The trajectory search in the SL advection scheme

RC LACE stay report
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1 Introduction

In a semi-Lagrangian model, the trajectory is computed in an iterative way in the time scheme. To compute the unknown position of the departure point at moment t we need to know the position of the final point (a known gridpoint) at moment $t + \Delta t$. The most common number of iterations used is small (2 or 3).

Recent studies show that forecast improvements can be determined by an increase of iterations, especially for extreme weather cases with high wind speed [1]. Furthermore, in global model IFS of ECMWF a dynamic approach of choosing different number of iterations for each gridpoint, depending on the convergence rate of the trajectory search for that specific point is implemented.

After integrating the LAM model configuration ALARO to a higher horizontal resolution, some improvement was observed in the verification scores when using 3 iterations instead of 2 for trajectory search. The purpose of this study was to assess the differences which may appear with higher number of iterations and represents a continuation of the previous stay.

In order to track changes when increasing the number of iterations, a convergence rate was defined. As a first step, the distances between the origin points obtained in consecutive iterations were computed. Using these definition of distances, a convergence criterion was introduced: the consecutive distances ratio.

2 Implementation in the cycle 43t1

All necessary changes were implemented in version cy43t1 of the model, in routine *elarmes*. Hence, routines *lapinea* and *call_sl* were modified accordingly. The distances were computed as follows:

$$\delta x_{ik}^{(l)} = \frac{|x_{ik}^{(l)} - x_{ik}^{(l-1)}|}{d_k}, \tag{1}$$

where ik refers to the $i-th$ gridpoint of the $k-th$ level. For a given gridpoint, $x_{ik}^{(l)}$ is the coordinate of the departure point after l iterations. Distances were computed separately in both horizontal and vertical components. For the horizontal part, d_k is the horizontal resolution Δx, while in vertical d_k represents the distance between two vertical levels of the model.

Having these values defined, the next step was to define a convergence rate for each gridpoint, as the following ratio:

$$cr_{ik}^{(l)} = \frac{\delta x_{ik}^{(l)}}{\delta x_{ik}^{(l-1)}} = \frac{|x_{ik}^{(l)} - x_{ik}^{(l-1)}|}{|x_{ik}^{(l-1)} - x_{ik}^{(l-2)}|}. \tag{2}$$

For both diagnostics, we considere that a value exceeding 0.5 may indicate a convergence problem of the iterative method for the considered gridpoint.

3 Results

These quantities were evaluated for a strong wind winter period (03 - 09 January 2017), in three horizontal resolutions: 1km, 2km and 4km. The vertical resolution was 87 levels. However, results are presented below for the 4th of January 2017, when the maximum values of wind speed occured (as well, higher distances), for 48 hours forecast (00 UTC run) and level 50.

Figure 1 shows the horizontal distances $\delta x_{ik}^{(l)}$ after several consecutive iterations (from 2 to 9), for 1km resolution. It can be observed that there is a clear decrease in distances with each iteration; for example, after 9 iterations, distances are less than 0.005 in all points. The same applies for distances computed in vertical, but in this case the values of distances are slightly larger (not shown).

The largest distances were observed in the high mountainous area in the south-western part of the domain. The pattern is similar in all resolutions (1km - Figure 1, 2km and 4km - Figure 2). This area is also characterized by large wind speed values for this date (Figure 3).

Figure 4 shows the histogram of distances, for 4km resolution (first column) and 1km resolution (second column) after 2 (first row), 3 (second row), 4 (third row) and 5 (fourth row) iterations. The distribution of values according to selected thresholds is quite similar in both resolutions. However, it seems that percentages of smaller values are bigger in higher resolution.

Distances were evaluated for all levels, the values having an incresing trend beginning with middle levels. Maximum values after two iterations were noticed mostly in bottom levels. The distances were evaluated hourly, for up to 48 hours forecast. It seems that distances increase after longer integration. In accordance with the results shown in Figures 1 and 2, it was observed that these distances decrease with more iterations, for most of the points. All distances are smaller than 1, but distances bigger than 0.5 may already indicate divergence.

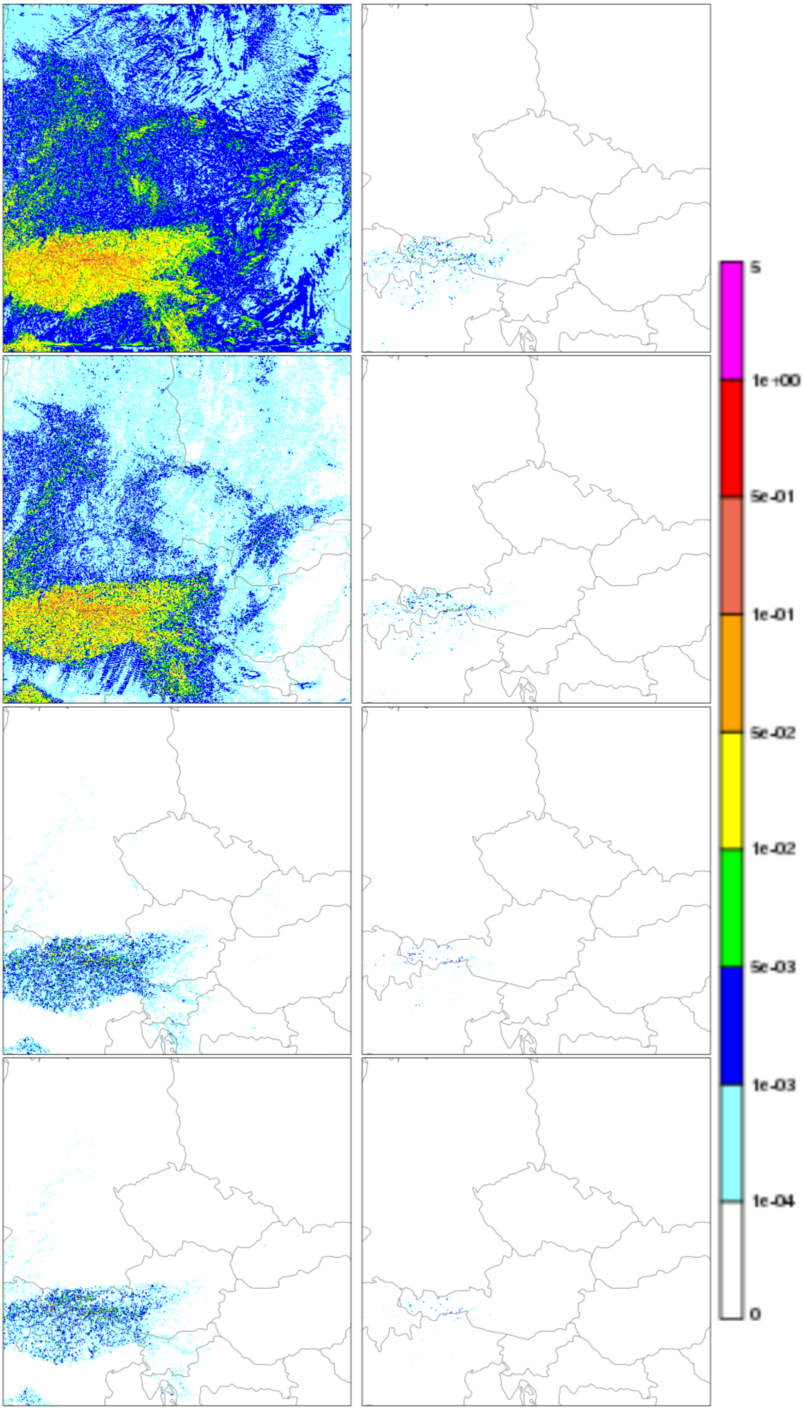

Figure 1. Distances after consecutive iterations: l=2, l=3, l=4, l=5 (first column), l=6, l=7, l=8, l=9 (second column); level 50, forecast from 04 January 2017, 00 UTC for 48 hours; 1km resolution

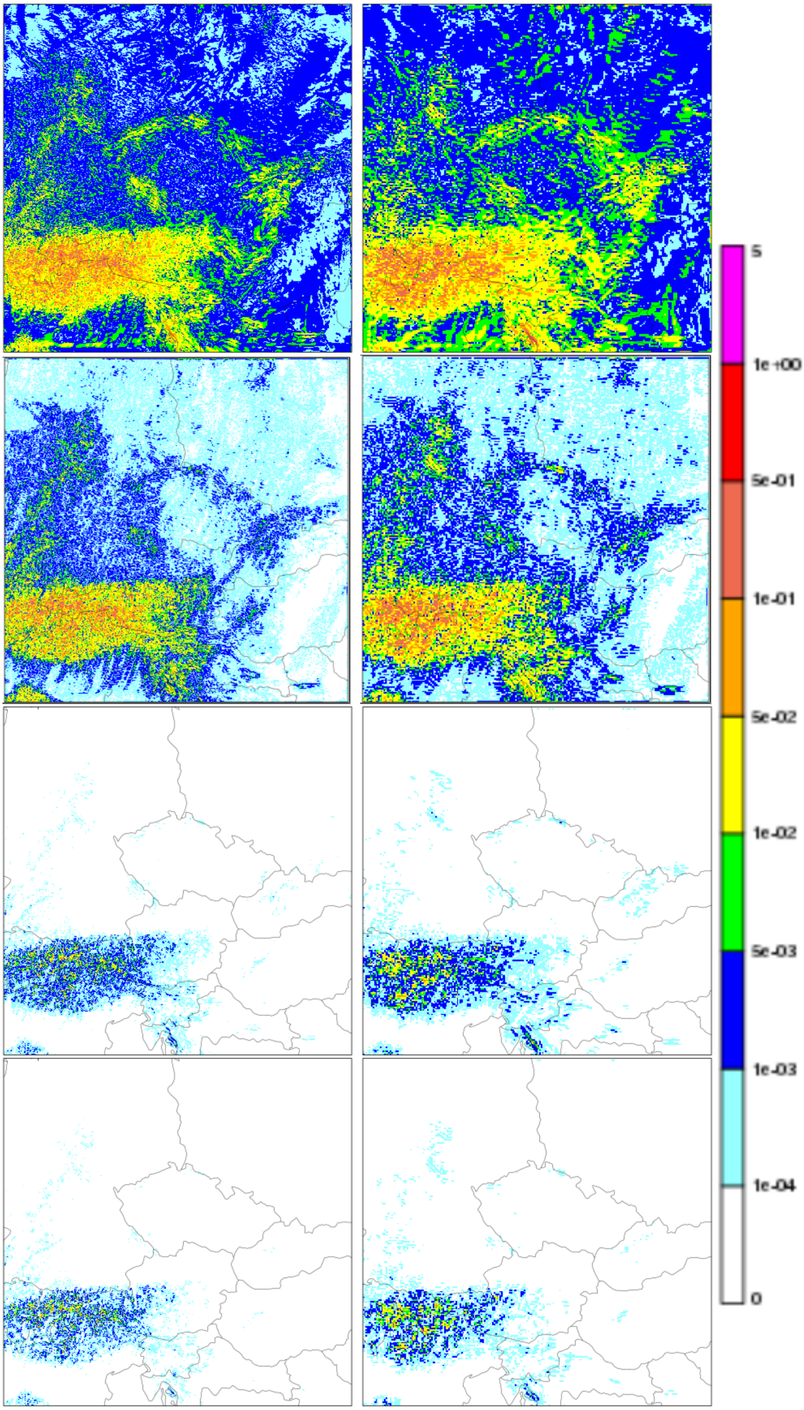

Figure 2. Distances after consecutive iterations, from top to bottom: l=2, 3, 4, 5, level 50, forecast from 04 January 2017, 00 UTC for 48 hours; 2km resolution - first column and 4km resolution - second column

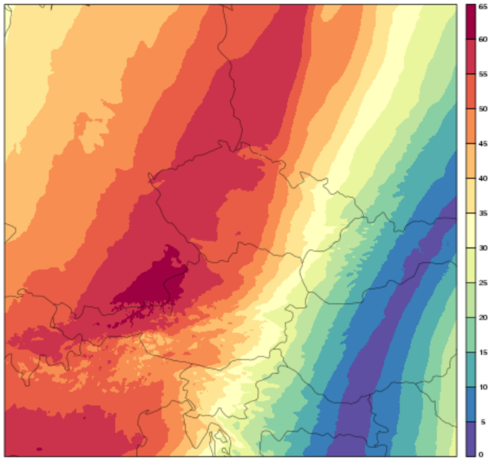

Figure 3. Wind speed at level 25, forecast from 04 January 2017, 00 UTC for 48 hours, 1km resolution

In the search for points with different behaviour than the apparent decrease in distance after each iteration, it was checked if there are points for which the distances increase after each iteration, in all ranges (up to 48 hours) and levels, for the whole period. It was found that such points exist: for example, in horizontal, in 4km resolution, for the 4^{th} of January, there are very few points with this behaviour (all in the same vicinity). In higher resolution experiments (2km and 1km) for the same day there are no points with this property anymore.

Figure 5 shows the values of the convergence rate as defined in equation (2). This definition can lead to different pattern for odd and even iterations, when very small distances occur in the denominator. To avoid this behaviour, points for which $|x_{ik}^{(l)} - x_{ik}^{(l-1)}| < \varepsilon$ are not represented in this figure, assuming the convergence was reached for them. The chosen value for this threshold was $\varepsilon = 10^{-2}$.

Considering that for a point, a convergence rate that exceeds 0.5 is a divergence indicator, it can be noticed that there are several areas with points that do not converge (blue and red points) in both resolutions. These areas seem to be reduced for finer resolution case (Figure 5 - second column).

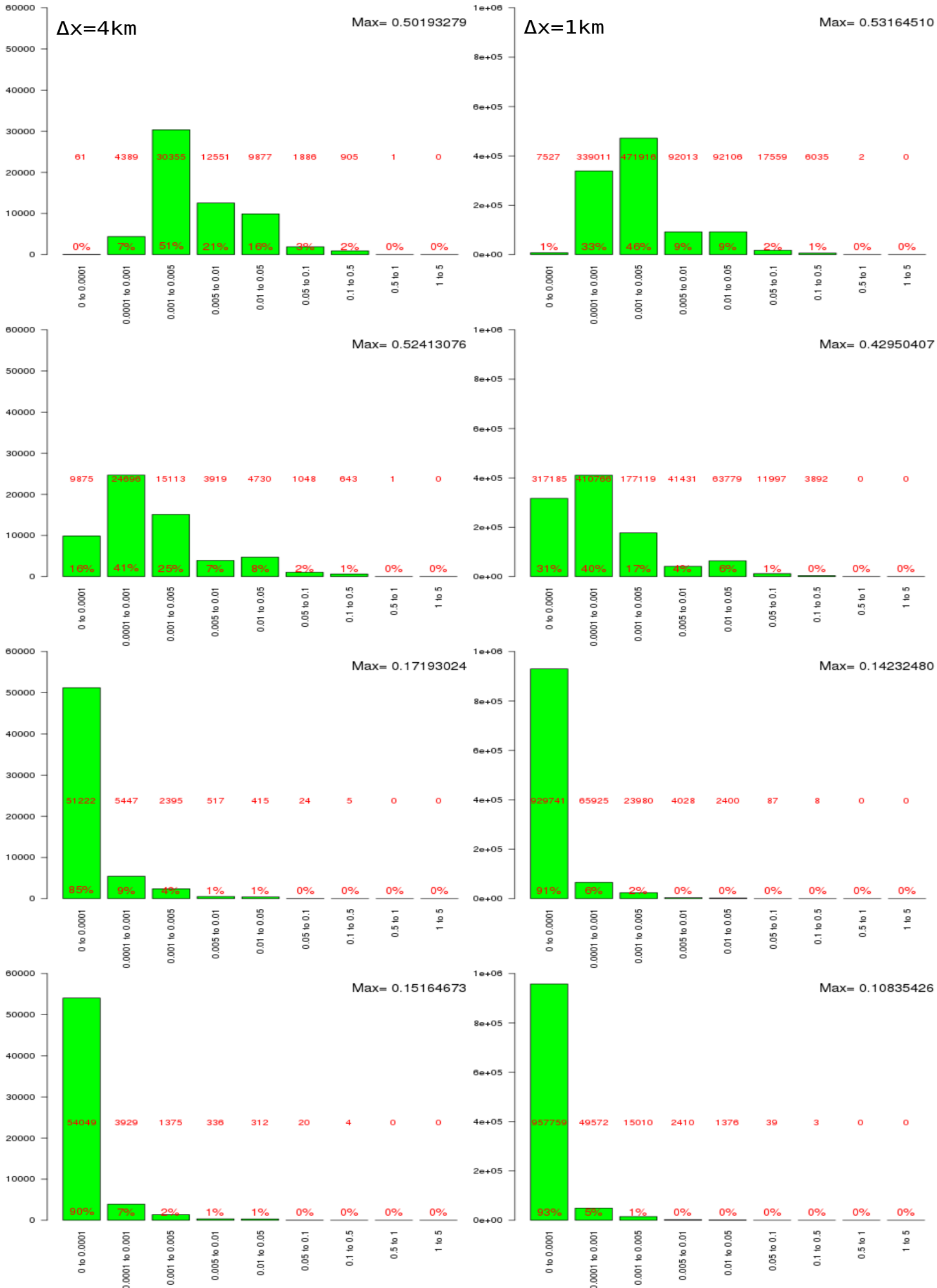

Figure 4. Histogram of distances after iterations, from top to bottom: l=2, l=3, l=4 and l=5, level 50, forecast from 04 January 2017, 00 UTC for 48 hours; first column - 4km resolution, second column - 1km resolution

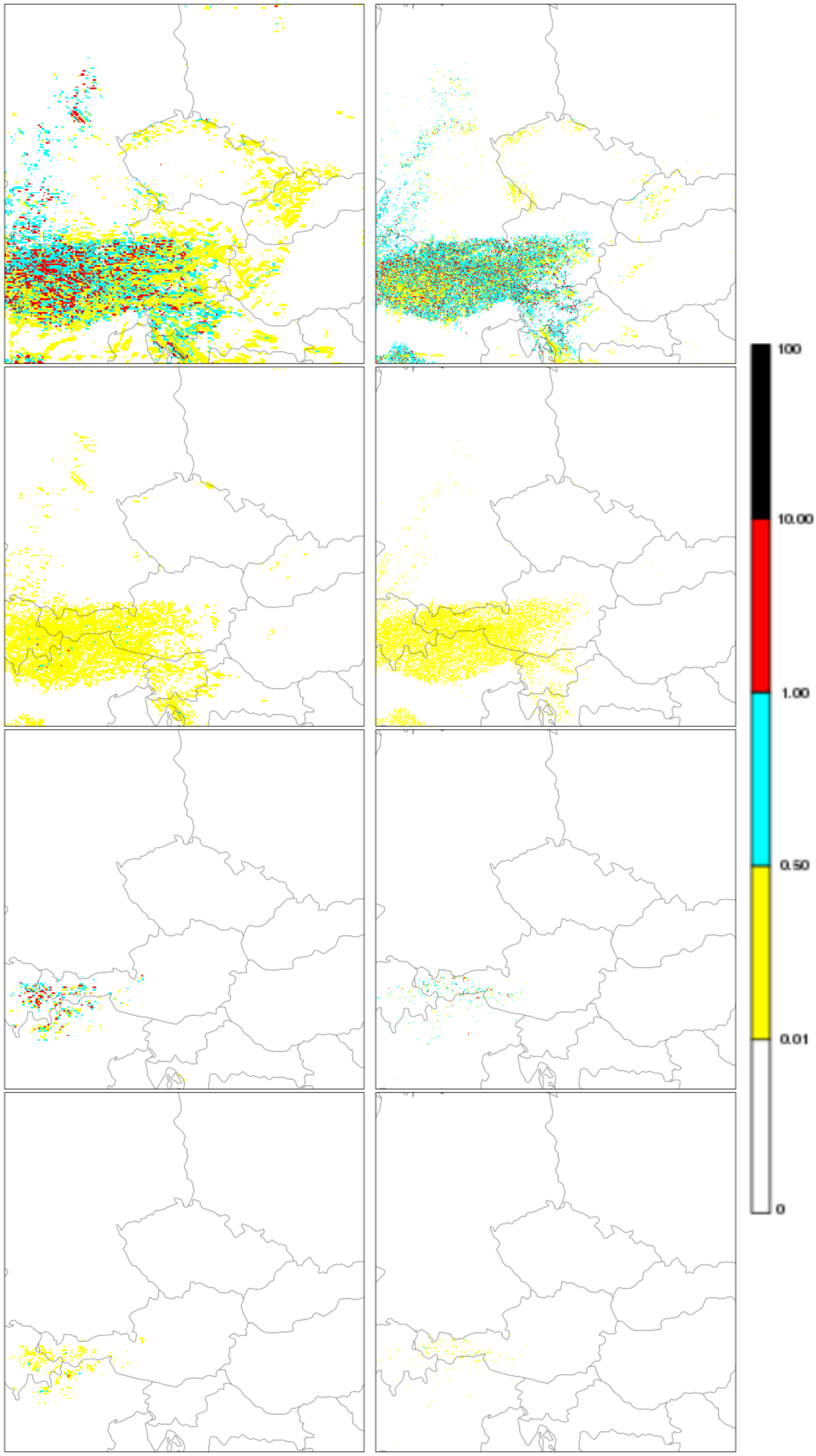

Figure 5. Convergence rate after consecutive iterations, from top to bottom: l=3, 4, 5, 6, forecast from 04 January 2017, 00 UTC for 48 hours, L50; first column: $\Delta x = 4km$, second column: $\Delta x = 1km$

Regarding this convergence rate (after 3, 4, 5 and 6 iterations) it was checked if there are points for which its value increases with each iteration and the value after the 6^{th} iteration is bigger than the 0.5 threshold. It was found that for 04.01.2017 there are 43 points in 4km resolution experiments, whereas in 1km runs there are 480 such points. Most of the points with this property are in levels closer to top and bottom in 1km case, but not in 4km (Figure 6).

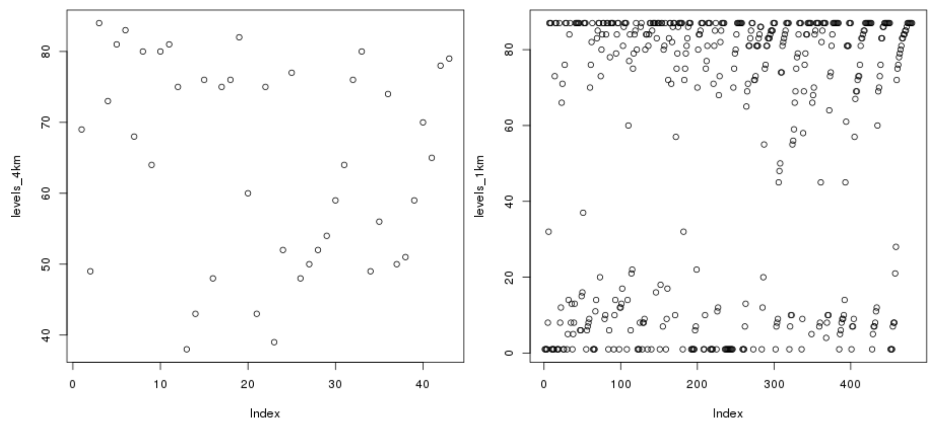

Figure 6. Levels in which the convergence rate increases with each iteration and the value after the 6th iteration is bigger than the 0.5, 4km - left and 1km - right, forecast from 04 January 2017, 00 UTC

When investigating the number of iterations needed for each point to reach a convergence rate smaller than 0.5 it was shown that in 4km run, the majority of the points "converge" after 3 iterations and a small number of points need 4 iterations to reach this threshold, while in 1km there are even few points that need more than 6 iterations to converge.

For the same date, there seems to be different behaviour in horizontal and vertical: the convergence is faster in vertical. The pattern is different along with levels: for example, in horizontal, the biggest number of points that converged after 3 iterations are in top and bottom levels, whereas in vertical are in top levels.

Another parameter for convergence assessment in the iterative algorithm is the Lipschitz number. This number was calculated in horizontal and vertical directions by Petra Smolíková and results show that this number has similar pattern in horizontal to the convergence indicators.

4 Conclusion

Several criteria for convergence of points in the iterative time scheme were evaluated. Large values of these parameters are obtained after longer integration and are associated with high altitude areas in the domain. Also, it was noticed that this amounts are in correspondence with strong wind speed. In addition, we may say that it is difficult to obtain an exact/absolute indicator of convergence in the iterative method.

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References

[1] M. Diamantakis and L. Magnusson, *Sensitivity of the ECMWF model to Semi-Lagrangian departure point iterations*, Mon. Wea. Rev., 144, 32333250, https://doi.org/10.1175/MWR-D-15-0432.1