

*Regional Cooperation for  
Limited Area Modeling in Central Europe*



# LACE : News in dynamics 2017

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thanks to Alexandra Craciun, Jozef Vivoda and other colleagues



## Dynamical core of ALADIN/HIRLAM system

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- fully compressible Euler equations (NH) or hydrostatic primitive equations (HPE)
- space discretization in horizontal: Fourier spectral method
- mass based vertical coordinate using Laprise hydrostatic pressure
- semi-implicit time scheme – direct solver for Helmholtz equation for one prognostic variable, vertical/horizontal direction separation
- semi-Lagrangian advection
- prognostic variables differ in grid-point space and in spectral space for stability and accuracy reasons; they are transformed every time step

# Outline

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## 1. Dynamic definition of the time scheme

(Jozef Vivoda, SHMU)

## 2. The trajectory search in the SL advection scheme

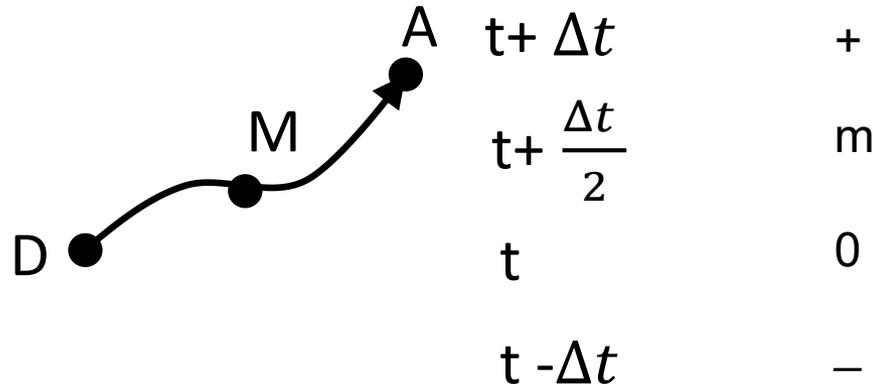
(Alexandra Craciun, Meteo Romania)

# Dynamic definition of the time scheme

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Advection equation  $\frac{df(t, x)}{dt} = N(t, x)$

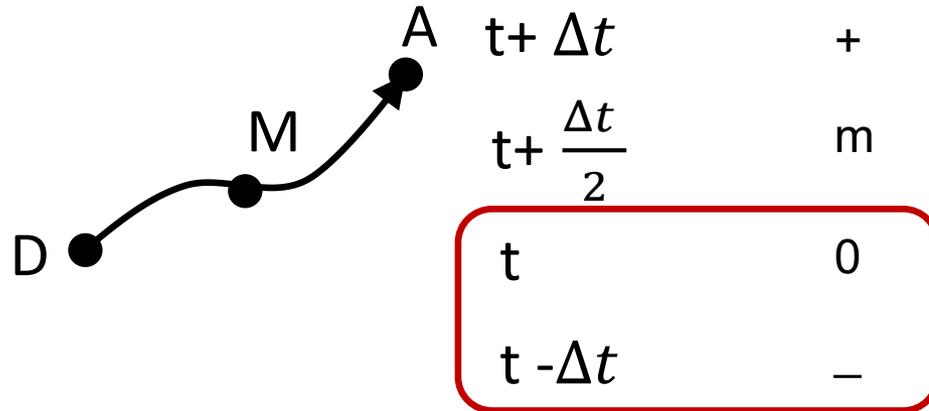
Time centered explicit  
semi-Lagrangian approach  $\frac{f_A^+ - f_D^0}{\Delta t} = N_M^m$



## Dynamic definition of the time scheme

Advection equation  $\frac{df(t, x)}{dt} = N(t, x)$

Time centered explicit  
semi-Lagrangian approach  $\frac{f_A^+ - f_D^0}{\Delta t} = N_M^m$



available information

## Dynamic definition of the time scheme

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First order treatment **NESC**: 
$$N_M^m = \frac{1}{2}(N_A^0 + N_D^0) + \mathcal{O}(\Delta t)$$

The time centered scheme using available information

$$N_M^m = a_1 N_A^0 + a_2 N_A^- + a_3 N_D^0 + a_4 N_D^-$$

For any  $\alpha$ :

$$N_M^m = \left(-\alpha + \frac{3}{4}\right)N_A^0 + \left(\alpha - \frac{1}{4}\right)N_A^- + \left(\alpha + \frac{3}{4}\right)N_D^0 + \left(-\alpha - \frac{1}{4}\right)N_D^- + \mathcal{O}(\Delta t^2)$$

For  $\alpha = \frac{1}{4}$  we get **SETTLS** (Hortal, 2002):

$$N_M^m = \frac{1}{2}(N_A^0 + N_D^0) + \frac{1}{2}(N_D^0 - N_D^-) + \mathcal{O}(\Delta t^2)$$

## Dynamic definition of the time scheme

**NESC**  $N_M^m = \frac{1}{2}(N_A^0 + N_D^0)$

**SETTLS**  $N_M^m = \frac{1}{2}(N_A^0 + N_D^0) + \frac{1}{2}(N_D^0 - N_D^-)$

**COMBINED SCHEME**  $N_M^m = \frac{1}{2}(N_A^0 + N_D^0) + \frac{\beta}{2}(N_D^0 - N_D^-)$

We may change  $\beta$  arbitrarily from 0 to 1.

We consider solution in the shape  $N(t, x) = (\lambda + i\omega)f(t, x)$   
and analyse single Fourier component advected with a constant  
wind  $U$

$$f(n\Delta t, j\Delta x) = A^n e^{ijUk\Delta t}$$

Stability reached when  $A \leq 1$ .

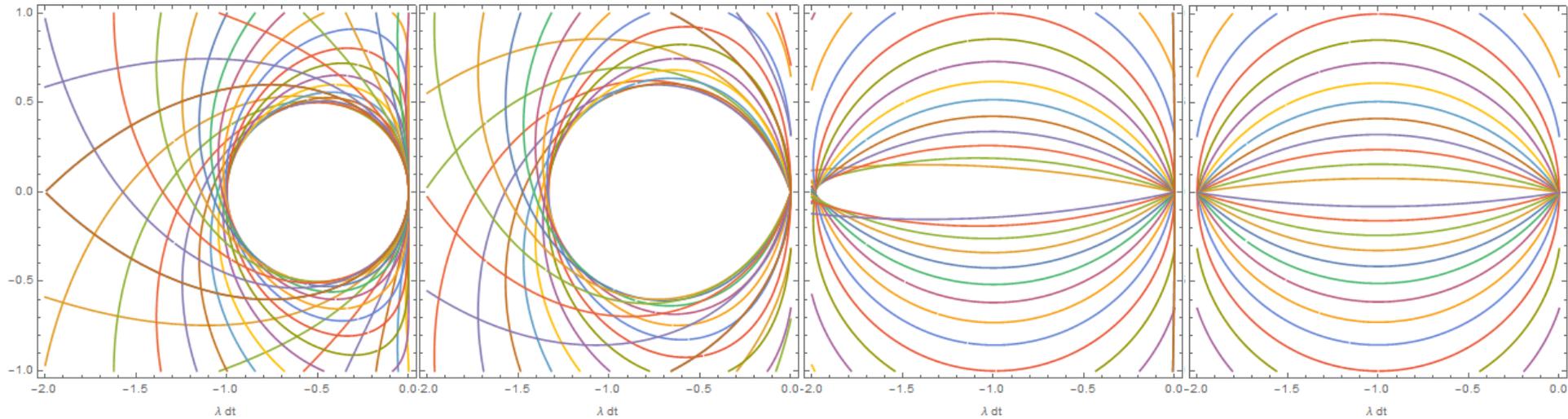
# Stability analysis

$\beta = 1$

$\beta = 1/2$

$\beta = 1/100$

$\beta = 0$



$\uparrow \omega \Delta t$

$\rightarrow \lambda \Delta t$

(Courtesy of J.Vivoda)

SETTLS:  $\lambda, \omega$  constraints

NESC:  $\omega=0$

# Dynamic definition of the time scheme

**Semi-implicit scheme:**

$$\frac{df(t, x)}{dt} = N(t, x) + L(t, x)$$

$$\frac{f_A^+ - f_D^0}{\Delta t} = N_M^m + \frac{1}{2}(L_A^+ + L_D^0)$$

Stability analysis for solution

$$L(t, x) = \delta i \omega f(t, x)$$

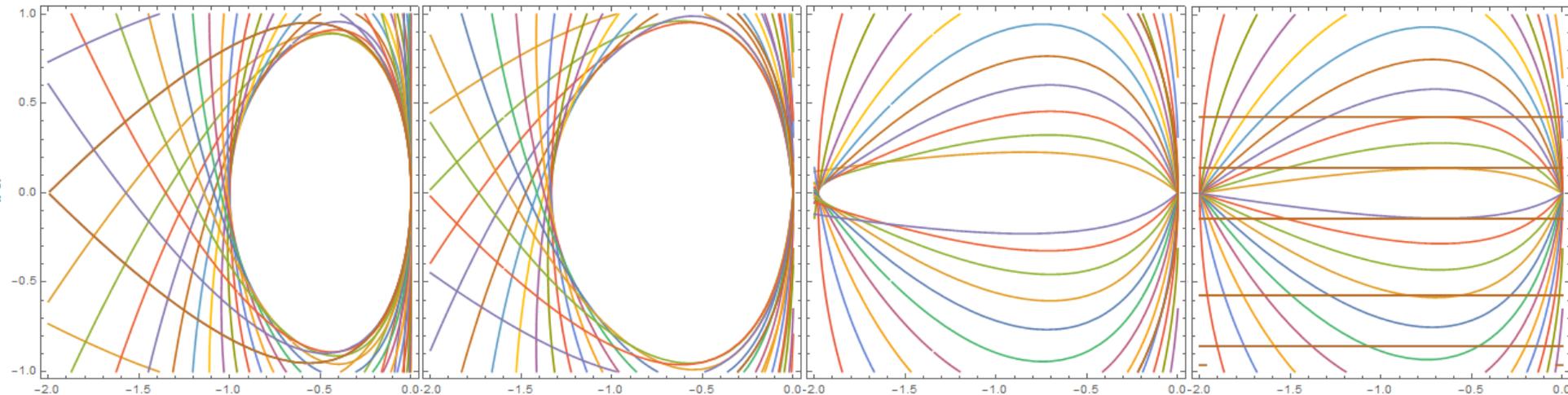
$$N(t, x) = (\lambda + (1 - \delta) i \omega) f(t, x)$$

$\beta = 1$

$\beta = 1/2$

$\beta = 1/100$

$\beta = 0$



SETTLS:  $\lambda, \omega$  constraints

NESC:  $\omega=0$

## Dynamic definition of the time scheme

Using explicit guess: 
$$N_M^m = a_1 N_A^0 + a_2 \widetilde{N}_A^+ + a_3 N_D^0 + a_4 N_D^-$$

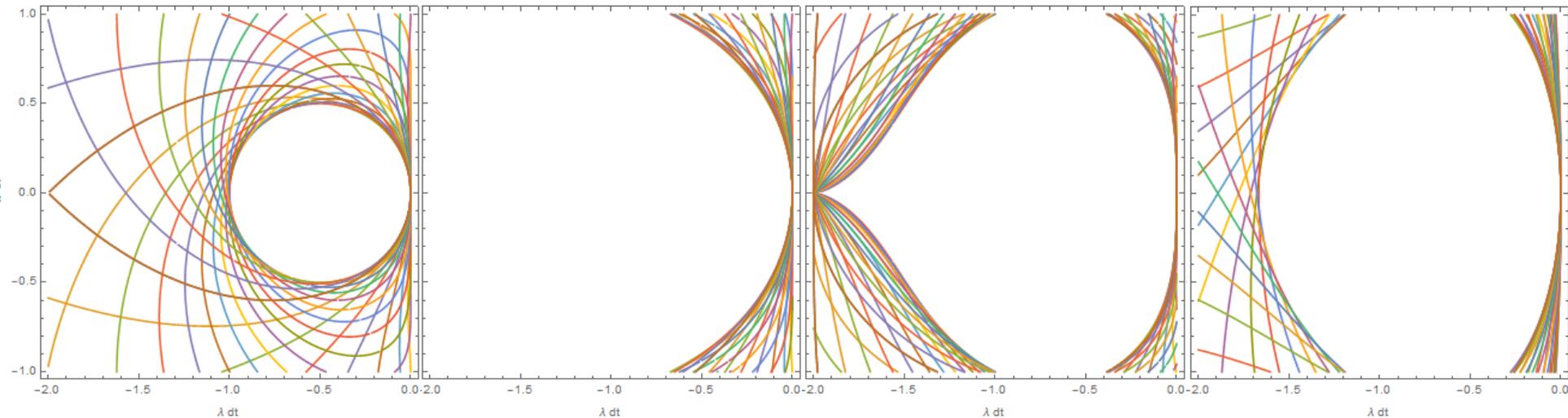
Stability analysis shows better stability in both,  $\omega$  and  $\lambda$  direction.

$\delta=0, \alpha = 1/4$

$\delta=0, \alpha = 0$

$\delta=0, \alpha = -1/4$

$\delta=3/4, \alpha = 1/10$



# Dynamic definition of the time scheme

## Iterative centered implicit scheme

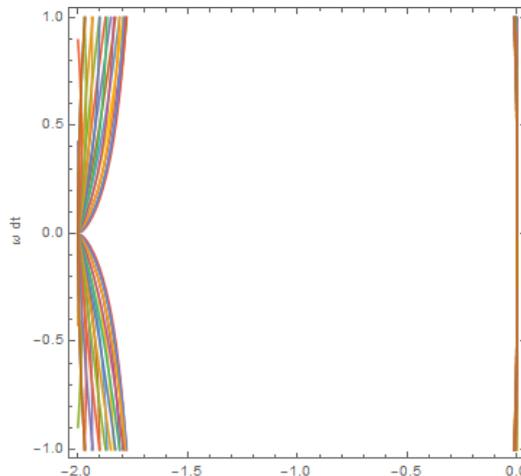
**Predictor step** using combined scheme:

$$\frac{f_A^{+(0)} - f_D^0}{\Delta t} = \frac{1}{2}(N_A^0 + N_D^0) + \frac{\beta}{2}(N_D^0 - N_D^-) + \frac{1}{2}(L_A^{+(0)} + L_D^0)$$

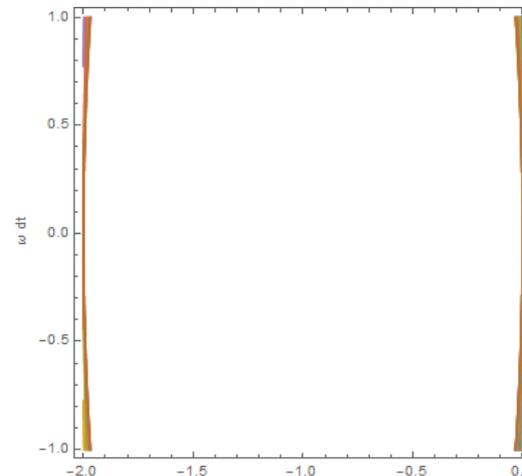
**Corrector step** using NESCS :

$$\frac{f_A^{+(n)} - f_D^0}{\Delta t} = \frac{1}{2}(N_A^{+(n-1)} + N_D^0) + \frac{1}{2}(L_A^{+(n)} + L_D^0)$$

$\beta = 1$   
SETTLS



$\beta = 0$   
NESCS



## Dynamic definition of the time scheme

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The NESC/SETTLS used

- 1) in SL trajectory search
- 2) for non-linear residuum in the SI scheme

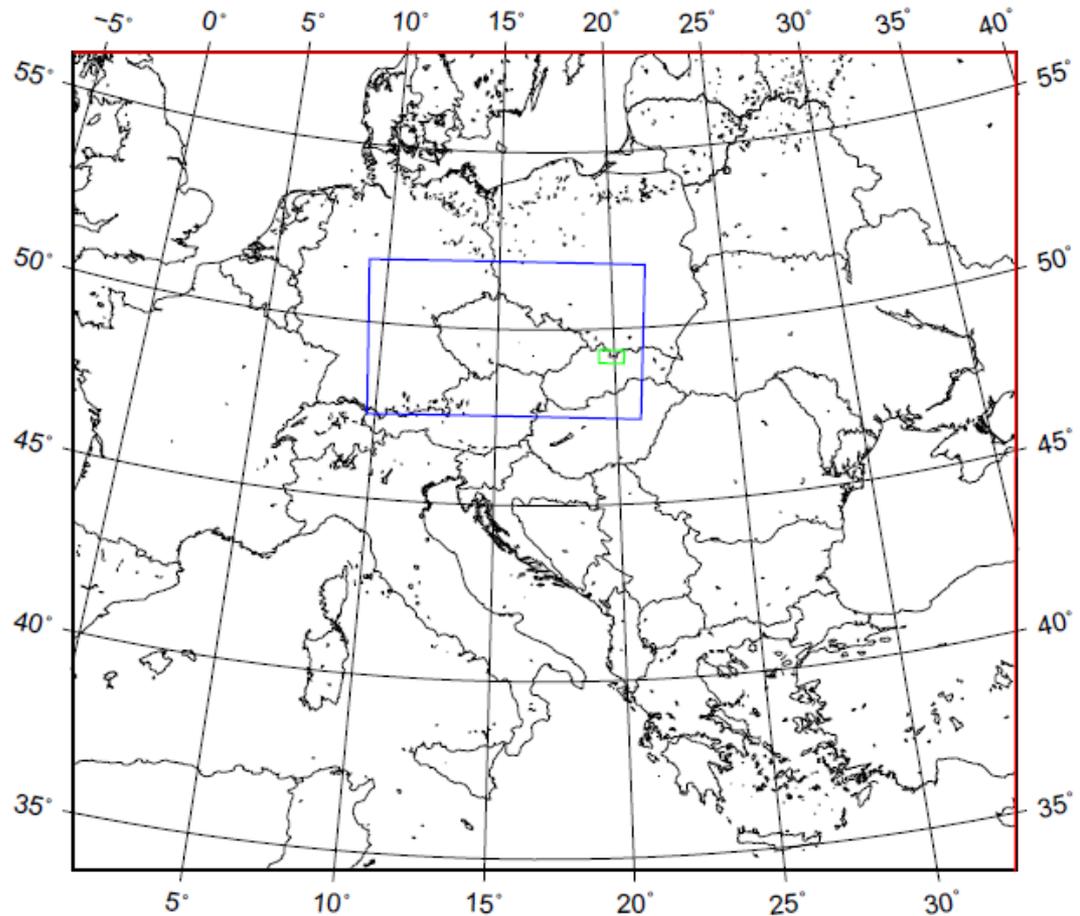
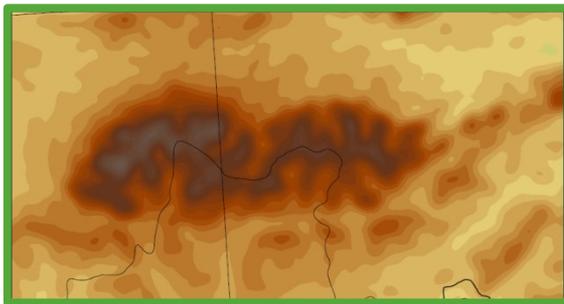
From experience:

- 1) SETTLS beneficial for trajectory search
- 2) Instability may occur when SETTLS applied on non-linear residuum, while NESC is stable (and only first order accurate)

The idea: To use SETTLS whenever possible, and switch to NESC if needed. If it is not “very often”, we keep sufficient accuracy and restore stability.

## Dynamic definition of the time scheme

Experiments  
on a small domain



## Experiments on small domain

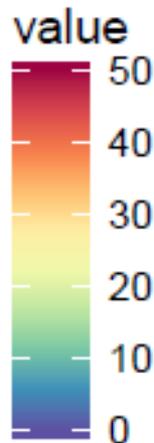
PC with one iteration:

$$\mu = 1 - \frac{|N^0 - N^-|}{|N^0| + |N^-|}$$

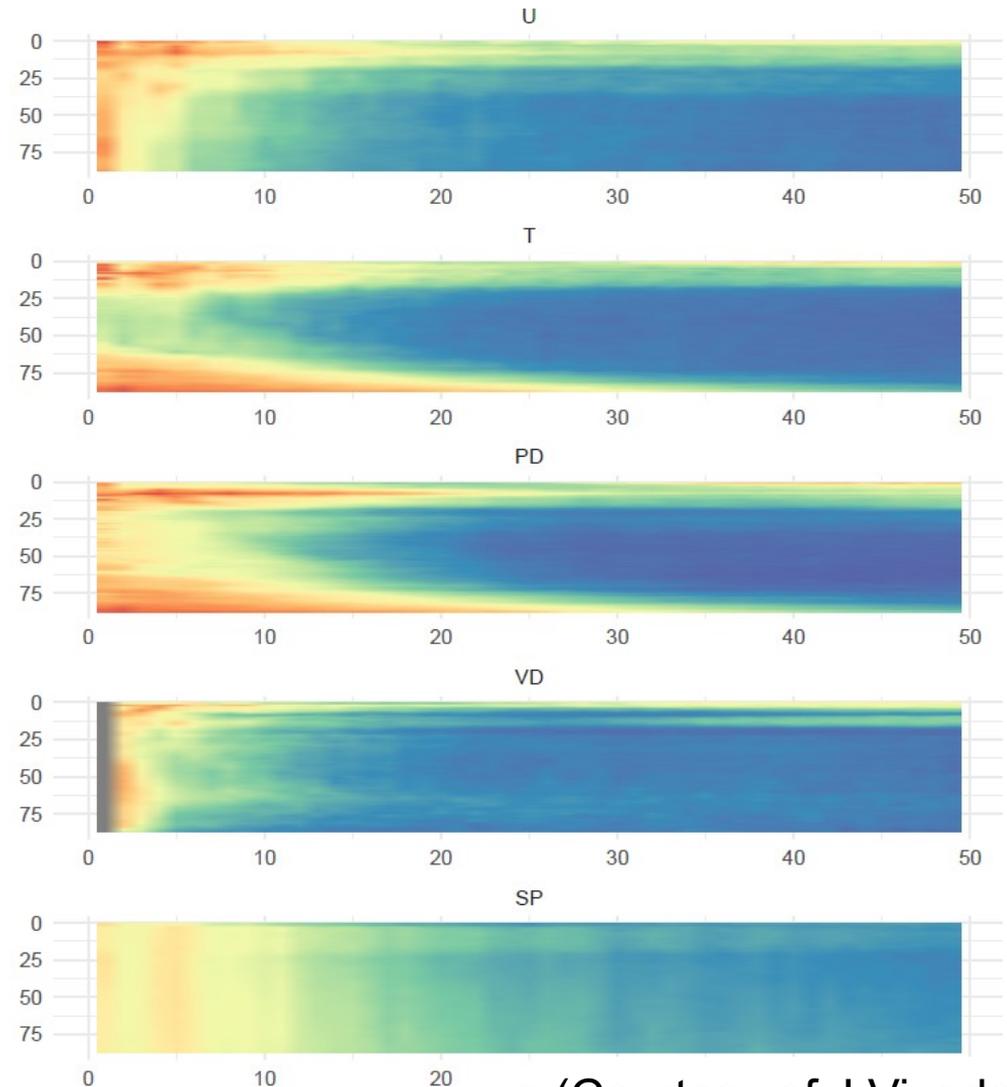
for pressure departure\*.

When  $\mu \approx 1$  use SETTLS,  
otherwise use NESCS  
in each grid point in  
predictor.

Number of points  
with NESCS  
calculation:



(\*adopted from M.Diamantakis,  
LSETTLSVF option)

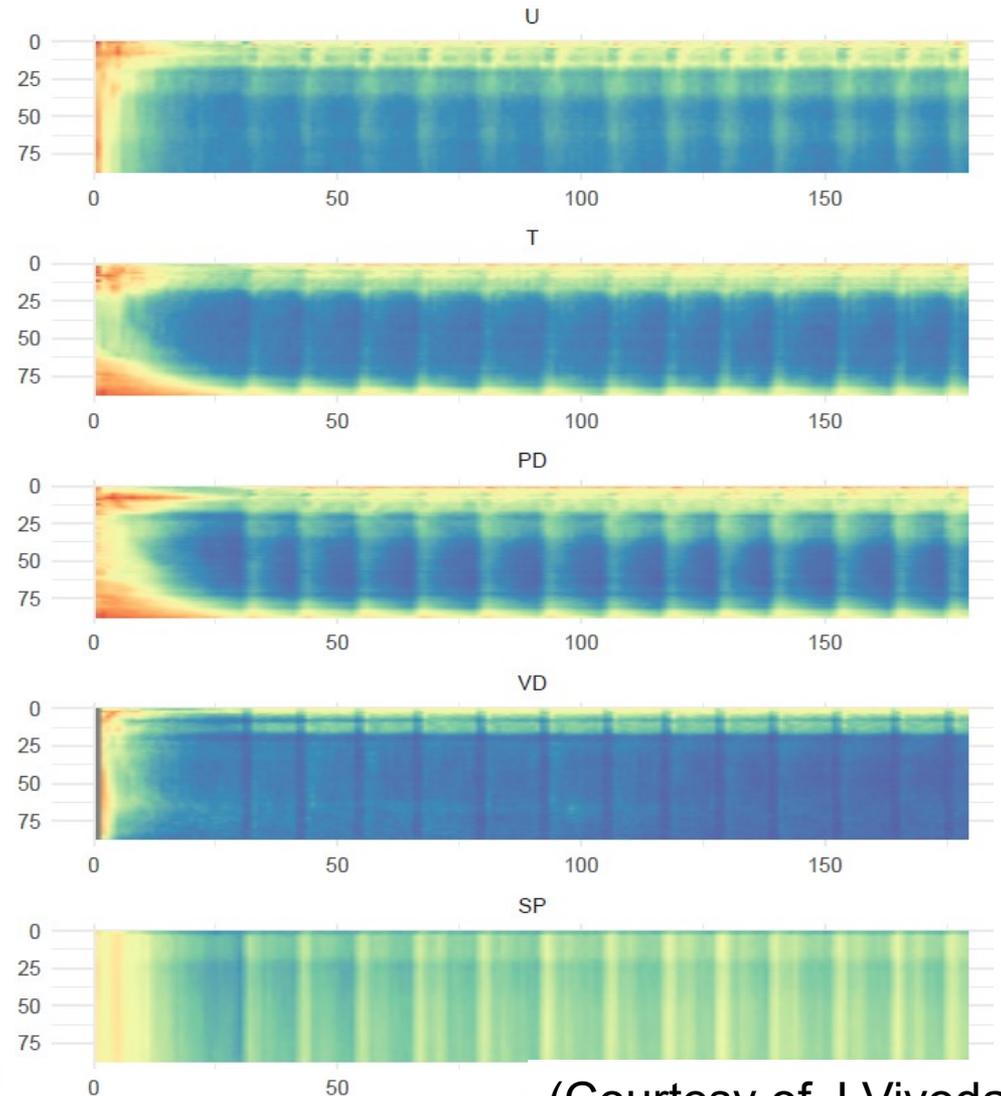
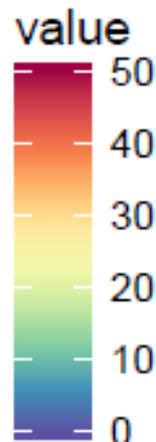


stc (Courtesy of J.Vivoda)

## Experiments on small domain

Collect globally information on the usage of NESC; if NESC applied in less than 10% of grid points, skip corrector.

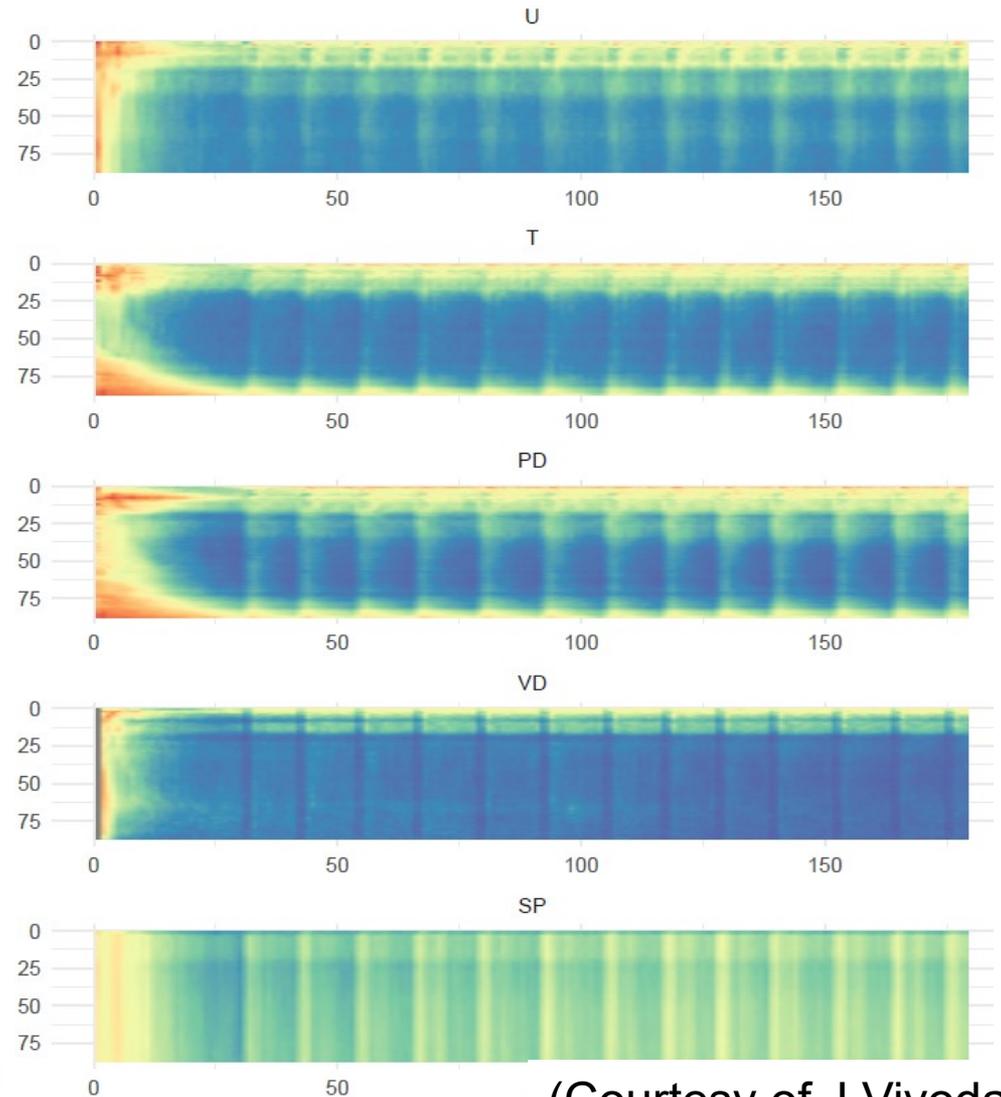
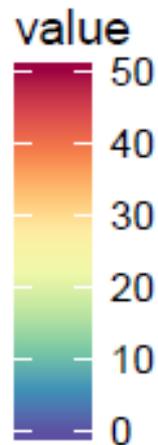
Number of points with NESC calculation:



## Experiments on small domain

Hence corrector is not applied in all time steps.

The accuracy and stability is restored.



## Dynamic definition of the time scheme

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### Conclusions:

- SETTLS scheme is enabled in predictor step of the PC scheme (For LPC\_CHEAP as well).
- Dynamic choice of the predictor used (SETTLS/NESC) and of the number of correctors applied may be an efficient answer to stability/accuracy/efficiency trade-off.
- When using dynamic definition of the time scheme we are not able to predict exactly in advance the time to results; some threshold may be established.

## The trajectory search in the SL advection scheme

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- ▶ PC scheme with reiteration of SL trajectories produces noisy solution in some cases.
- ▶ If model horizontal resolution increased => local divergence may increase => Lipschitz criteria may be broken locally => divergent algorithm for searching SL origin point => increase in the number of iterations may lead to even less accurate solutions.
- ▶ Similar problems have been identified at ECMWF in IFS and fixed by local change of the computation of the half level wind (M.Diamantakis).

## The trajectory search in the SL advection scheme

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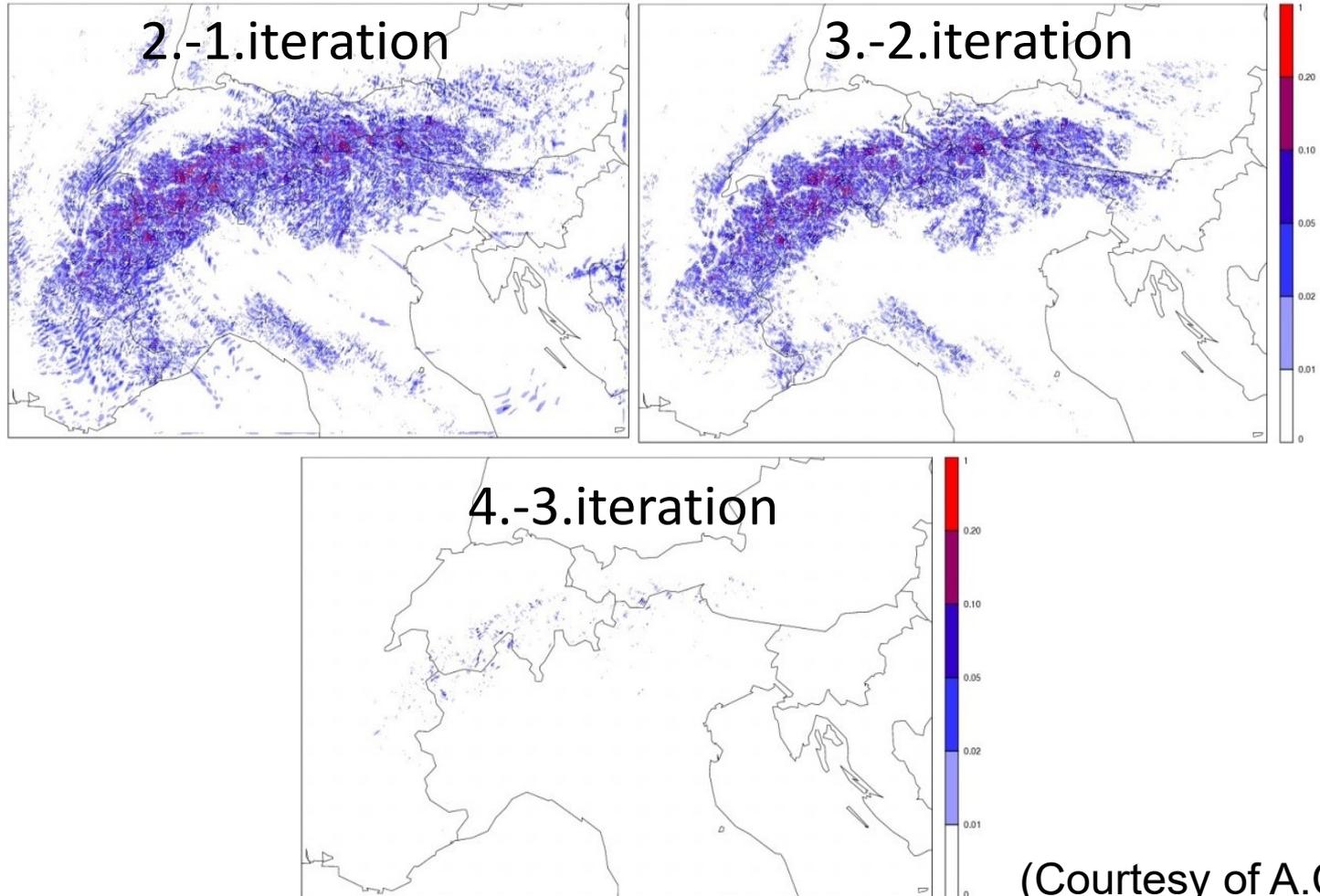
### Current work:

- ▶ to calculate distances between two points representing estimations of the origin point from two successive iterations
- ▶ applied separately for horizontal and vertical components
- ▶ applied on several real cases

### Conclusions:

- ▶ Second iteration already very close to the first one.
- ▶ Depending on the criteria we may find some divergent grid points, but the origin points are not moved by more than dms.
- ▶ More systematic testing on longer period is needed.

# The trajectory search in the SL advection scheme



(Courtesy of A.Craciu)

# Thank you for your attention!