Idealized tests and real simulations with finite elements used in vertical discretization of ALADIN-NH (cy40t1)

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The topic of the design of vertical finite elements scheme for ALADIN-NH is being solved since 2006 mainly in the frame of RC LACE. The main objective of this task is to have a stable and robust vertical finite elements (VFE) discretization to be used in high resolution real simulations with orography with the expected benefit being the enhanced accuracy for the same vertical resolution when comparing with vertical finite differences (VFD) method. We want to stick as much as possible to the existing choices in the design of dynamical kernel (SI time scheme, mass based vertical coordinate) and to stay close to the design of VFE in hydrostatic model version (according to Untch and Hortal).

As the most important task for the year 2014 it was identified the need to explain in details why the currently implemented version of VFE in NH model (cy40t1) works and what are its benefits. For this reason we have concentrated ourselves on the theoretical explanations and studies of the simple cases with known analytical solution to see the impact of vertical discretization method choice.

A slightly modified version of the idealized test setup used by Skamarock and Klemp was proposed by Baldauf and Brdar: the quasi linear 2-dimensional expansion of sound and gravity waves in a channel induced by a weak warm bubble. The modification allows derivation of an exact analytical solution for the compressible, nonhydrostatic Euler equations that are the basis for ALADIN-NH model. The derived analytical solution is supposed to be used as a benchmark to assess compressible dynamical cores. This test is designed for usual height based vertical coordinate models with vertical velocity imposed to be zero at bottom and top of the domain. These boundary conditions are natural for height based vertical coordinate models, but not for mass based vertical coordinate models as ALADIN-NH. In the original solution, the evolution of the perturbation is a set of waves that propagate horizontally.

However, for a mass based model, the vertical velocity is imposed to be zero at the bottom but at the top the model is open and there are not boundary conditions for the vertical velocity: atmosphere can evolve freely and can move up and down at the top. As a consequence, in a mass based vertical coordinate model, the evolution of the initial perturbation is a set of waves that propagate both in the horizontal and vertical directions. Trying to fix vertical velocity to zero at a given height is not an easy task, as the model itself is not prepared nor designed for such an imposition. Our simple proposition to solve this difficulty was to impose vertical velocity to be zero by the sponge instantaneously and directly on the upper boundary of 4km. However, the results show that mass is being lost through the upper boundary and the evolution of wave is smooth down by this fact.

As a consequence, the difference between the vertical velocity field of the experiment with FE used in vertical discretization and the one with FD used is order of magnitude smaller than the overall error of both experiments compared to the analytical solution. See Figure 1 for vertical velocity fields and Figure 2 for comparison of vertical velocity value in the height of 0.5km, 5km (middle of the domain) and 9.5km. The curves for VFE and VFD in various levels are almost indistinguishable, except for the higher level, where VFE seem to work better in the middle of the domain. There is some noise generated with VFE close to lateral boundaries.

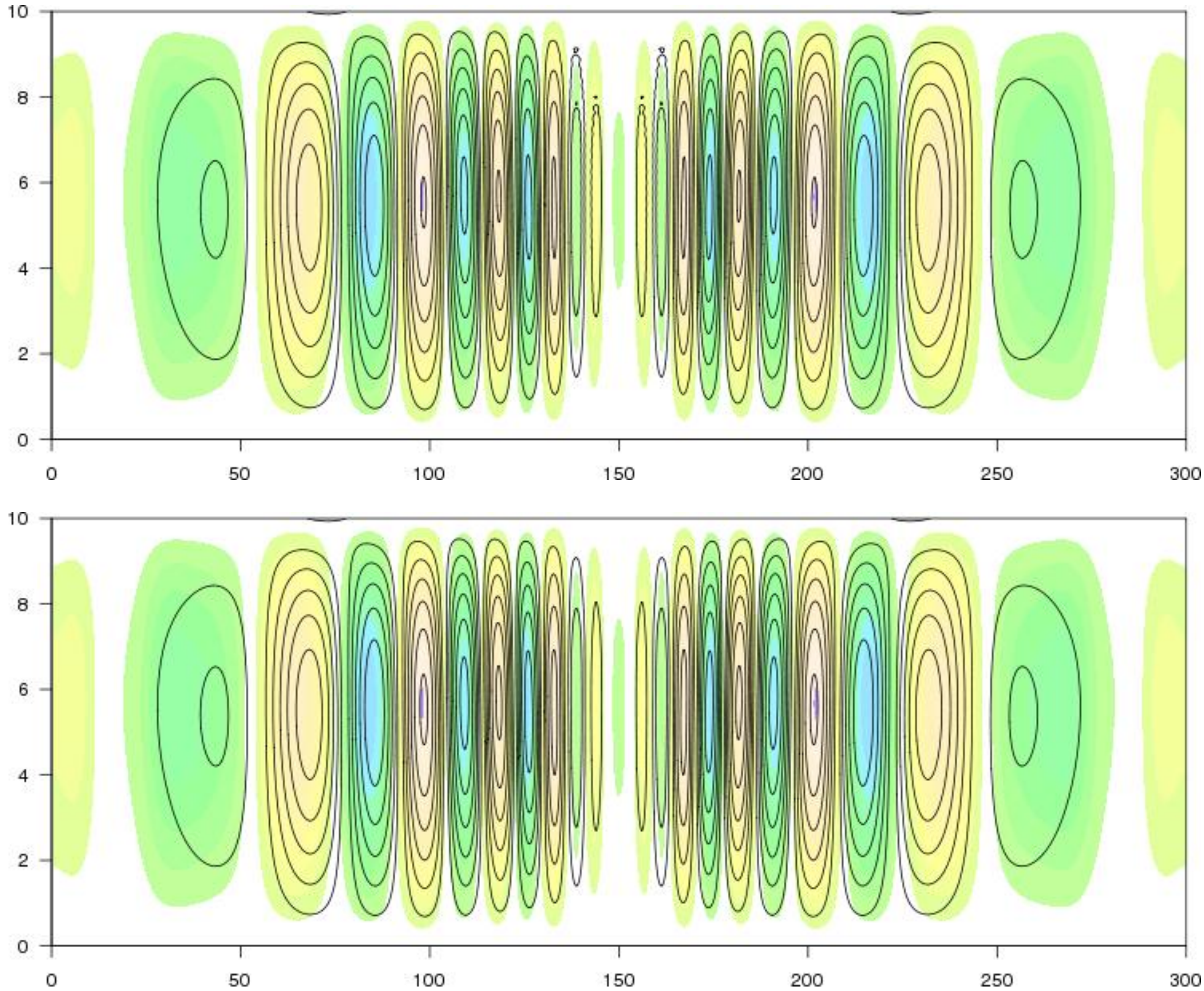

Figure 1: *Potential temperature field in Baldauf-Brdar test (gravity waves in a channel); shading shows analytical solution, while contours represent solution with VFE (top) and VFD (bottom).*

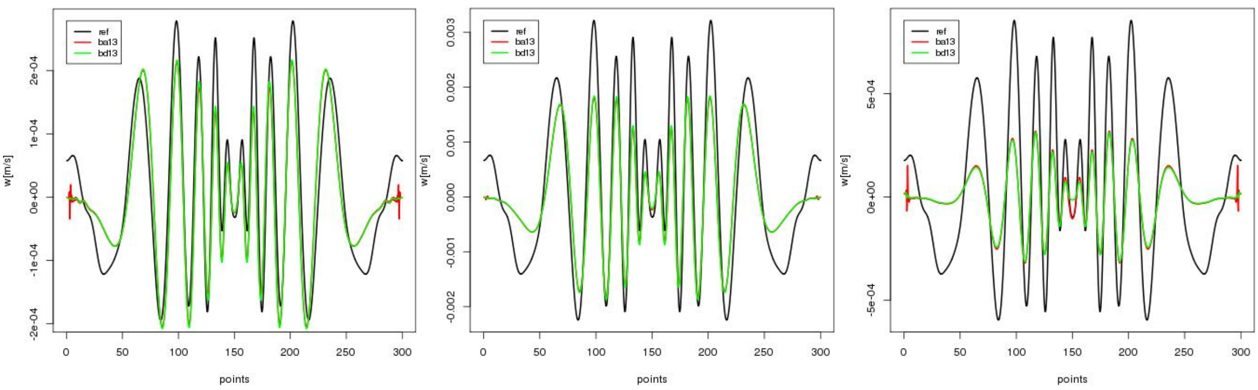

Figure 2: *Potential temperature in Baldauf-Brdar test for distinct vertical levels in 125m vertical resolution; left: 4th level in 0.5km; middle: 40th level in 5km (middle of the domain in vertical); right: 76th level in 9.5km. Analytical value is in black, VFE in red and VFD in green.*

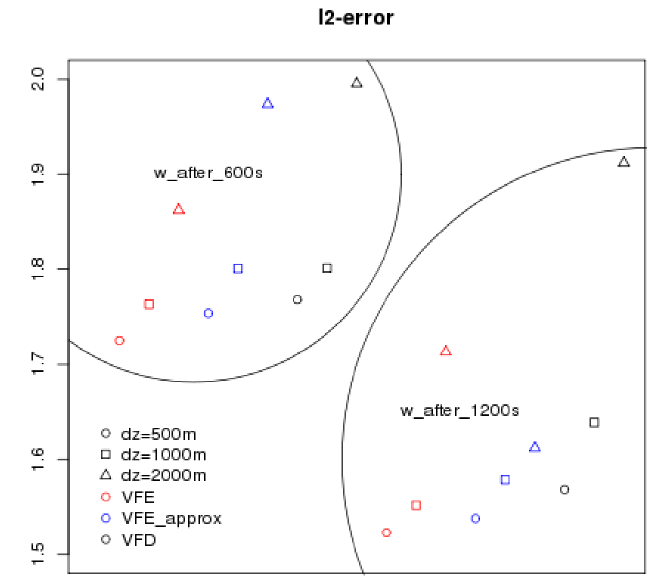

Figure 3: *Relative l2-error for vertical velocity field in distinct experiments with modified Baldauf-Brdar test after 300s and 600s.*

The idea of Juan Simarro has emerged that we may modify the initial perturbation of temperature to localize its maximum in the lower atmosphere. This perturbation would evolve as a set of waves propagating horizontally and vertically, and, because the initial perturbation is located in the lower atmosphere, it will take some minutes to get the upper atmosphere. During this time, the vertical velocity at the top will be zero in the analytical solution and, therefore, also should be zero in the numerical solution for any model, open or not at the top. He has modified the initial condition in the analytical solution calculation and we have run another set of experiments for this new simple case.

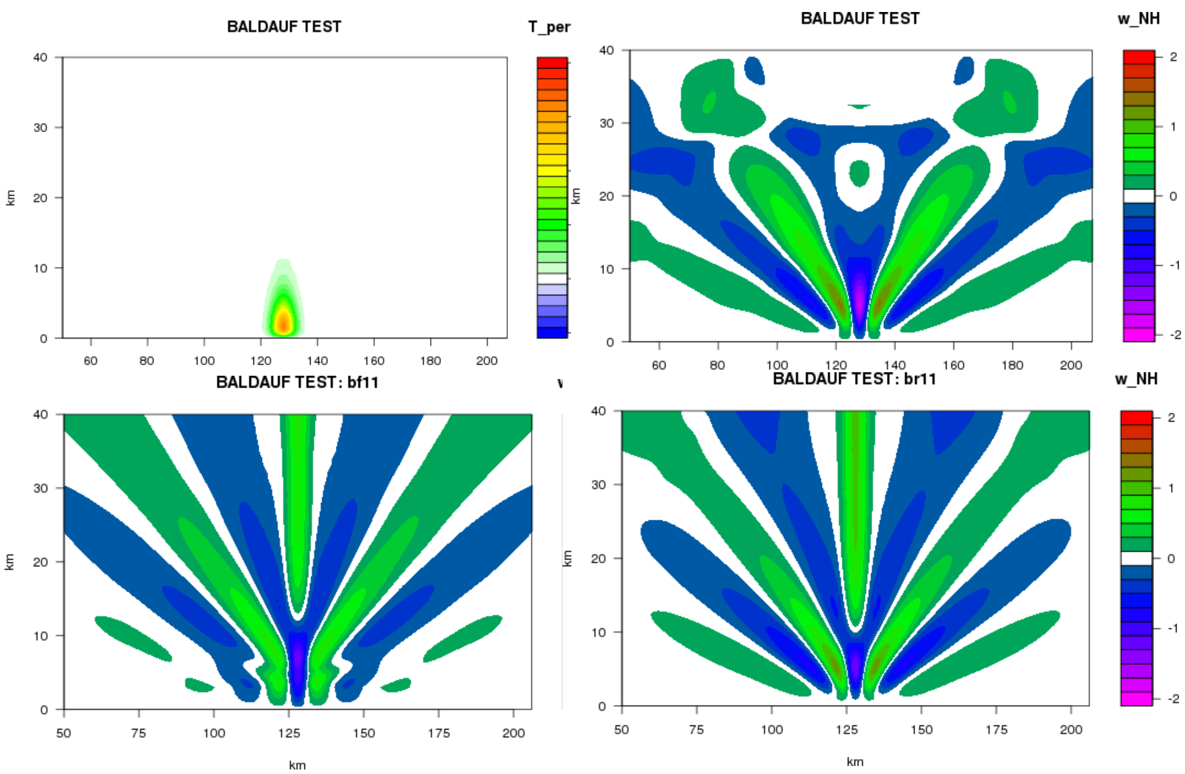

Figure 4: *Potential temperature perturbation in the lower atmosphere according to Simarro. Top left: initial field of potential temperature; top right: vertical velocity field after 600s – analytical solution; bottom: numerical solutions for VFE (left) and VFD (right).*

In this experiment the maximum perturbation was 0.01K from the basic value 250K at the middle of the domain in horizontal and in the height of 2km. The horizontal resolution was 500m with 256 points in horizontal and successively, 500m, 1000m and 2000m in vertical with corresponding number of vertical levels to get 40km of vertical extent of the whole domain. The timestep used was 0.5s to avoid errors of time discretization to influence the results and the integration has continued until 600s has been reached.

Unfortunately, it is not true that the vertical velocity at the top of the model will remain zero. In the mass based vertical coordinate model as ALADIN-NH is, the sink of mass through the top is unavoidable and the solution is again distorted. On the other hand, we could notice that the accuracy of the experiment with FE was enhanced compared to the one with FD in vertical discretization. And this claim holds for all three resolutions used, 500m, 1000m and 2000m, see Figures 3 and 4 for an illustration of this fact.

As conclusion, it is difficult to show benefit coming from the usage of FE in the vertical discretization.

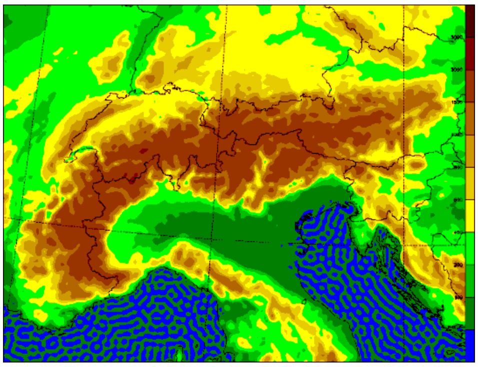

Figure 5: Orography of 3D simulations: the domain over Alpine region.

Another important task for VFE topic was to show that FE may be as stable as FD in 3D real simulations in high horizontal resolutions. We have chosen 2 months of experiments, January 2014 and July 2014, over the domain covering the Alpine region. Consult Figure 5 for the orography used. The horizontal resolution was 1.25km and vertically we have used Czech operational setting of 87 levels. We have run one integration per day from 00UTC until +24hours. Results obtained have confirmed previous results obtained with coarser resolution of 2.2km over Czech domain. We may conclude:

1) VFE scheme used in NH with proper setting of FE parameters and proper setting of vertical levels may be as stable as FD scheme; the time step used in our experiments was 50s.

2) It is difficult to find any benefit from FE used in vertical discretization concerning objective scores.

3) The precipitation field is modified by FE in such a way that there is bigger number of grid points without rain (cumulated precipitations for 1 hour < 0.1mm) and bigger number of grid points with highest values of cumulated precipitations (>30mm/hour). Consequently, there is smaller number of grid points with modest rain between 0.1 and 30mm. We consider this trend as beneficial since the field of precipitation is more sharpened and the rain locations are more restricted. We should admit that this phenomenon is clearly present but not as intensive as to be observed easily. Our conclusion comes from the statistical analysis of results; see Figure 6 and 7 for histograms of cumulated precipitations for each hour in each

forecast made. Concerning the intensity of observed trend, we may summaries: we have 24 events per one day in the months, i.e. 744 events per grid point; we have 403x694 grid points in the area for histogram calculation; thus we have 2.10^{8} events in the area, with our 17 histogram intervals the total amount of events in which the two experiments series (FE and FD) differs is 6.10^{5}; we see that two experiment series differ only in 0.3% of events when using our histogram intervals. Nevertheless, the phenomenon is observed in both, summer and winter series, and it is clearly pronounced.

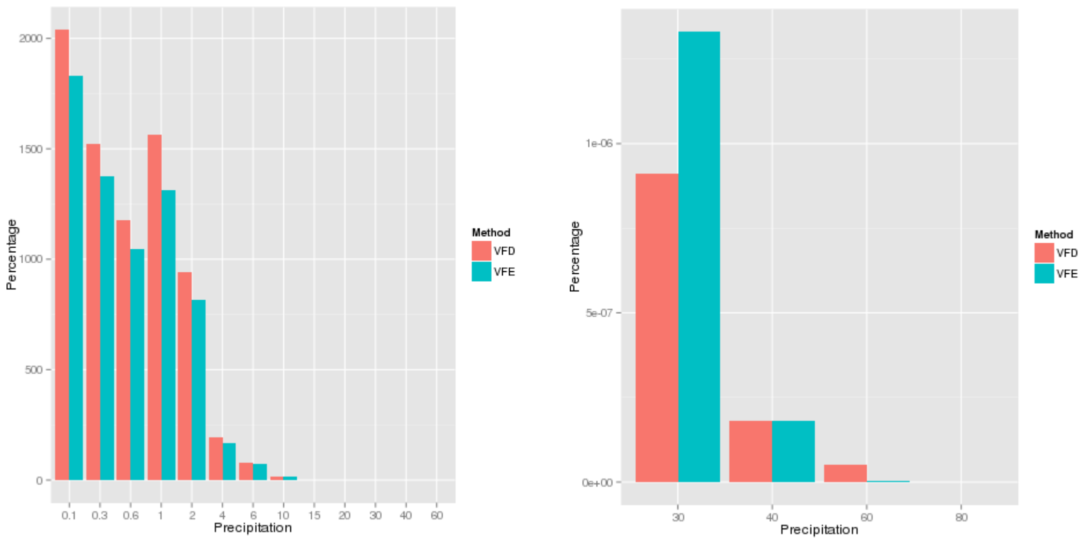

Figure 6: *Histograms for winter period of 3D simulations (January 2014). To be able to visualize the difference between FE and FD experimental series, we have rescaled the results in such a way that only 10% of common events are taken with the remaining number of events in which two series differ.*

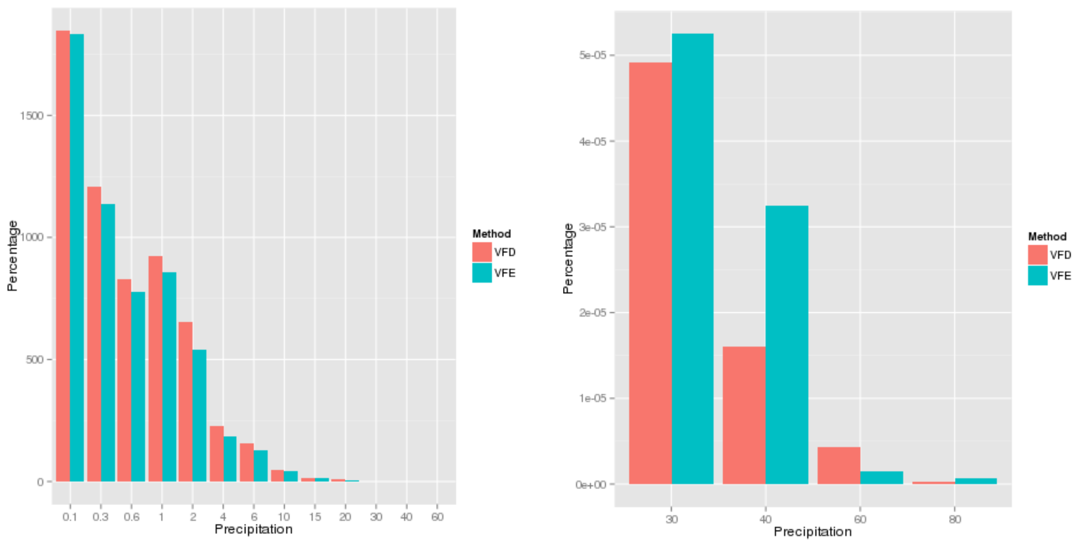

Figure 7: *Same as Figure 6 but for summer period of experiments (July 2014).*

References

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