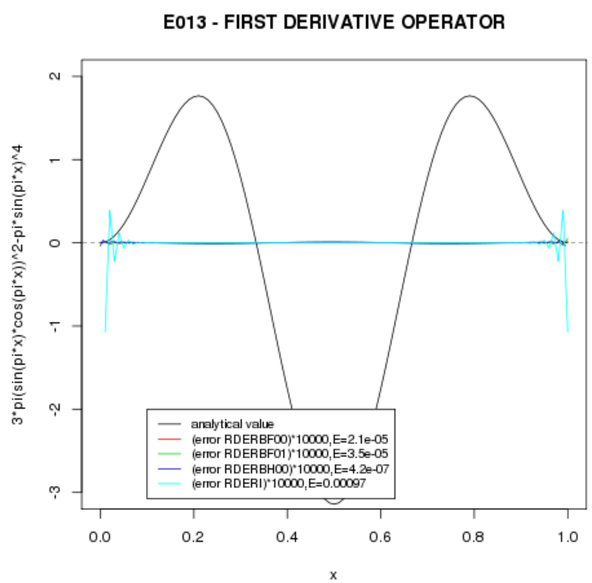
Each time step we perform 2 kind of transformations:

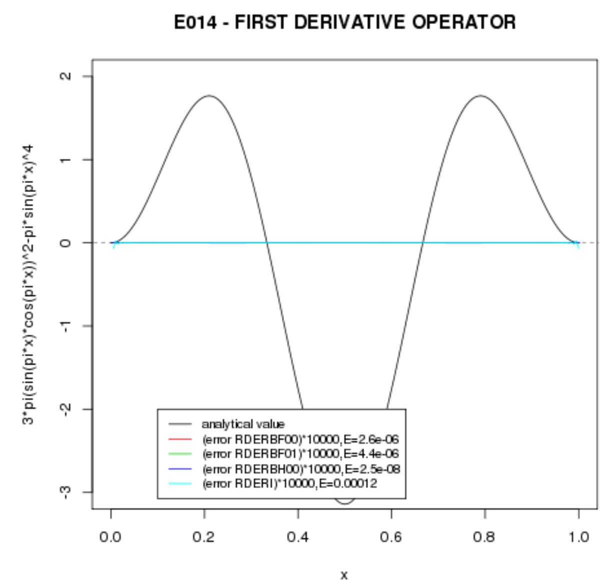
$$gw = gw_s + \mathbf{T}_i\left(\frac{mRT}{p}(d - X)\right)$$ at time t

$$d = \frac{p}{mRT}\mathbf{T}_d(gw) + X$$ at time t+dt (explicit guess)

we need to preserve steady state => $\mathbf{T}_i\mathbf{T}_d$ = Id, not possible with FE operators => $\mathbf{T}_i$, $\mathbf{T}_d$ are **FD operators**

Tests: Theoretical accuracy of vertical operators





100 uniformally distributed levels 200

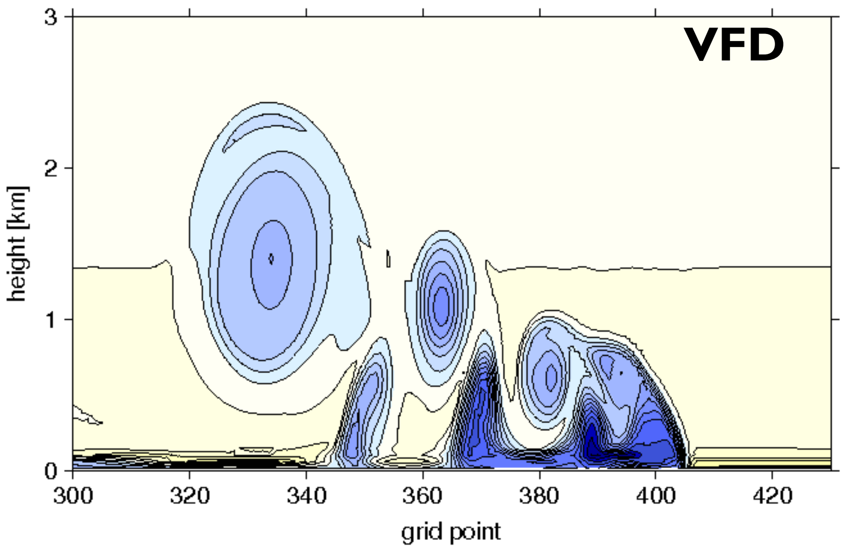
Compared to the analytical value of the function satisfying the boundary conditions

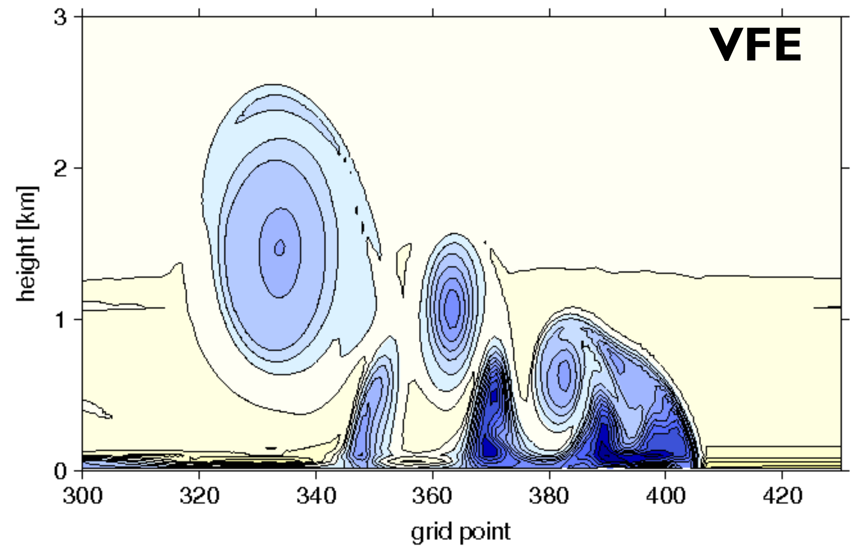
$$\frac{\partial}{\partial x}\left(\sin^3(\pi x)\cos(\pi x)\right)$$

Sensitivity in idealized experiments

2D vertical plane model experiments:

1) Flow over Agnesi shape orography – NLNH regime
2) Potential flow
3) Density current (Straka test): $\Delta x=\Delta z=50m$, $\Delta t=3s$, 300 time steps, symmetric temperature perturbation -15K, only half of domain shown



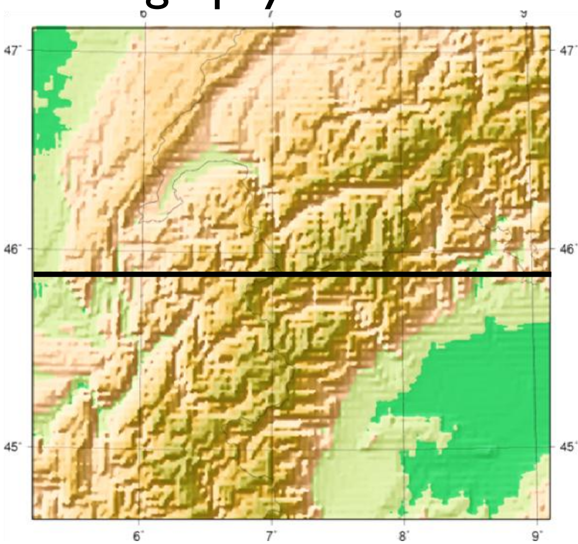


Potential temperature field, contour interval 1K

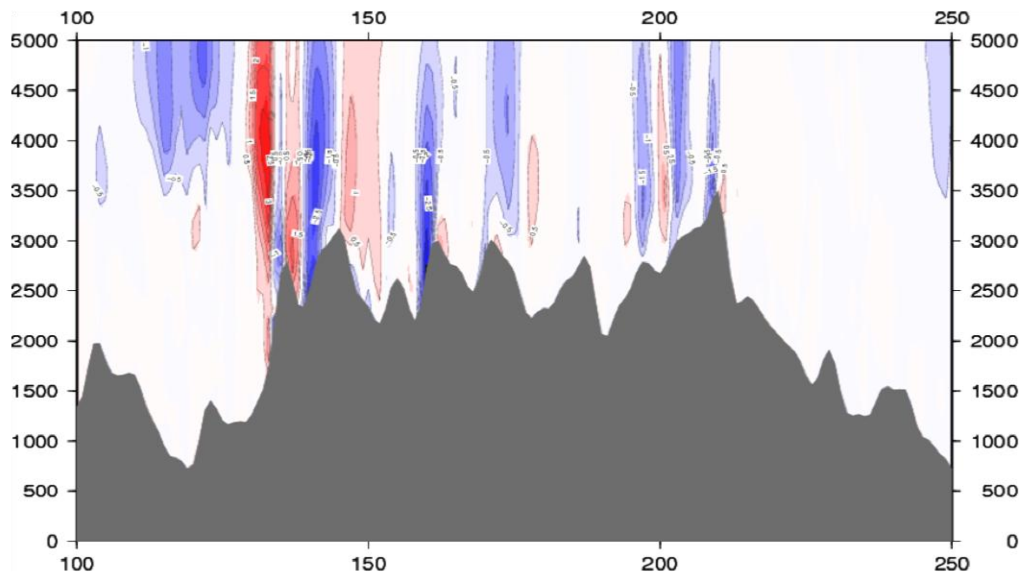
3D academic tests

- steep orography (Alpine ridge), 1km horizontal resolution
- 28 Feb 2012 00UTC, +24hours
- adiabatic run
- timestep 30s

Orography



Vertical cross section through the middle of the domain

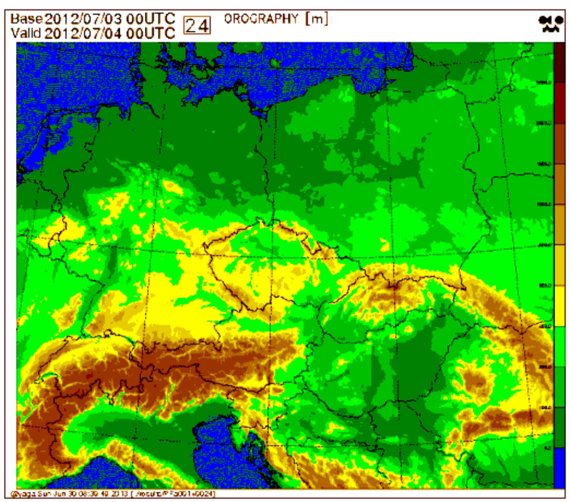


3D real cases

Experiments: summer (July 2012) and winter (Dec 2012) 10days series, +24hours once per day from 00UTC

- horizontal resolution of 2.2km
- timestep 90s, 2tl PC_NESC + 1 iteration, SL advection
- 1 hour cumulated precipitation
- ALARO physics with no deep convection parameterization, microphysics applied to resolved clouds and precipitation

6 July 2012 00 UTC, 22 hours of integration



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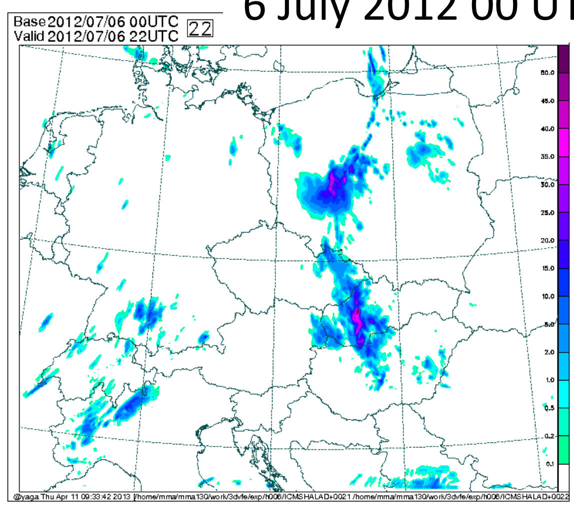
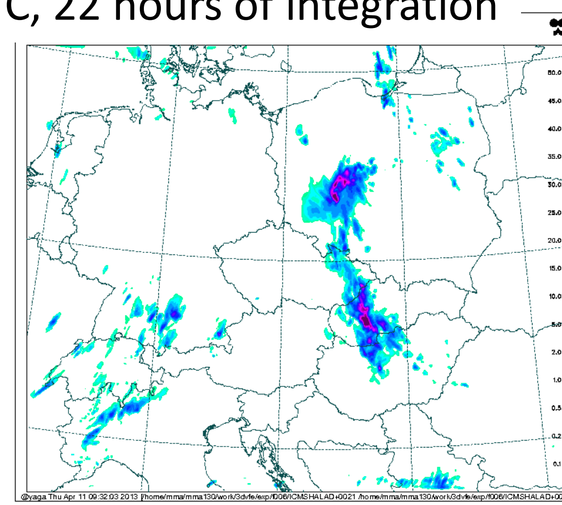


VFD

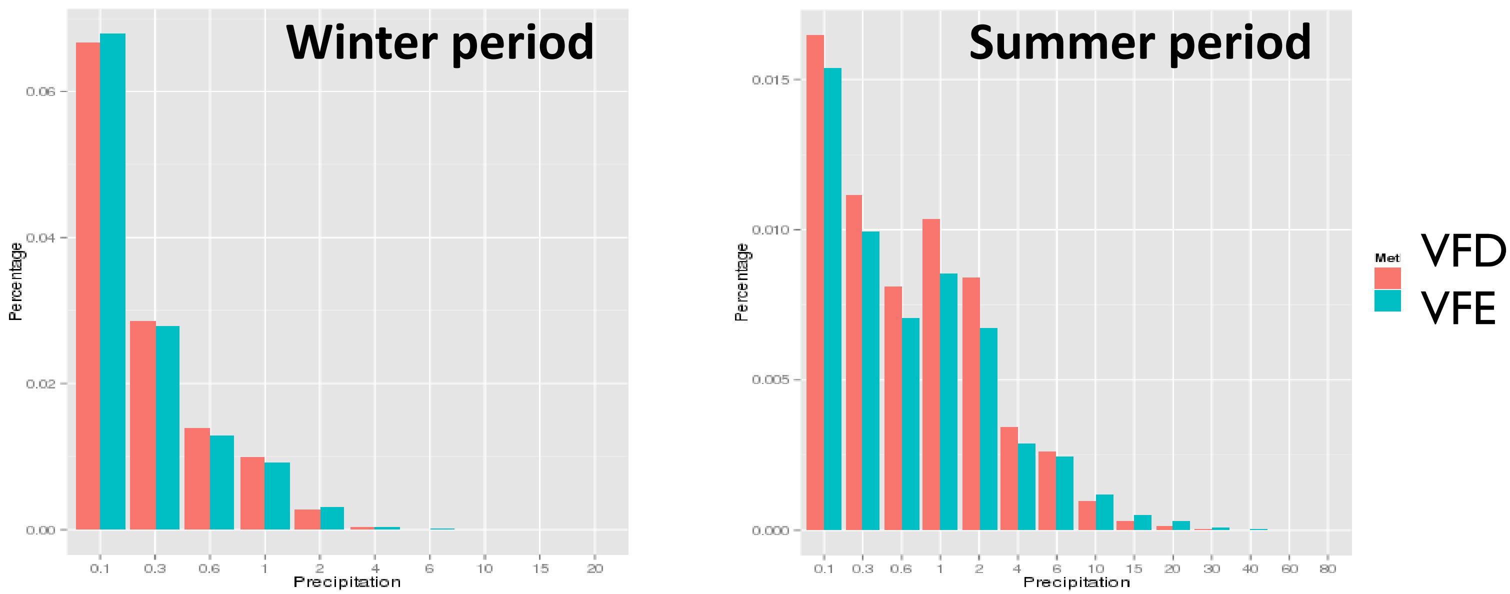


VFE

Further results

- Objective scores neutral to the change of vert. discretization
- An interaction of the vert. discretization with the resolved convection detected => with VFE the weak precipitations occur less often, while there is a shift to more intense precipitations

1 hour cumulated precipitation histograms



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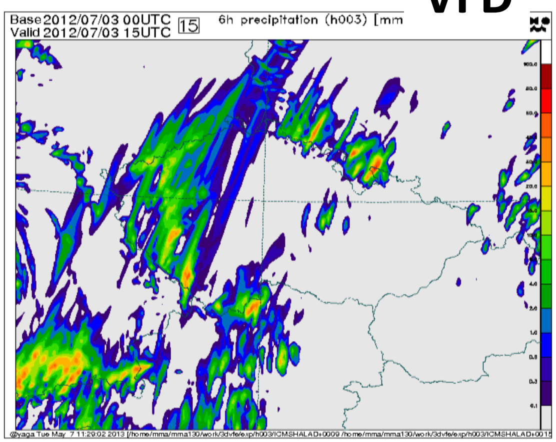
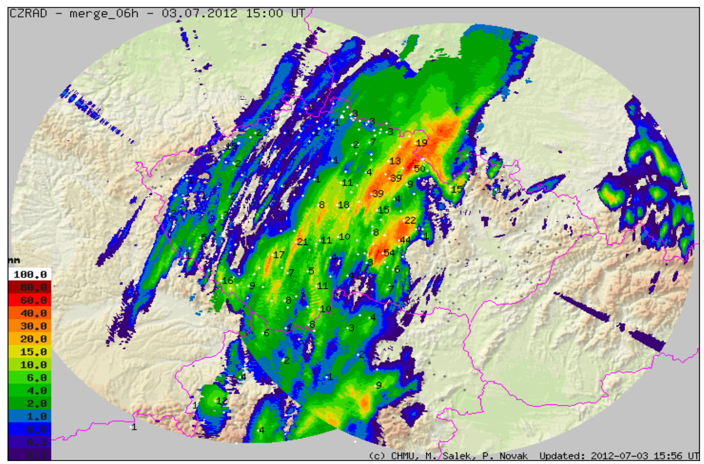
3D real cases verification against radar

The change in physical parameterizations (deep convection par. added according to ALARO-0) has stronger impact then the change in vertical discretization.

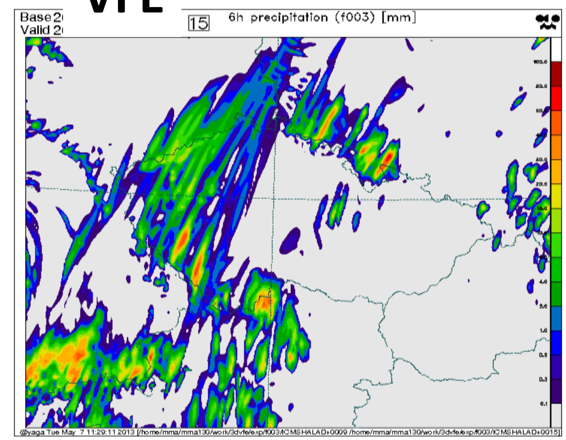
VFD

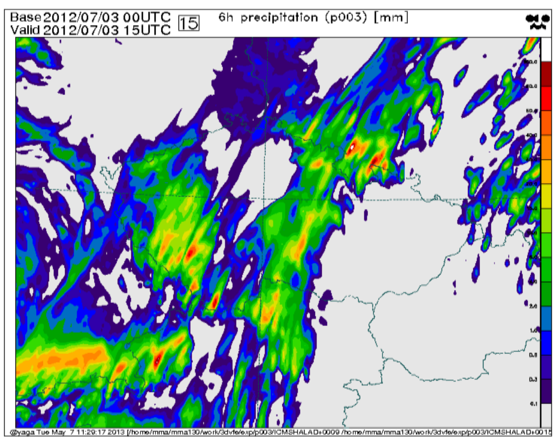


RADAR

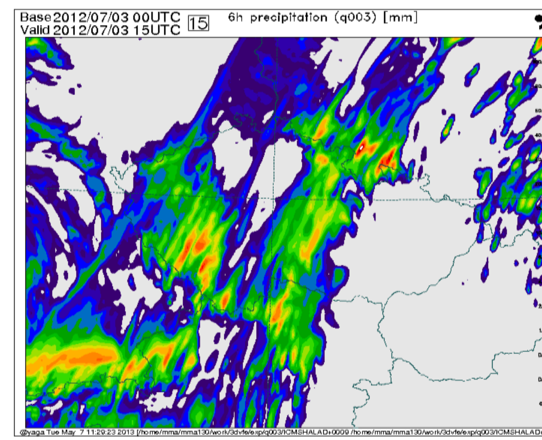


VFE





ALARO-0



Working plans

- to phase the existing working VFE implementation into the official IFS/ARPEGE/ ALADIN cycle
- to adapt the current implementation to the global model ARPEGE/IFS – in cooperation with HIRLAM
- thorough testing of the VFE implementation - stability and accuracy properties, convergence of the SI solver and its speed
- to study the influence of the B-spline order on the accuracy and the time stepping stability of the whole system

Thank you for your attention !

İlginiz için teşekkür ederiz !