Further research on including orography in the vertical Laplacian of the linear model used in the ICI time scheme

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The Semi-Implicit (SI) time scheme with constant coefficients typically assumes a flat orography in the basic state. However, it is not strictly necessary to adhere to linearization around this basic state. An alternative approach involves initially performing linearization, then approximating orographic terms and applying some simplifications. This method has already proven effective in cases like the two-temperature SITR and SITRA, where the reference temperature varies across different terms of the linear model after linearization. Similarly, in blended hydrostatic and non-hydrostatic systems, several control parameters are introduced in the full model, with their values generally chosen independently in both the linear and full models after linearization. This report continues last year's work on addressing geopotential in the implicit part of the system. We defined several discretizations for the vertical Laplace operator and compared the results

1 Orogrphy approximation in the linear model

We begin with the full Euler equations expressed in hybrid η-coordinates:

$$\frac{dT}{dt} = \frac{\kappa}{\kappa - 1} T(D + \hat{d}) \tag{1a}$$

$$\frac{d\vec{v}}{dt} = -RT\frac{\nabla\pi}{\pi} - RT\nabla\hat{q} - \nabla\phi - \frac{1}{m}\frac{\partial(p-\pi)}{\partial\eta}\nabla\phi \tag{1b}$$

$$\frac{dgw}{dt} = \frac{g^2}{m}\frac{\partial(p-\pi)}{\partial\eta} \tag{1c}$$

$$\frac{d\hat{q}}{dt} = \frac{1}{\kappa - 1}(D + \hat{d}) - \frac{\dot{\pi}}{\pi} \tag{1d}$$

$$\frac{\partial q_s}{\partial t} = -\frac{1}{\pi_s}\int_0^1 \nabla \cdot m\vec{v}d\eta, \tag{1e}$$

where

$$\hat{d} = \frac{p}{mRT}\left(\frac{\partial \vec{v}}{\partial \eta}\nabla\phi - \frac{\partial gw}{\partial \eta}\right) \tag{2a}$$

$$\dot{\pi} = \vec{v}\cdot\nabla\pi - \int_0^\eta \nabla\cdot m\vec{v}d\eta' \tag{2b}$$

$$\phi = \phi_s + \int_\eta^1 \frac{mRT}{p}d\eta'. \tag{2c}$$

Assuming the linear approximation of pressure perturbation $p - \pi \approx \pi^*\hat{q}$ the linear model is written as:

$$\frac{dT}{dt} = \frac{\kappa}{\kappa - 1}T^*(D + \hat{d}) \tag{3a}$$

$$\frac{d\vec{v}}{dt} = -RT^*\frac{\nabla\pi}{\pi^*} - RT^*\nabla\hat{q} - \nabla\phi - \frac{1}{m^*}\frac{\partial\pi^*\hat{q}}{\partial\eta}(\nabla\phi)^* \tag{3b}$$

$$\frac{dgw}{dt} = \frac{g^2}{m^*}\frac{\partial\pi^*\hat{q}}{\partial\eta} \tag{3c}$$

$$\frac{d\hat{q}}{dt} = \frac{1}{\kappa - 1}(D + \hat{d}) + \frac{1}{\pi^*}\int_0^\eta m^*Dd\eta' \tag{3d}$$

$$\frac{\partial q_s}{\partial t} = -\frac{1}{\pi_s}\int_0^1 m^*Dd\eta. \tag{3e}$$

Then the definition of $\hat{d}$ can be linearized as follows:

$$\hat{d} = \frac{\pi^*}{m^*RT^*}\left(\frac{\partial\vec{v}}{\partial\eta}(\nabla\phi)^* - \frac{\partial gw}{\partial\eta}\right). \tag{4}$$

1.1 Lineariztaion of the gradient of geoptential $\nabla\Phi$

The discretized diagnostic relation for the geopotential is:

$$\Phi_l = \Phi_s + \sum\frac{(RT)_k}{e^{\hat{q}_k}}\delta_k + \frac{(RT)_l}{e^{\hat{q}_l}}\alpha_l, \tag{5}$$

which we can linerize as:

$$\Phi_l^* = \Phi_s^* + RT^*\left(\sum_{k=l+1}^{L}\delta_k^* + \alpha_l^*\right) \tag{6}$$

Then linearized gradient of the geopotential is:

$$\nabla\Phi_l^* = \nabla\Phi_s^* + RT^*\left(\sum_{k=l+1}^{L}\nabla\delta_k^* + \nabla\alpha_l^*\right) \tag{7}$$

and

$$\nabla\alpha_l^* = -\frac{\alpha_l^*C_l}{\pi_l^*\delta\pi_l^*}\nabla\pi_s, \tag{8}$$

$$\nabla\delta_l^* = -\frac{\delta_l^*C_l}{\pi_l^*\delta\pi_l^*}\nabla\pi_s, \tag{9}$$

where

$$C_l = A_{\tilde{l}}B_{\tilde{l}-1} - A_{\tilde{l}-1}B_{\tilde{l}} \text{ and } C_1 = 0. \tag{10}$$

To later solve the characteristic equation as a Helmholtz equation, we need to approximate the gradient of the hydrostatic pressure of the basic state ($\nabla\pi_S^*$) to be horizontally independent.

$$\nabla\pi_s = \frac{g\pi_s^*\Lambda}{\sqrt{2}}, \tag{11}$$

where $\frac{\Lambda}{\sqrt{2}}$ represent some constant slope.

You can find the subroutine SUNHEESI_GEOP in Arp/sunheesi_geop.F90, which computes the linearized gradient of the geopotential on full and half levels.

Let's rewrite the equations from the linear system that contain the term $(\nabla\Phi)^*$:

$$\begin{aligned}\frac{\partial \vec{v}}{\partial t} &= -R\mathcal{G}^*\nabla T - RT^*\nabla q_s + RT^*(\mathcal{G}^*-1)\hat{q} - \frac{1}{m^*}\frac{\partial \pi^*\hat{q}}{\partial \eta}(\nabla\Phi)^* \\ \frac{\partial gw}{\partial t} &= g^2\frac{1}{m^*}\frac{\partial \pi^*\hat{q}}{\partial \eta} \\ \hat{d} &= \frac{\pi^*}{m^*RT^*}\left[\frac{\partial \vec{v}}{\partial \eta}(\nabla\Phi)^* - \frac{\partial gw}{\partial \eta}\right] = \frac{1}{RT^*}\left[(\nabla\Phi)^*\partial^*\vec{v} - \partial^* gw\right]. \qquad (12)\end{aligned}$$

We denote $\mathbb{A} = -R\mathcal{G}^*\nabla T - RT^*\nabla q_s + RT^*(\mathcal{G}^*-1)\nabla\hat{q}$ and $\mathbb{B} = \frac{1}{m^*}\frac{\partial\pi^*\hat{q}}{\partial\eta}$. Then

$$\frac{\partial \vec{v}}{\partial t} = \mathbb{A} - (\nabla\Phi)^*\ \mathbb{B} \qquad (13)$$

$$\frac{\partial gw}{\partial t} = g^2\mathbb{B}. \qquad (14)$$

We apply time differentiation on (12) and substitute with (13) and (14) to get:

$$\frac{\partial d}{\partial t} = -\frac{1}{RT^*}\left[g^2\partial^*\mathbb{B} + \left(\,(\nabla\Phi)^*\,\right)^2\partial^*\mathbb{B} - (\nabla\Phi)^*\partial^*\mathbb{A} + \left((\nabla\Phi)^*\partial^*(\nabla\Phi)^*\right)\mathbb{B}\right]. \qquad (15)$$

Since the slope Λ can be either positive (uphill) or negative (downhill), it does not make sense to represent its value in the linear model with a non-zero number. On the other hand, in second-order terms in Λ, the sign no longer matters, and the linear slope can be represented by its maximum value within the domain, regardless of the sign. Thus, we retain only the terms that are second-order in Λ and neglect the term $(\nabla\Phi)^*\partial^*\mathbb{A}$. Consequently, all the operators on the right side of equation (15) are applied to the variable $\hat{q}$, allowing us to redefine the vertical Laplacian, previously defined as $\mathcal{L}_v^* = \partial^*(1+\partial^*)$

2 Discretizations

Sibce

$$\partial^* X = \frac{\pi^*}{m^*}\frac{\partial X}{\partial \eta} = \pi^*\frac{\partial X}{\partial \pi^*}. \qquad (16)$$

the equation (15) can be rewritten as

$$\frac{\partial d}{\partial t} = -\frac{\pi^*}{RT^*}\left[\nabla\Phi^*\frac{\partial\nabla\Phi^*}{\partial\pi^*}\mathbb{B} + (\nabla\Phi)^2\frac{\partial\mathbb{B}}{\partial\pi^*} + g^2\frac{\partial\mathbb{B}}{\partial\pi^*}\right], \qquad (17)$$

which could be descretizated in several ways.

1. First, we will consider the option where the product $\pi^*\hat{q}$ is treated as a single variable, thereby ignoring the product rule for derivatives. In this case, the term $\pi^*\frac{\partial\mathbb{B}}{\partial\pi^*}$ corresponds to the previous version of $\mathcal{L}_v^*$. Each term of equation (17) will be discretized, and then we will sum the corresponding terms to obtain the coefficients for the new vertical Laplacian operator $\mathcal{L}_{new}^*$.

   Since the pressure field is defined only at full levels, we need to interpolate it to half levels in order to compute the first derivative of $\pi^*\hat{q}$

   $$\mathbb{B}_l = \frac{\pi^*_{\tilde{l}}\hat{q}_{\tilde{l}} - \pi^*_{\tilde{l-1}}\hat{q}_{\tilde{l-1}}}{\pi^*_{\tilde{l}} - \pi^*_{\tilde{l-1}}} \qquad (18)$$

   which contains $\hat{q}$ on half levels. Therefore we define interpolate operator I_{FH} such that:

   $$(\hat{q}_H)_{\tilde{l}} = (I_{FH}\hat{q}_F)_{\tilde{l}} = \frac{1}{\beta_{l+1}+\alpha_l}\left(\beta_{l+1}\hat{q}_l + \alpha_l\hat{q}_{l+1}\right), \qquad (19)$$

   $$(\hat{q}_H)_{\tilde{0}} = \hat{q}_1, \qquad (20)$$

   $$(\hat{q}_H)_{\tilde{L}} = \hat{q}_L \qquad (21)$$

and $\beta_l = \delta_l - \alpha_l$. The subroutine SUDEV_INTFH which gives coefficients of interpolate operator or interpolated vector from full levels to half levels, could be find in the module yomdev.

Let be:

$$W1(l) = \frac{\beta_{l+1}}{\beta_{l+1} + \alpha_l}, W2(l) = \frac{\alpha_l}{\beta_{l+1} + \alpha_l}, \tag{22}$$

assuming that $\pi^*_{\tilde{l}} - \pi^*_{l\tilde{-}1} = \delta_l \pi^*_l$ we can write the discretized $\pi^* \mathbb{B}$ as:

$$\pi^*_l \mathbb{B}_l = \frac{\pi^*_{\tilde{l}} W2(l) \hat{q}_{l+1} + \left(\pi^*_{\tilde{l}} W1(l) - \pi^*_{\tilde{l}-1} W2(l-1)\right) \hat{q}_l + \pi^*_{\tilde{l}-1} W1(l-1) \hat{q}_{l-1}}{\delta_l} \tag{23}$$

Coefficients of operator $\pi^* \mathbb{B}$ are then

$$\tilde{A}_l = -\frac{\pi^*_{\tilde{l}-1} W1(l-1)}{\delta_l} \tag{24}$$

$$\tilde{B}_l = \frac{\pi^*_{\tilde{l}} W1(l) - \pi^*_{\tilde{l}-1} W2(l-1)}{\delta_l} \tag{25}$$

$$\tilde{C}_l = \frac{\pi^*_{\tilde{l}} W2(l)}{\delta_l} \tag{26}$$

The discretization of $\pi^* \frac{\partial \mathbb{B}}{\partial \pi^*}$ is :

$$\frac{\partial \mathbb{B}}{\partial \pi^*} = \frac{(\pi^*_l - \pi^*_{l-1}) \pi^*_{l+1} \hat{q}_{l+1} - (\pi^*_{l+1} - \pi^*_{l-1}) \pi^*_l \hat{q}_l + (\pi^*_{l+1} - \pi^*_l) \pi^*_{l-1} \hat{q}_{l-1}}{(\pi^*_{l+1} - \pi^*_l)(\pi^*_l - \pi^*_{l-1})(\pi^*_{\tilde{l}} - \pi^*_{l\tilde{-}1})} \tag{27}$$

and it's coefficients are:

$$\overline{A}_l = \frac{\pi^*_{l-1}}{\delta_l (\pi^*_l - \pi^*_{l-1})} \tag{28}$$

$$\overline{B}_l = -\frac{(\pi^*_{l+1} - \pi^*_{l-1}) \pi^*_l}{\delta_l (\pi^*_{l+1} - \pi^*_l)(\pi^*_l - \pi^*_{l-1})} \tag{29}$$

$$\overline{C}_l = \frac{\pi^*_{l-1}}{\delta_l (\pi^*_{l+1} - \pi^*_l)}, \tag{30}$$

which are the same as coefficients of $\mathcal{L}^*_v$ A, B, and C. Then we can write the elements of the tridiagonal matrix which represent $\mathcal{L}^*_{new}$ as:

$$A_l = \frac{2}{g^2} \left((\nabla \Phi^*)_l \left(\frac{\partial \nabla \Phi^*}{\partial \pi^*} \right)_l \tilde{A}_l + (\nabla \Phi^*_F)^2_l \, \overline{A}_l \right) + \overline{A}_l, \tag{31}$$

$$B_l = \frac{2}{g^2} \left((\nabla \Phi^*)_l \left(\frac{\partial \nabla \Phi^*}{\partial \pi^*} \right)_l \tilde{B}_l + (\nabla \Phi^*_F)^2_l \, \overline{B}_l \right) + \overline{B}_l, \tag{32}$$

$$C_l = \frac{2}{g^2} \left((\nabla \Phi^*)_l \left(\frac{\partial \nabla \Phi^*}{\partial \pi^*} \right)_l \tilde{C}_l + (\nabla \Phi^*_F)^2_l \, \overline{C}_l \right) + \overline{C}_l \tag{33}$$

and:

$$(\mathcal{L}^*_{new} X)_l = A_l X_{l-1} + B_l X_l + C_l X_{l+1}$$

On the top levels we have:

$$\tilde{A}_1 = \overline{A}_1 = 0, \tag{34}$$

$$\tilde{B}_1 = \frac{\pi^*_{\tilde{1}} W1(1) - \pi^*_{\tilde{0}} W2(0)}{\delta_1}, \quad \overline{B}_1 = -\frac{\pi^*_2}{\delta_1 (\pi^*_2 - \pi^*_1)}, \tag{35}$$

$$\tilde{C}_1 = -\frac{\pi^*_{\tilde{1}} W2(1)}{\delta_1}, \quad \overline{C}_1 = \frac{\pi^*_2}{\delta_1 (\pi^*_2 - \pi^*_1)} \tag{36}$$

and on the bottom level we have:

$$\tilde{A}_L = -\frac{\pi^*_{\tilde{L}} W1(L)}{\delta_L}, \quad \overline{A}_L = \frac{\pi^*_{L-1}}{\delta_L(\pi^*_L - \pi^*_{L-1})}, \tag{37}$$

$$\tilde{B}_L = \frac{\pi^*_{\tilde{L}} W1(L) - \pi^*_{\tilde{L}-1} W2(L-1)}{\delta_L}, \quad \overline{B}_L = \frac{\pi^*_{L-1}}{\delta_L(\pi^*_L - \pi^*_{L-1})}, \tag{38}$$

$$\tilde{C}_L = \overline{C} = 0. \tag{39}$$

As we have the gradient of geopotential on full and half levels we can simply define:

$$\left(\frac{\partial \nabla \Phi^*}{\partial \pi^*}\right)_l = \frac{(\nabla \Phi^*)_{\tilde{l}} - (\nabla \Phi^*)_{\tilde{l-1}}}{\delta_l} \tag{40}$$

2. For the second option we rewrite the equation (17) to the form

$$\frac{\partial d}{\partial t} = -\frac{\pi^*}{RT^*}\left[\nabla \Phi^*_F \frac{\partial \nabla \Phi^*_H \mathbb{B}_H}{\partial \pi^*} + \left(\frac{\partial \mathbb{B}}{\partial \pi *}\right)_F\right] \tag{41}$$

and ignore the product rule for derivatives in both terms on the right-hand side. In this case, we will treat $\mathbb{B}$ as a variable at half levels since $\hat{q}$ is defined at full levels. This approach allows us to avoid interpolation and use the following discretizations:

$$\mathbb{B}_{\tilde{l}} = \frac{\pi^*_{l+1}\hat{q}_{l+1} - \pi^*_l \hat{q}_l}{\pi^*_{l+1} - \pi^*_l} \tag{42}$$

$$\left(\frac{\partial \nabla \Phi^*_H \mathbb{B}_H}{\partial \pi^*}\right)_l = \frac{\nabla \Phi^*_{\tilde{l}} \mathbb{B}_{\tilde{l}} - \nabla \Phi^*_{\tilde{l}-1} \mathbb{B}_{\tilde{l}-1}}{\pi^*_{\tilde{l}} - \pi^*_{\tilde{l}-1}} = \tag{43}$$

$$= \frac{1}{\delta_l \pi^*_l}\left(\frac{\nabla \Phi^*_{\tilde{l}} \pi^*_{l+1} \hat{q}_{l+1}}{\pi^*_{l+1} - \pi^*_l} - \pi^*_l \left(\frac{\nabla \Phi^*_{\tilde{l}}}{\pi^*_{l+1} - \pi^*_l} + \frac{\nabla \Phi^*_{\tilde{l}-1}}{\pi^*_l - \pi^*_{l-1}}\right)\hat{q}_l + \frac{\nabla \Phi^*_{\tilde{l}-1} \pi^*_{l-1} \hat{q}_{l-1}}{\pi^*_l - \pi^*_{l-1}}\right). \tag{44}$$

Then we can define

$$\tilde{A}_l = \frac{\nabla \Phi^*_{\tilde{l}-1} \pi^*_{l-1}}{\delta_l(\pi^*_l - \pi^*_{l-1})}, \quad \tilde{A}_1 = 0, \quad \tilde{A}_L = \frac{\nabla \Phi^*_{\tilde{L}-1} \pi^*_{L-1}}{\delta_L(\pi^*_L - \pi^*_{L-1})} \tag{45}$$

$$\tilde{B}_l = -\frac{\pi^*_l}{\delta_l}\left(\frac{\nabla \Phi^*_{\tilde{l}}}{\pi^*_{l+1} - \pi^*_l} + \frac{\nabla \Phi^*_{\tilde{l}-1}}{\pi^*_l - \pi^*_{l-1}}\right), \tilde{B}_1 = -\frac{1}{\delta_1}\left(\frac{\nabla \Phi^*_{\tilde{1}} \pi^*_1}{\pi^*_2 - \pi^*_1} + \nabla \Phi^*_{\tilde{0}}\right), \tilde{B}_L = -\frac{1}{\delta_L}\left(\frac{\nabla \Phi^*_{\tilde{L}-1} \pi^*_L}{\pi^*_L - \pi^*_{L-1}} + \nabla \Phi^*_{\tilde{L}}\right) \tag{46}$$

$$\tilde{C}_l = \frac{\nabla \Phi^*_{\tilde{l}} \pi^*_{l+1}}{\delta_l(\pi^*_{l+1} - \pi^*_l)}, \quad \tilde{C}_1 = \frac{\nabla \Phi^*_{\tilde{1}} \pi^*_2}{\delta_1(\pi^*_2 - \pi^*_1)}, \quad \tilde{C}_L = 0. \tag{47}$$

Th coefficients $\overline{A}_l$, $\overline{B}_l$ and $\overline{C}_l$ are the same as in the first case. The operator $\mathbf{L}^*_{new}$ is defined with following diagonals elements:

$$A_l = \frac{2}{g^2} \nabla \Phi^*_l \tilde{A}_l + \overline{A}_l, \tag{48}$$

$$B_l = \frac{2}{g^2} \nabla \Phi^*_l \tilde{B}_l + \overline{B}_l, \tag{49}$$

$$C_l = \frac{2}{g^2} \nabla \Phi^*_l \tilde{C}_l + \overline{C}_l \tag{50}$$

3. In the third options of discretization we will treat the term $\pi^* \hat{q}$ as product of functions, so we have:

$$\mathbb{B}_F = \hat{q}_l + \pi^*_l \frac{\hat{q}_{\tilde{l}} - \hat{q}_{\tilde{l-1}}}{\pi_{\tilde{l}} - \pi_{\tilde{l-1}}} \tag{51}$$

$$(\frac{\partial \mathbb{B}}{\partial \pi^*})_l = 2\frac{\hat{q}_{\tilde{l}} - \hat{q}_{\widetilde{l-1}}}{\pi_{\tilde{l}} - \pi_{\widetilde{l-1}}} + \pi^*_l \left(\frac{\partial^2 \hat{q}}{\partial \pi^{*2}}\right)_l \tag{52}$$

$$\frac{\partial^2 \hat{q}}{\partial \pi^{*2}} = \frac{(\pi^*_l - \pi^*_{l-1})\hat{q}_{l+1} - (\pi^*_{l+1} - \pi^*_{l-1})\hat{q}_l + (\pi^*_{l+1} - \pi^*_l)\hat{q}_{l-1}}{(\pi^*_{l+1} - \pi^*_l)(\pi^*_l - \pi^*_{l-1})(\pi^*_{\tilde{l}} - \pi^*_{\widetilde{l-1}})}. \tag{53}$$

This give us:

$$\tilde{A}_l = -\frac{\pi^*_l}{\delta_l}\beta_l \tag{54}$$

$$\tilde{B}_l = \frac{\pi^*_l}{\delta_l}\left(\beta_{l+1} - \alpha_{l-1}\right) + \pi^*_l \tag{55}$$

$$\tilde{C}_l = \frac{\pi^*_l}{\delta_l}\alpha_l \tag{56}$$

and

$$\overline{A}_l = 2\tilde{A}_l + \frac{\pi^*_l}{(\pi^*_l - \pi^*_{l-1})\delta_l} \tag{57}$$

$$\overline{B}_l = 2\left(\tilde{B}_l - \pi^*_l\right) + \frac{\pi^*_l(\pi^*_{l+1} - \pi^*_{l-1})}{(\pi^*_{l+1} - \pi^*_l)(\pi^*_l - \pi^*_{l-1})\delta_l} \tag{58}$$

$$\overline{C}_l = 2\tilde{C}_l + \frac{\pi^*_l}{(\pi^*_{l+1} - \pi^*_l)\delta_l}. \tag{59}$$

All three above dicretizations are implemented in siseve.F90. The switch for them is LDNOMUL, with value 1 for the first option, value 2 for the second option and value 0 for the third option.

3 Results

3.1 Code

The new vertical Laplacian was implemented in the code base on CY46t1mp_op2_nhxhypc, it is available in Prague on kazi1:/local/mma268/CY46t1mp_op2_nhxhypc_nika. The main change appears in SISEVE under switch LINPHI=.T.. The slope Λ in implicit scheme can be defined through the value of namelist parameter RINPHI_SLP.

3.2 2D Experiments

We did the 2D experiments with flat and Schär et al. [1] orography.

3.2.1 Flat orography

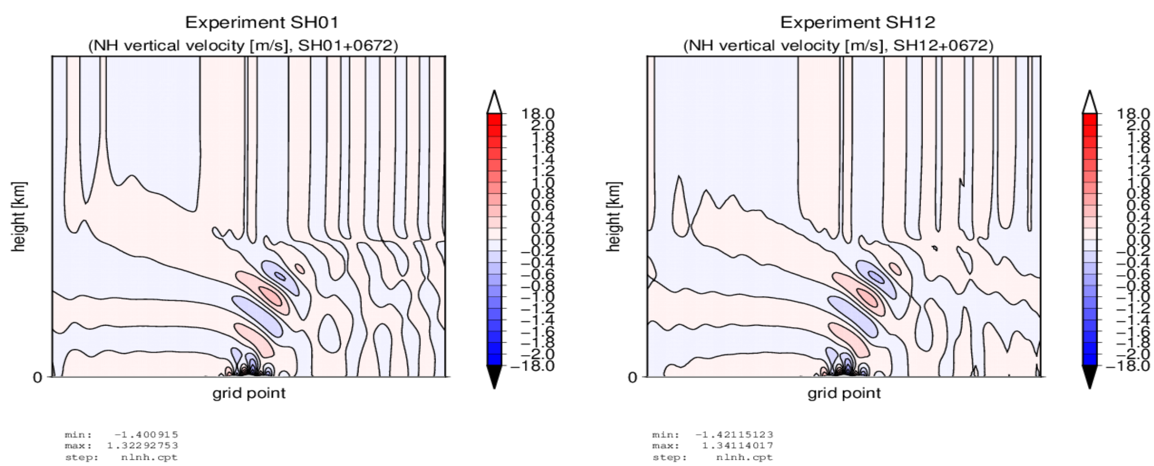
We tested all three discretizations on simple 2D case with flat orography. The resolution was $dx = 500$m and $dz = 250$m with 400 horizontal points and 180 vertical levels. The initial horizontal wind was $u_0 = 4ms^{-1}$. We ran the experiments with PC scheme and SITRA temperature of 50K, without sponge and with NVDVAR=5. Time step was set to 16s and we run the forecast for 6h.

It was found that the third option for discretization did not work even for a slope of 0. The eigenvalues of the vertical Laplacian in this case had non-zero imaginary parts, which led to an unsolvable Helmholtz equation. For this reason, we did not test this discretization further. The second discretization option worked for both slopes 0 and 1.

The first discretization option worked only with a slope of 0, where it produced the same operator as the old Laplacian. By "works," we mean that no vertical wind was generated, which is expected with no orography. However, when we set the slope to 1, the system crashed after 12 time steps due to an explosion in the horizontal wind. This is somewhat surprising, as there were no significant differences between the eigenvalues of the Laplacian with the first and the second discretizations and later we will see the similar results for the first and the second dicretizations in 3D experiments.

3.2.2 Schar orography

For Schar test we used schar orography with maximal height 250m, parameter $a = 5000$ and parameter $\lambda = 4000$. We had 400 horizontal points with $dx = 500$m, 150 vertical levels and $dz = 250$m. The horizontal wind was $10m/s$. We ran the experiments with the PC scheme and SITRA temperature of 100K, with sponge and NVDVAR=5. Time step was 32s. Since the first and the third decretizations did not even operate on flat orography, we decided to test only the second discretization option with slope equal to 0 and 1.



(a) $\Lambda = 0$ (b) $\Lambda = 1$

Figure 1: Vertical wind after 6h of integration with different value of Λ

The results of this experiment is illustrated on Figure 1. It is hard to say which result are more stable. Then we increased slope to 2 and got unstable forecast with too much vertical wind. See Figure 2

We also made some experiment with potential flow. In this experiments we have $dx = dy = 200$m $dt = 1$s and $u_0 = 15m/s$. We use PC scheme with SITRA= 50 K and NVDVAR=5, without sponge. For first two discretizations we got the expected results with slope=1 (see Figure 3).

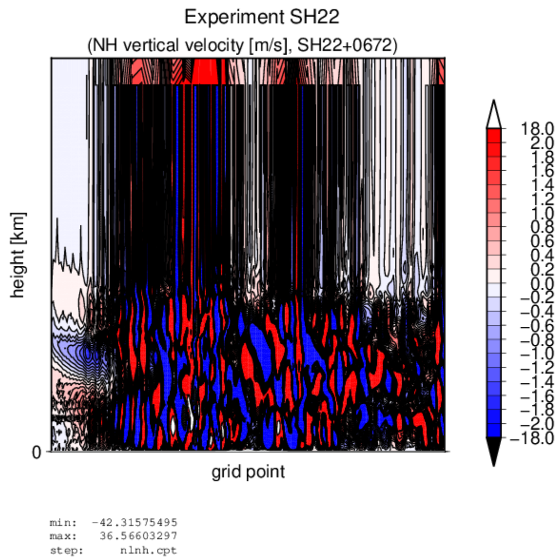
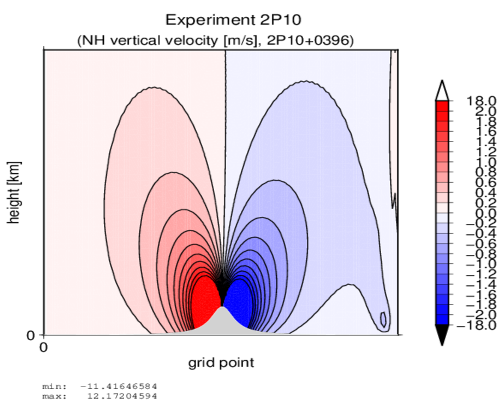
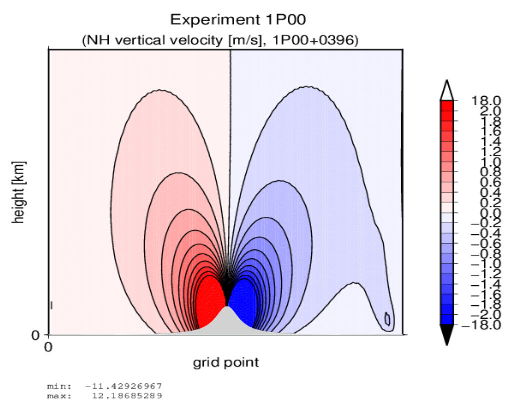


Figure 2: Vertical wind after 6h of inegration with different value of $\Lambda = 2$



(a) $\Lambda = 1$



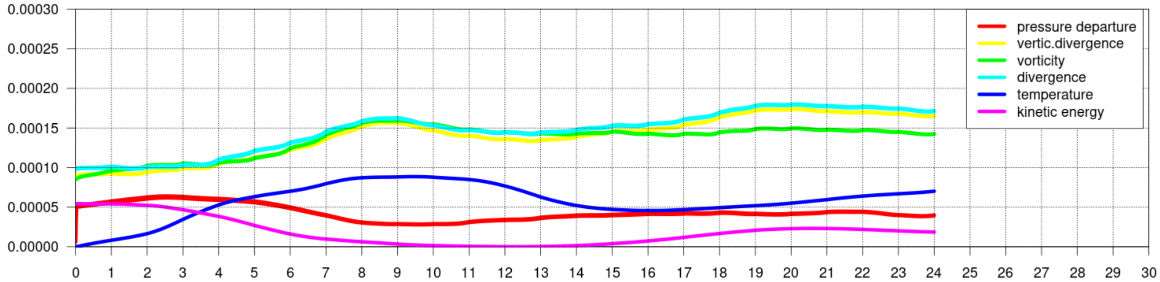
(b) $\Lambda = 0$

Figure 3: Vertical wind propagation computed with the second discretization

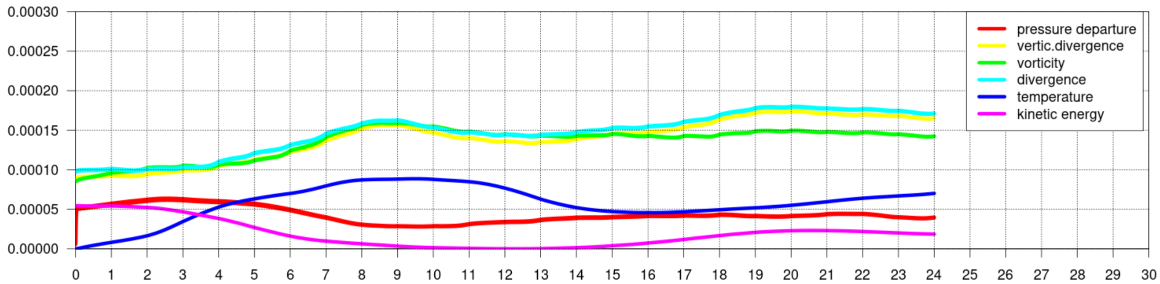
3.3 3D experiment

We focus on the case of strong winds from 19 August 2022 00 UTC, forecast for 24 hours in high resolution (200m) over the Alps. We use the Czech operational setting of the model ALARO for 3D simulations (PC scheme with one iteration and SITRA=100K, SLHD, 3MT, ACRANEB2 and TOUCANS, model 2, as physics parameterizations) and NVDVAR=5. we use lateral boundary conditions from the Czech operational run and the initial file interpolated from the initial file of the Czech operational run (2.325km of horizontal resolution, 87 vertical levels and the time step of 90s) with lateral boundaries provided by Météo France based on ARPEGE. The initial file is provided using 3DVar and CANARI.) and get balanced with DFI. Here, we cut spectral orography with cubic truncation. The time step used is 8s. Here, the usage of $\Lambda = 0$ (for reference) and the second discretization with $\Lambda = 1$ results in stable runs whose spectral norms evolution.

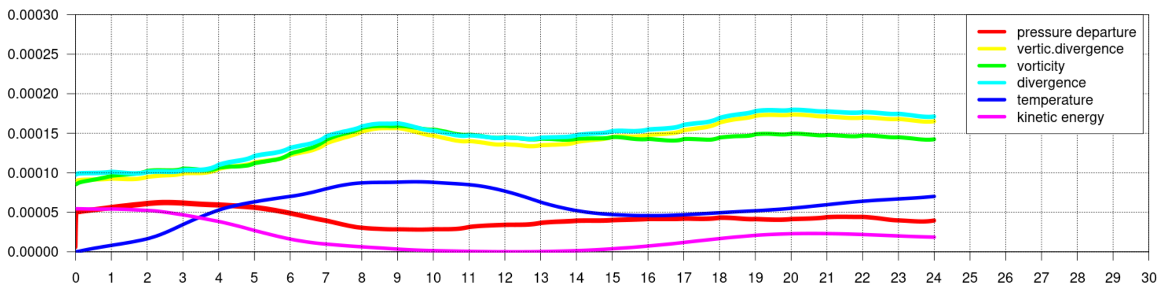
Then we compare first discretization with $\Lambda = 0.5$, $\Lambda = 1$ and $\Lambda = 2$. We observe no significant difference between runs with $\Lambda = 0.5$ and $\Lambda = 1$, but $\Lambda = 2$ results in instability. On the Figure 4 are norms of the runs with the second dicretization and the different Λ. We have not presented the results with the first discretization here, since even in the 2d experiment it turns out that we do not get stable results with it.



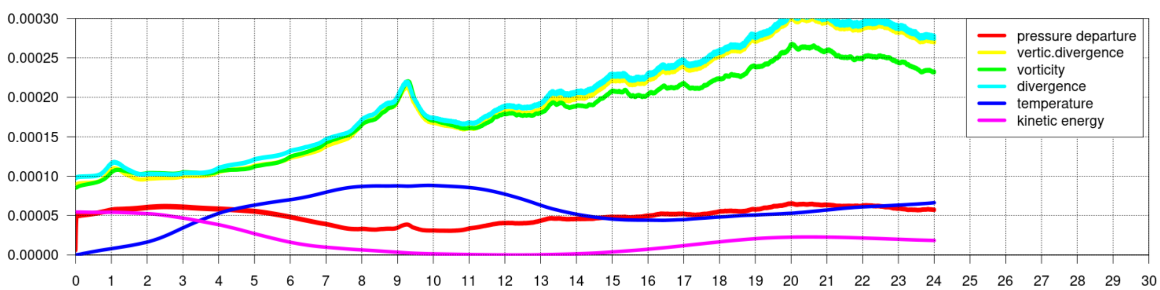
(a) $\Lambda = 0$



(b) $\Lambda = 0.5$, the second discretization



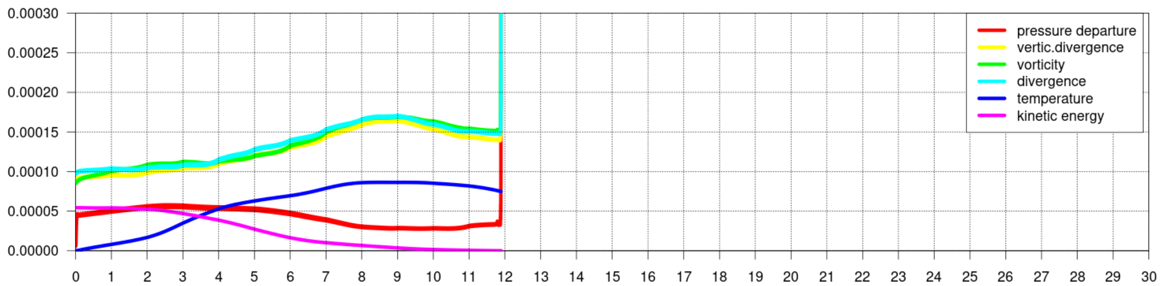
(c) $\Lambda = 1$, the second discretization



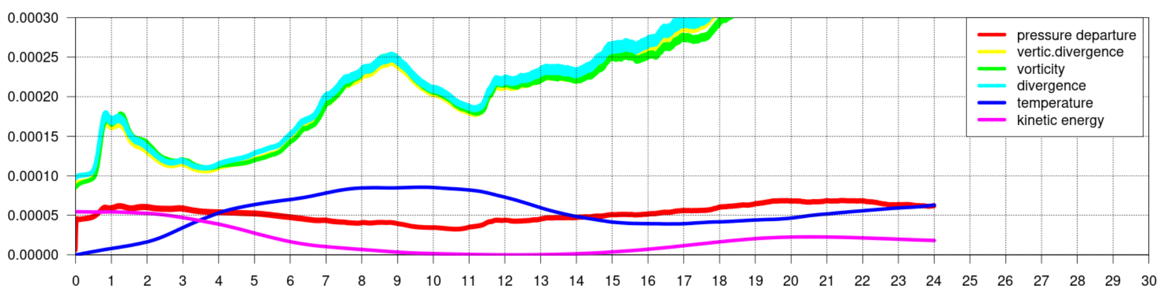
(d) $\Lambda = 2$, the second discretization

Figure 4: The spectral norms of model variables, PC scheme, horizontal resolution 200m, time step 8 s, the second discretization with different Λ

We also tried to increase the time step to 16s and in this case the 24h of forecast with $\Lambda = 1.5$ was completed but not in case of $\Lambda = 0$ (old vertical Laplace). This could be a sign of better stability, but anyway there appears a lot of noise in the norms (Figure 5) and 16s is too large for the other respects as well.



(a) $\Lambda = 0$



(b) $\Lambda = 1.5$

Figure 5: The spectral norms of model variables, PC scheme, horizontal resolution 200m, time step 16 s

4 Conclusion

In summary, only the second discretization method, where the product rule for differentiation is ignored in all terms, has proven to be effective. The failure of the first discretization method, even in the case of flat orography, suggests a potential error in the code implementation. Furthermore, the representation of geopotential in the implicit part of the system shows promise for improved stability. To confirm this, it will be necessary to test the model on more unstable cases.

References

[1] Ch. Schär et al., 2002: *A New Terrain-Following Vertical Coordinate Formulation for Atmospheric Prediction Models*, Month. Weath. Rev. 130, 2459–2480, doi: 10.1175/1520-0493(2002)130<2459:ANTFVC>2.0.CO;2.