Second order accurate time scheme for the physics-dynamics interface based on SETTLS technique

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1 Preface

The presented work has been inspired by the ideas of Mariano Hortal and thoroughly discussed with Filip Váňa. We thank for the concept proposal and the helping advices.

The present coupling of physics to dynamics offers very stable and robust solution. The physical tendencies are calculated once per time step at its beginning from the values in time t and then interpolated to the origin point. This explicit treatment shows the unconditional stability for a wide range of linear problems. The price to pay for it is its only first order accuracy in time. Using the SETTLS technique for the physical tendencies the present time-stepping should be easily extensible to a second order accuracy coupling without a need to change the timestep organization. As shown in Appendix 6 this is paid by more restricted stability properties. However, the SETTLS type scheme still remains unconditionally stable for a group of linear problems. It is successfully used for the non-linear part of dynamical kernel of the AAA model. This qualifies it as a promising solution for the physics-dynamics coupling as well. We implement it to the cycle cy36t1ope of the AAA model code (see Section 4 for the description). Then we realize a set of real case simulations with the LAM model and ALARO physics and resume the results in Section 3. In Appendix 6 we give a summary of the stability analyses of the current and proposed scheme. In Section 5 we draw up some conclusions.

2 Proposed time scheme

Let assume that the evolution of the system writes

$$\frac{d\psi}{dt} = \mathcal{R}(\psi), \tag{1}$$

It was shown in [2] that all the $\mathcal{O}\left((\Delta t)^2\right)$ accurate approximations of (1) using time levels t and $t-\Delta t$ and space locations of the origin point O and of the final point F for the evaluation of the right hand side term $\mathcal{R}(\psi)$ may be expressed by the following formula

$$\frac{\psi_F^+ - \psi_O^0}{\Delta t} = \left(\frac{3}{4}-\alpha\right)\mathcal{R}(\psi_F^0) + \left(\frac{3}{4}+\alpha\right)\mathcal{R}(\psi_O^0) - \left(\frac{1}{4}-\alpha\right)\mathcal{R}(\psi_F^-) - \left(\frac{1}{4}+\alpha\right)\mathcal{R}(\psi_O^-)$$

with an undetermined parameter α. It was shown in [1] that the choice of $\alpha = \frac{1}{4}$ which corresponds to the SETTLS scheme designed in [3] yields the best stability properties in comparison with other possible choices of α. Since the stability depends on the Courant number its quality is difficult to judge. A region which renders absolutely stable solutions independent of the Courant number is assigned to each α. The size of this region varies dramatically with α and has its maximum size for $\alpha = \frac{1}{4}$ (see Appendix 6 for more details).

Let now assume the separation of $\mathcal{R}$ into two parts as done in the design of the AAA model

$$\frac{d\psi}{dt} = \mathcal{M}(\psi) + \mathcal{P}(\psi), \tag{2}$$

where $\mathcal{M}$ denotes the model dynamics and $\mathcal{P}$ the model physics, i.e. unresolved atmospheric processes. Then the model dynamics is treated in a SI manner by an arbitrary separation of the evolution terms between a linear part $\mathcal{L}$, treated in a centered implicit way, and a non-linear residual $\mathcal{N} = \mathcal{M} - \mathcal{L}$, treated explicitly.

We shall consider now the treatment of the physics part $\mathcal{P}$. In a current operational version of the AAA model the two-time-level SI SL scheme with the SETTLS type discretization of the non-linear residual according to [3] is used. It writes

$$\frac{\psi_F^+ - \psi_O^0}{\Delta t} = \mathcal{L}\left[\frac{1}{2}\left(\psi_F^+ + \psi_O^0\right)\right] + \mathcal{N}\left[\frac{1}{2}\left(2\psi_O^0 - \psi_O^- + \psi_F^0\right)\right] + \mathcal{P}\left(\psi_O^0\right), \tag{3}$$

where $\mathcal{P}$ is calculated explicitly in the origin point. The proposed SETTLS type treatment of the physics-dynamics coupling writes

$$\frac{\psi_F^+ - \psi_O^0}{\Delta t} = \mathcal{L}\left[\frac{1}{2}\left(\psi_F^+ + \psi_O^0\right)\right] + (\mathcal{N} + \mathcal{P})\left[\frac{1}{2}\left(2\psi_O^0 - \psi_O^- + \psi_F^0\right)\right]. \tag{4}$$

where $\mathcal{P}$ is treated similarly as the non-linear residual $\mathcal{N}$.

3 Experiments

We run the real model simulations starting from the ARPEGE results from the 9th of November 2010, 00UTC, on the operational domain for the Czech Republic with 4.7km resolution in horizontal and 87 vertical levels. The configuration was the operational one, i.e. two time level SL SI time scheme with the SETTLS extrapolation for the non-linear part of the dynamical kernel and pseudo-second order decentering of 0.05, in the hydrostatic regime, with the physics of the ALARO (cy36t1ope) package. We run the forecast for 54 hours, with the time step of 180s. We denote "*reference*" the described experiment with the current explicit coupling of physics to dynamics. And we speak about "*settlsp*" experiment when the SETTLS type physics-dynamics interface is used.

Vertical diffusion treatment

In the *settlsp* experiment we faced the difficulties first in the lowest model levels. We have encountered a lot of "SMILAG TRAJECTORY UNDERGROUND" messages and we could see big changes in the temperature budget near the ground. See the DDH graphics on the Fig.1. The time oscillations in the physical tendencies of all the advected variables were generated, see the central plots of Fig.2 for the oscillations in the temperature tendencies. We suspected the vertical transport for these oscillations and hence we tried to cure this problem by excluding the vertical diffusion from the SETTLS type coupling. Thus in the following experiment all the tendencies from physics were treated by SETTLS type scheme, while the tendencies from vertical diffusion contributing to the final tendency of the temperature, the horizontal wind components and the specific humidity q, were coupled explicitly. See the right hand side of Fig.1 for the DDH plots and the bottom of Fig.2 for the time evolution of temperature tendencies. The diagnostic near the ground were substantially reduced towards more satisfactory values and this specific kind of time oscillations in physical tendencies was eliminated.

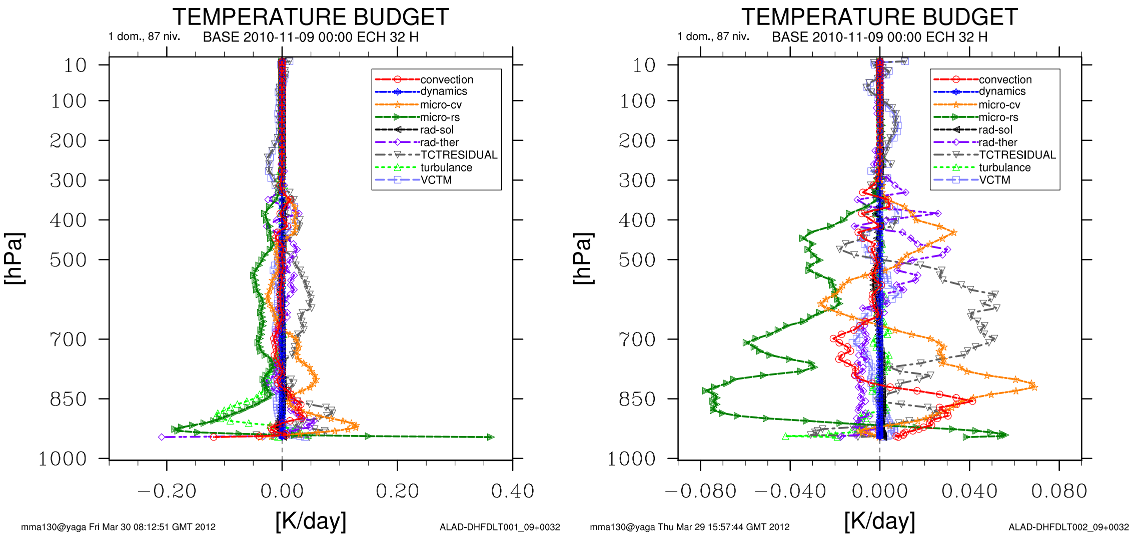

Figure 1: *DDH difference exp-ref for temperature. Left: settlsp applied on all the parameterizations, right: settlsp applied on all the parameterizations except the vertical diffusion.*

This is in correspondence with the results of Wedi [7]. In the ECMWF global model IFS, the vertical diffusion was also excluded from the averaging of the physical tendencies along the trajectory because large errors have been found in the vicinity of orography in the lowest model level.

General results

Unfortunately, other oscillations situated mostly in the lower parts of the domain were detected in the results of the *settlsp* experiment not sensitive to the vertical diffusion treatment. These oscillations appear and disappear in many cases, not destroying the meteorological relevance of the results. They remain local, restricted to few points only. But in some cases, these oscillations are amplified in the model time evolution and may cause the blow up of the particular run of the model.

We spotted the problematic points for the particular run of 9th November 2010 described earlier and we show the time evolution of the physical tendencies of temperature in such a single point for the whole vertical dimension, see Fig.3.

Then we show the same physical tendencies evolution of temperature, specific humidity respectively, for the 77th and the 87th vertical level on Fig.3 and Fig.4. The *settlsp* experiment was aborted after 645 time steps, i.e. after 32 hours and 15 minutes of integration. The reason was in unrealistic values of temperature and other variables appearing in the Eastern Europe, over the Belarus territory. See Fig.6 for the localization of the problematic spots. The meteorological results on other localities have reasonable values. The incident oscillations are of $2\Delta t$ period and may appear during the execution with the amplifying amplitude and then disappear again as in the first 250 steps of our experiment, or they may rapidly intensify as at the end of our experiment causing the abort. In other words they have a character of fibrillations.

We investigated the question which model variables are crucial for appearance of these phenomena and found out that the physical tendencies of GMV variables as temperature and horizontal wind components are immune against the SETTLS technique application while GFL variables as specific humidity q and falling species are sensitive to this technique. See Fig.3 and Fig.4 for the time evolution

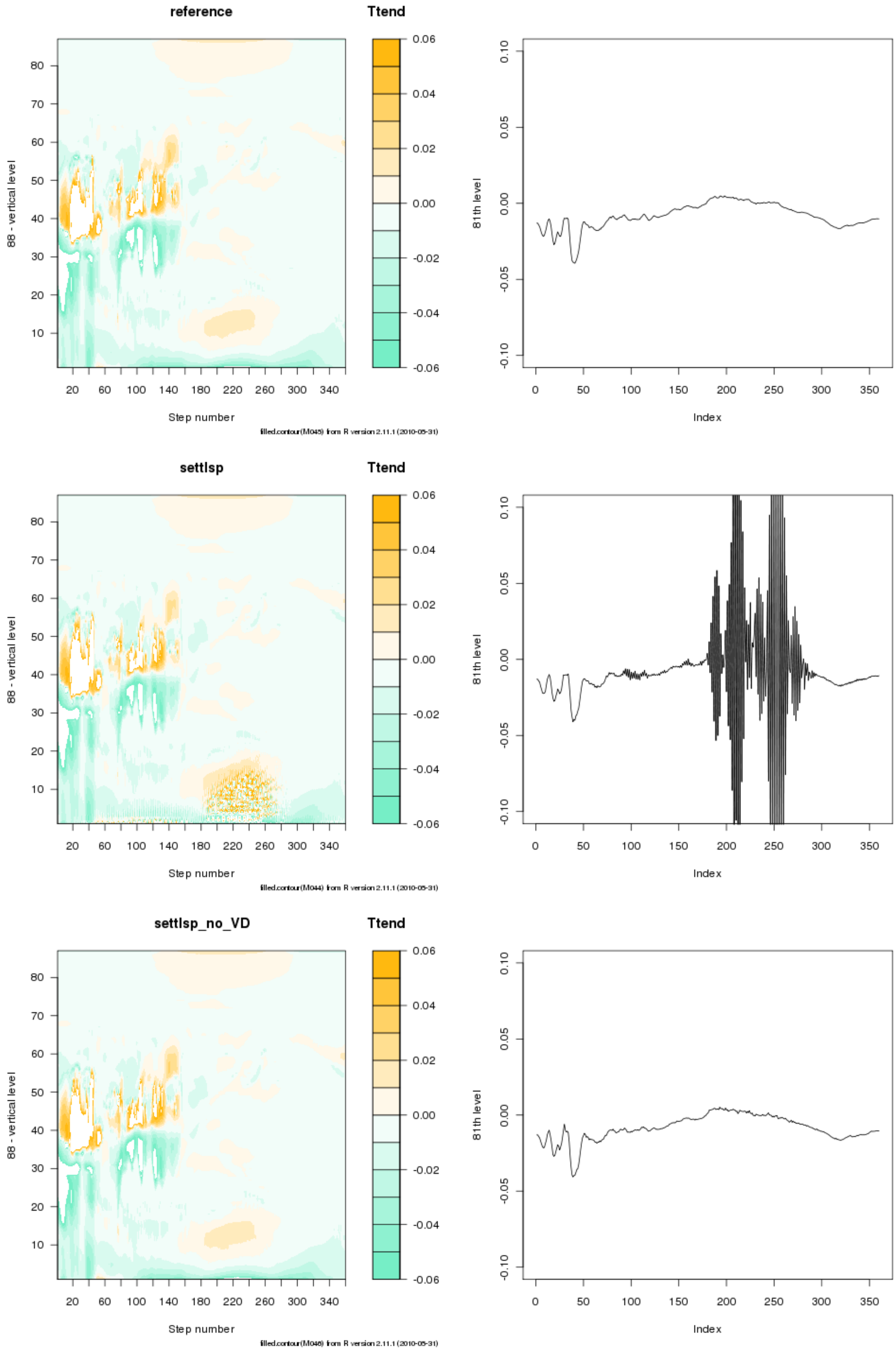

Figure 2: *Time evolution of the physical tendency of temperature in a single point, Left: vertical cross section, right: the 81level. From top to bottom: the reference, settlsp, settlsp on all variables but not on the tendencies from vertical diffusion.*

of temperature, q respectively, if SETTLS is applied only on GMV variables and Fig.5 if applied on GMV and q.

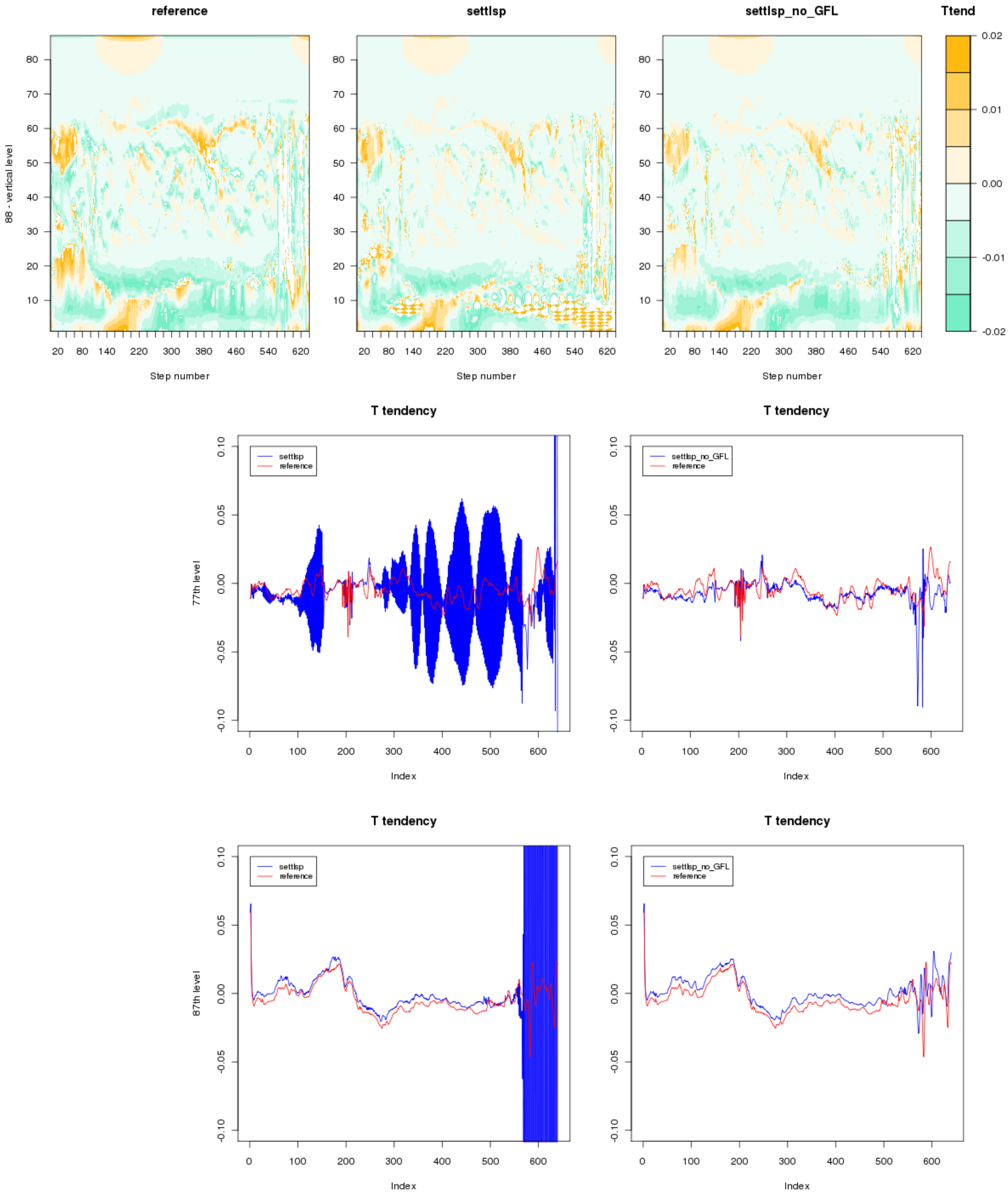

Figure 3: *Time evolution of T tendencies; left: the reference; center: settlsp on GMV and q, q_i, q_l, TKE; right: settlsp on GMV only. Top: vertical cross section; center: 77th level; bottom: 87th level.*

We assume that the alternating character of the GFL fields in comparison to the much smoother pattern of the field of temperature induces this behaviour. The application of SETTLS technique supposes implicitly the smooth nature of the original fields.

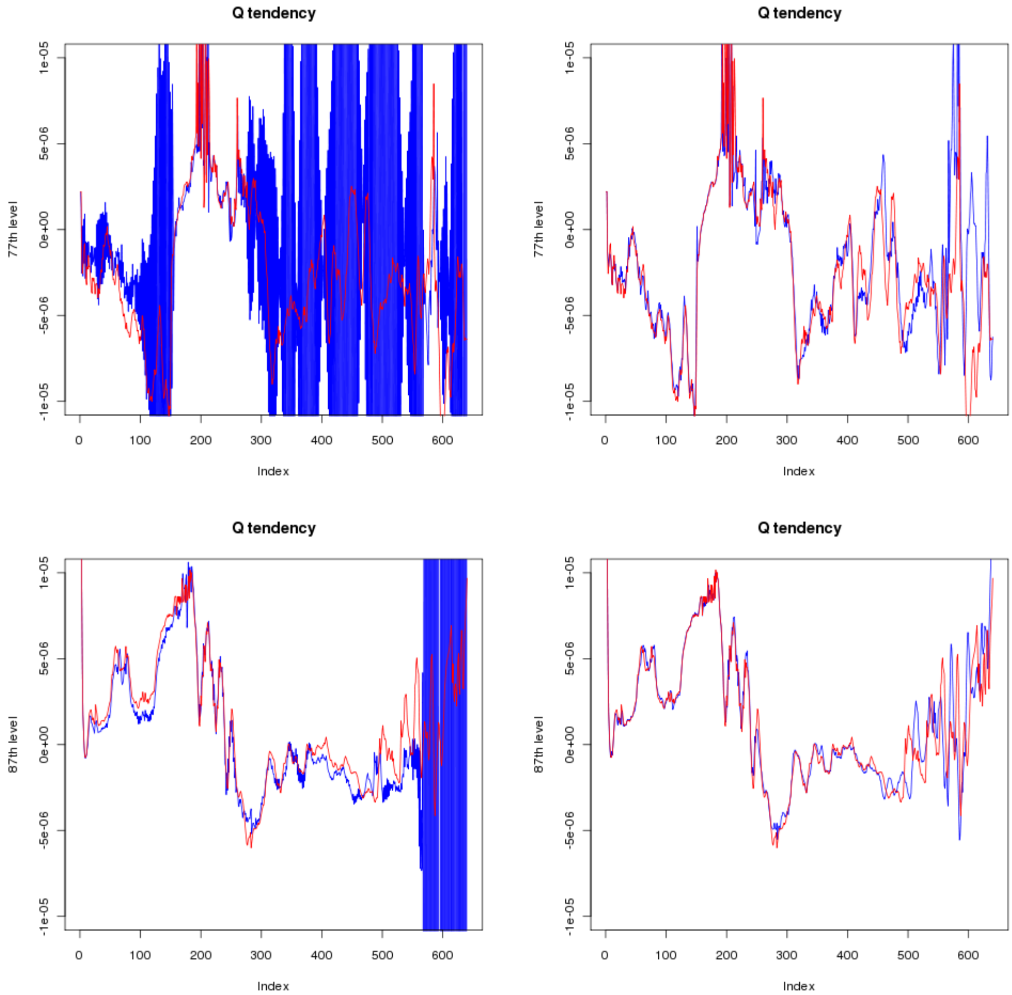

Figure 4: *Time evolution of q tendencies; left: settlsp on GMV and q, q_i, q_l, TKE; right: settlsp on GMV only.*

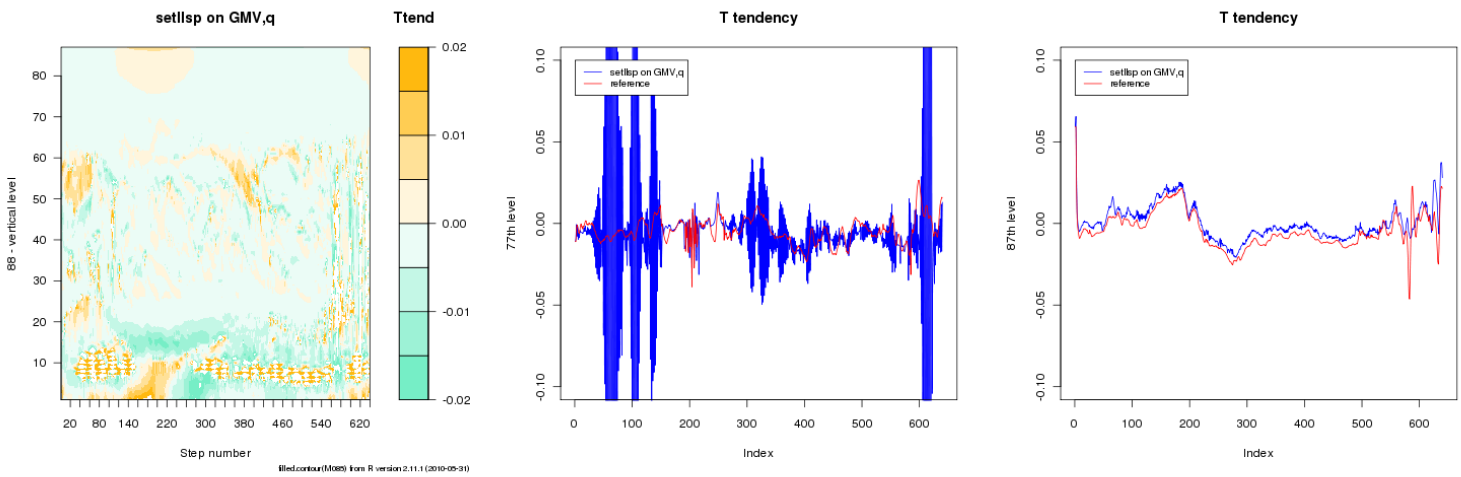

Figure 5: *Time evolution of T tendencies, settlsp on GMV and q only.*

Figure 6 shows the differences in the temperature field on the lowest model level between the *settlsp* and *reference* experiments. The red circle denotes the spot in which time oscillations appear causing finally the variation in temperature field of $\pm 10K$ magnitude.

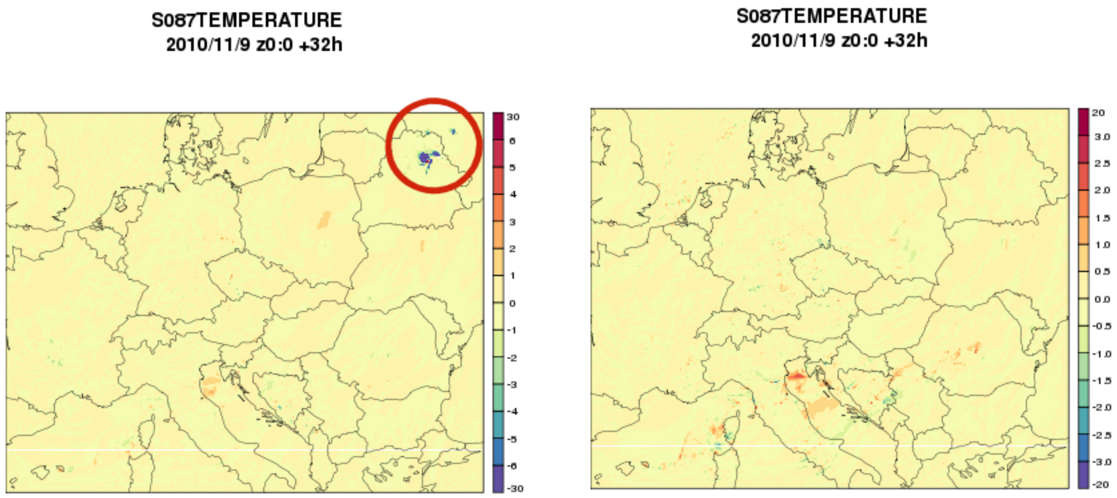

Figure 6: *Temperature at 87th level after 32 hours of integration - the difference between an experiment and the reference. Left: settlsp; right: settlsp applied only on GMV.*

Grid point representation of moisture

The specific humidity as a global model variable is represented in spectral space by means of spectral coefficients. We asked the question if there will be a significant difference in results if the specific humidity will be represented as a grid point field similarly as the other GFL variables. We run an experiment with the same setting as before for *settlsp* experiment but with grid point representation of q. The overall character of the results is very similar for both the experiments, the time oscillations appear on the same spot and the vertical oscillations in the field of temperature have a similar character. The results of the experiment with the grid point representation of q are not shown here.

Verification of the scores

As described in the previous paragraph we were not able to apply successfully the SETTLS type coupling on the physical tendencies of all the advected variables, but we found interesting the question if we reach better accuracy in the real simulations by applying this technique only on the GFL variables (for the hydrostatic approximation this means the temperature and the horizontal wind components). For one special case of the 9th of November 2010, we get promising results. See the left hand side of Fig.7 for the standard deviation of the geopotential field. For the whole month (November 2010), the scores of the *settlsp* experiment show similar quality as the reference experiment, or manifest even weak decay. The other parameters of validation (RMSE, bias) and other meteorological fields studied are not shown here, but they show the same behaviour.

4 Implementation

The modification introducing the SETTLS type treatment of the physics tendencies can be switched on by setting LSETTLSP=TRUE in the NAMDYNA namelist. For the decentering, we add the real

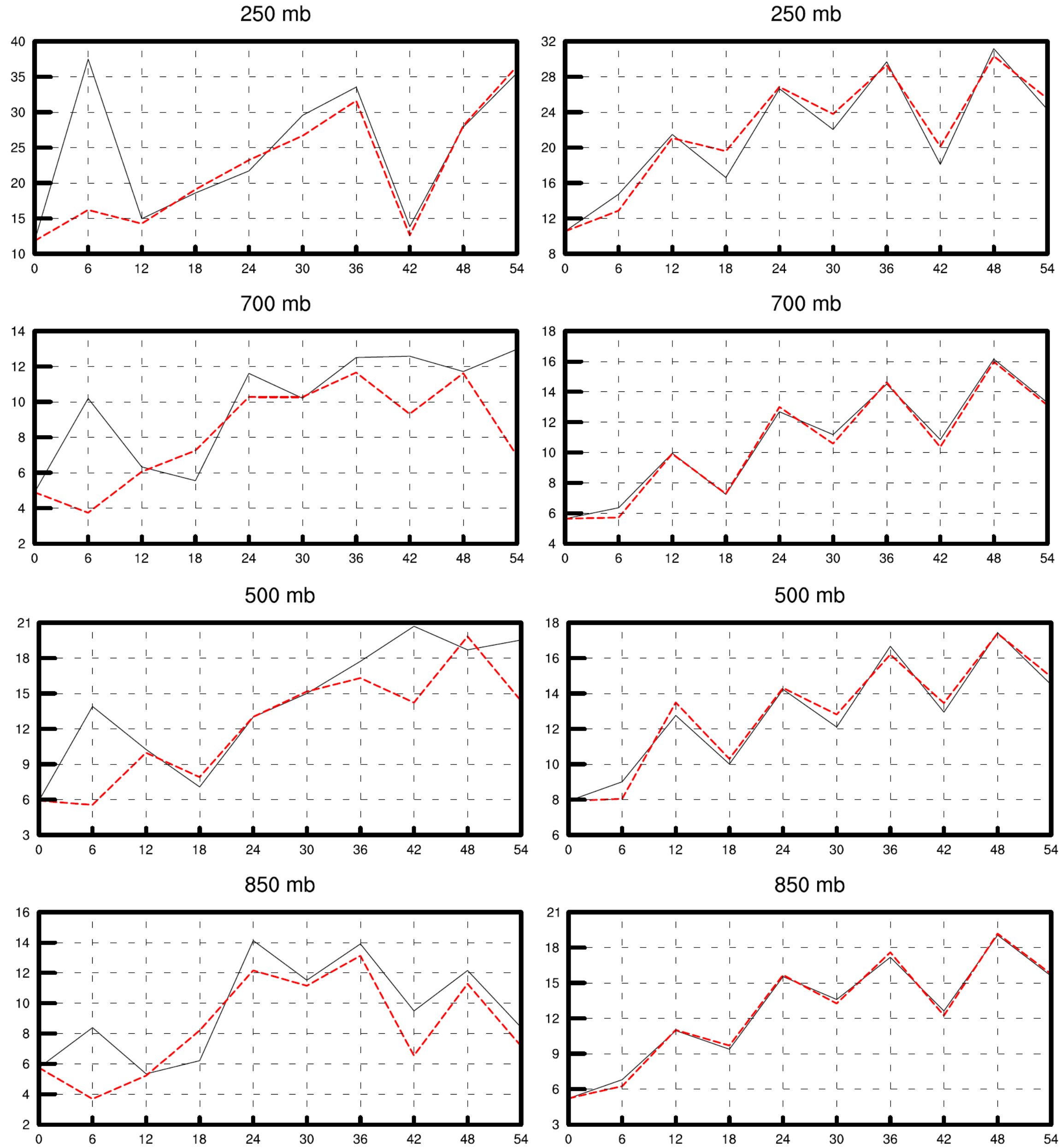

Figure 7: *Verification of the geopotential scores (STDE): black - reference, red - settlsp; left: 09/11/2010, right: November 2010.*

parameter VEPH=δ $(0 < \delta < 1)$ again to the NAMDYNA namelist. The routine most affected by the modification is CPG_PT. The total tendencies P^0 are saved here to be used in the following time step as P^-. We use for this purpose the already existing arrays PGMV(:,:,YT9%MC[X]PT) for X=U,V,T (and non-hydrostatic PD,VD eventually), PGMVS(:,:,YT9%MCSPPT) for surface pressure and PGFLPT(:,:,Y[X]%MPPT) for the GFL variables. Then $\frac{\Delta t}{2}\left(\delta P^0 + P^-\right)$ is subtracted from the original tendency.

The tendencies of horizontal winds are changed after they are used for the trajectory research, i.e. in LAVENT. In CPG_PT, we only prepare the contributions in the local arrays PSETTLSU, PSETTLSV. The part of the physical tendency $\frac{\Delta t}{2}\left(1 + \delta\right) P^0$ to be used in the final point of trajectory is simply added to the T1 component of the model variable.

Some preparation is needed for separation of vertical diffusion contribution to physical tendencies in CPTEND_NEW and CPUTQY. The tendencies from the vertical diffusion are saved to the local arrays PTEND[X]VD for X=U,V,H,Q,L,I and subtracted from the total physical tendencies before the SETTLS modification in CPG_PT is done.

5 Conclusions

The current explicit first order in time accurate coupling of physics to dynamics was easily extended to second order accuracy by using the SETTLS type technique. The theory in this case says that we will lose little bit on stability but gain on accuracy.

However, the real case simulations with the settings used in the current operational version of the model ALARO for the Czech domain showed poor stability of this configuration. If the SETTLS type coupling is applied on all the advected variables (but the moisture is enough to produce this phenomena) significant time oscillations appear in the field of temperature mostly near the ground, but not exclusively restricted to this area.

If applied only on prognostic GMV variables as temperature and the horizontal wind components, the stability was recovered but the expected enhanced accuracy was not detected in a one month validation (the forecast for 54 hours once per day).

We conclude from these tests that we shall stay with the current explicit technique of coupling the physics to dynamics and we resign for the time being on the second in time accuracy.

References

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6 Appendix: Stability analysis

We assume the forced one-dimensional semi-Lagrangian equation

$$\frac{d\psi}{dt} + i\omega\psi = -\beta\psi, \tag{5}$$

where $\beta > 0$ and ω are real constant determining a damping physical parametrization process and respectively an oscillatory dynamical process of the simplified model. Let assume the constant advecting velocity U with $\frac{d\psi}{dt} = \frac{\partial\psi}{\partial t} + U\frac{\partial\psi}{\partial x}$.

We are looking for a solution of (5) in the shape $\psi(x,t) = f(t)e^{ikx}$. The exact solution $\psi(x,t) = \psi_0 e^{ikx}e^{-\beta t}e^{-i(\omega+kU)t}$ will be considered as a reference. For a temporally discretised version with the time step Δt we denote $\epsilon = e^{-ikU\Delta t}$. At the origin point O the value of ψ at time t writes $\psi_O^t = \epsilon\psi_F^t$ with F being the arrival point of the SL trajectory.

We quantify the atmospheric state by ω while the reference state is represented by ω^* generally distinct from ω. Von Neumann method is used for stability analysis. The amplification factor $\mathcal{A} = \frac{\psi(x,t+\Delta t)}{\psi(x,t)}$ is computed and the stability condition of the shape $|\mathcal{A}| \leq 1$ is explored.

Explicit coupling

In the case of the explicit time scheme applied on the physical processes coupled to the dynamical part of the model the discretization is done according to (3). We get

$$\frac{\psi_F^+ - \psi_O^0}{\Delta t} + \frac{i}{2}\omega^*\left(\psi_F^+ + \psi_O^0\right) + \frac{i}{2}(\omega - \omega^*)\left(2\psi_O^0 - \psi_O^- + \psi_F^0\right) = -\beta\psi_O^0.$$

Let denote $K = \frac{i}{2}(\omega - \omega^*)\Delta t$. Then the amplification factor $\mathcal{A}$ satisfies the quadratic equation

$$\mathcal{A}^2\left(1 + \frac{i}{2}\omega^*\Delta t\right) + \mathcal{A}\left(K - \epsilon\left(1 - 2K - \beta\Delta t - \frac{i}{2}\omega^*\Delta t\right)\right) - \epsilon K = 0.$$

Without loss of generality we may assume $\omega^*\Delta t = 1$. Then the value of $|\mathcal{A}|$ depends in fact on three parameters $\beta\Delta t$, $\omega\Delta t$ and $kU\Delta t$ and we may explore it in dependence on these parameters. Notice that $\mathcal{A}$ is periodic in kU and hence we may restrict the study to $kU\Delta t \in < 0, 2\pi)$.

First, if there is no physical forcing, i.e. $\beta = 0$, we have unconditional stability with respect to $kU\Delta t$ for $0 \leq \omega \leq \omega^*$. Hence we will restrict all the following considerations to cases satisfying this condition.

If there is no dynamical forcing as for advectable GFL variables as moisture in Aladin, i.e. $\omega = \omega^* = 0$, we get stable scheme for β satisfying $0 \leq \beta\Delta t \leq 2$. Hence we will restrict to these values in the following. We depict the area of unconditional stability with respect to $kU\Delta t$ for the scheme under these two limits put on $\beta\Delta t$ and $\omega\Delta t$ on Fig.8b.

Without advection, i.e. if $kU = 0$, the maximal $\beta\Delta t$ for which the scheme is still stable increases from 1 to 2 as ω comes closer to ω^*. See Fig.8b. For $kU \neq 0$ the stability is always better.

The region of values of $kU\Delta t$ and $\beta\Delta t$ ensuring stability for any ω, $0 \leq \omega \leq \omega^*$, is depicted on Fig.8c. Similarly, the region of values of $\omega\Delta t$ and $kU\Delta t$ ensuring stability for any β, $0 \leq \beta\Delta t \leq 1$, is depicted on Fig.8d.'

The contours on Fig.8b,c,d show the borders of the region of unconditional stability for distinct values of the third parameter, i.e. $kU\Delta t$ for b, $\omega\Delta t$ for c and $\beta\Delta t$ for d. They have an illustrative character.

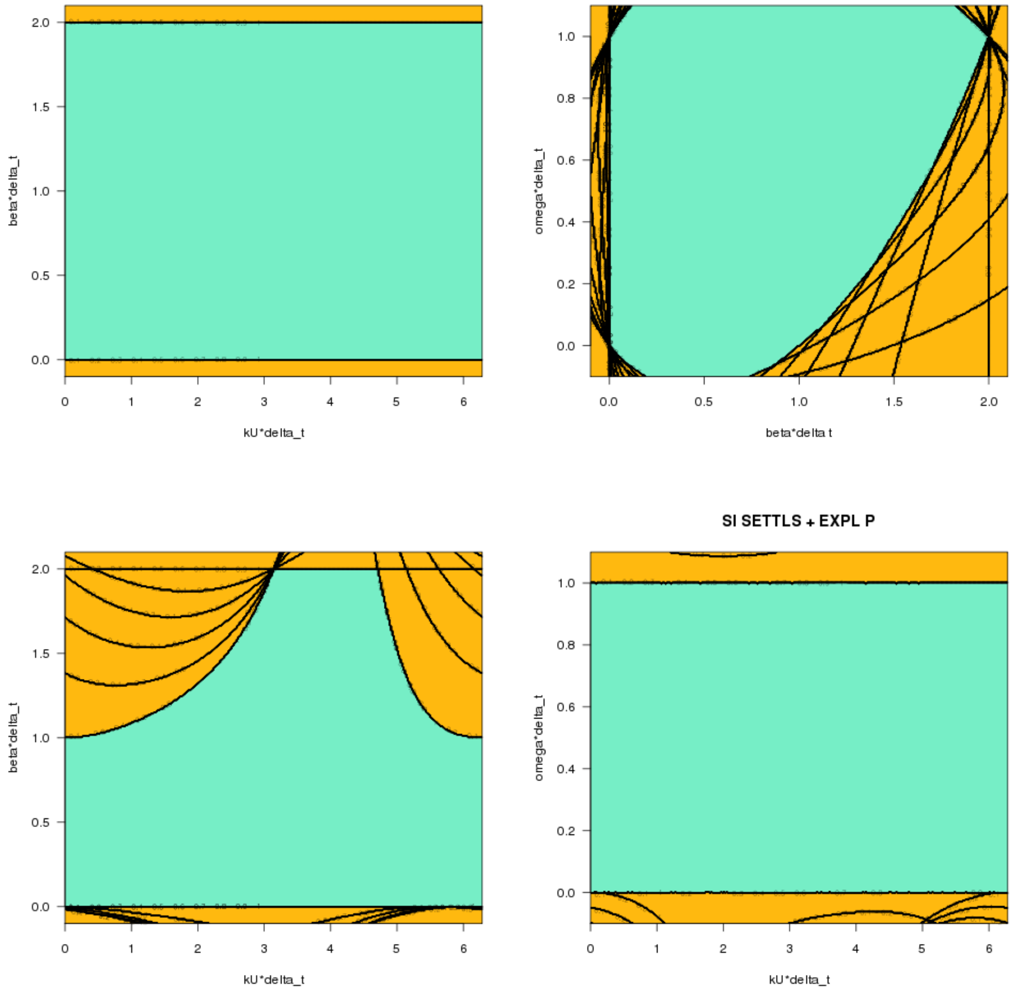

Figure 8: *Explicit coupling: a) $\omega = \omega^* = 0$; b) $kU\Delta t$ is varying from 0 to 2π and only points with $|\mathcal{A}(\beta\Delta t, \omega\Delta t)| \leq 1$ for all $kU\Delta t$ from this range are colored green; c) ω is varying from 0 to ω^* and again only points with stability ensured for any ω from this range are colored green; d) $\beta\Delta t$ is varying from 0 to 1 and only points with stability ensured for any $\beta\Delta t$ from this range are colored green.*

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SETTLS type coupling

In the case of SETTLS type coupling the discretization follows (4) and writes

$$\frac{\psi_F^+ - \psi_O^0}{\Delta t} + \frac{i}{2}\omega^* \left(\psi_F^+ + \psi_O^0\right) + \frac{i}{2}(\omega - \omega^*) \left(2\psi_O^0 - \psi_O^- + \psi_F^0\right) = -\frac{\beta}{2} \left(2\psi_O^0 - \psi_O^- + \psi_F^0\right).$$

Let denote $K' = \frac{i}{2}(\omega - \omega^*)\Delta t + \frac{1}{2}\beta\Delta t$. Then the amplification factor $\mathcal{A}$ can be determined from

$$\mathcal{A}^2 \left(1 + \frac{i}{2}\omega^* \Delta t\right) + \mathcal{A}\left(K' - \epsilon\left(1 - 2K - \frac{i}{2}\omega^* \Delta t\right)\right) - \epsilon K = 0.$$

The criteria of stability become more restrictive in this case.

Without any dynamical forcing, the stability is ensured only for $0 \leq \beta\Delta t \leq 1$ if expected to hold for any $kU\Delta t$. See Fig.9a.

The zone of stability is restricted to $0 \le \beta\Delta t \le \frac{3}{4}$ if ensured for all $kU\Delta t \in < 0, 2\pi)$ and all ω, $0 \le \omega \le \omega^*$, see Fig.9c. Surprisingly for the case $kU = 0$, i.e. no advection, the stability with respect to β is not supreme for $\omega = \omega^*$ where $0 \le \beta\Delta t \le \frac{3}{4}$, but for $\omega = \frac{\omega^*}{2}$ where $0 \le \beta\Delta t \le 1$, see Fig.9b.

One may think about pictures 8a,b,c and 9a,b,c respectively as projections of the 3D field $\mathcal{A}$ depending on the three parameters $\beta\Delta t$, $\omega\Delta t$ and $kU\Delta t$ on 3 perpendicular planes.

Let us resume that the diminution of stability due to the application of the second order accurate scheme of the SETTLS type on physical forcing may be considered as reasonable and does not necessarily indicate a fatal influence on the results in practical implementations.

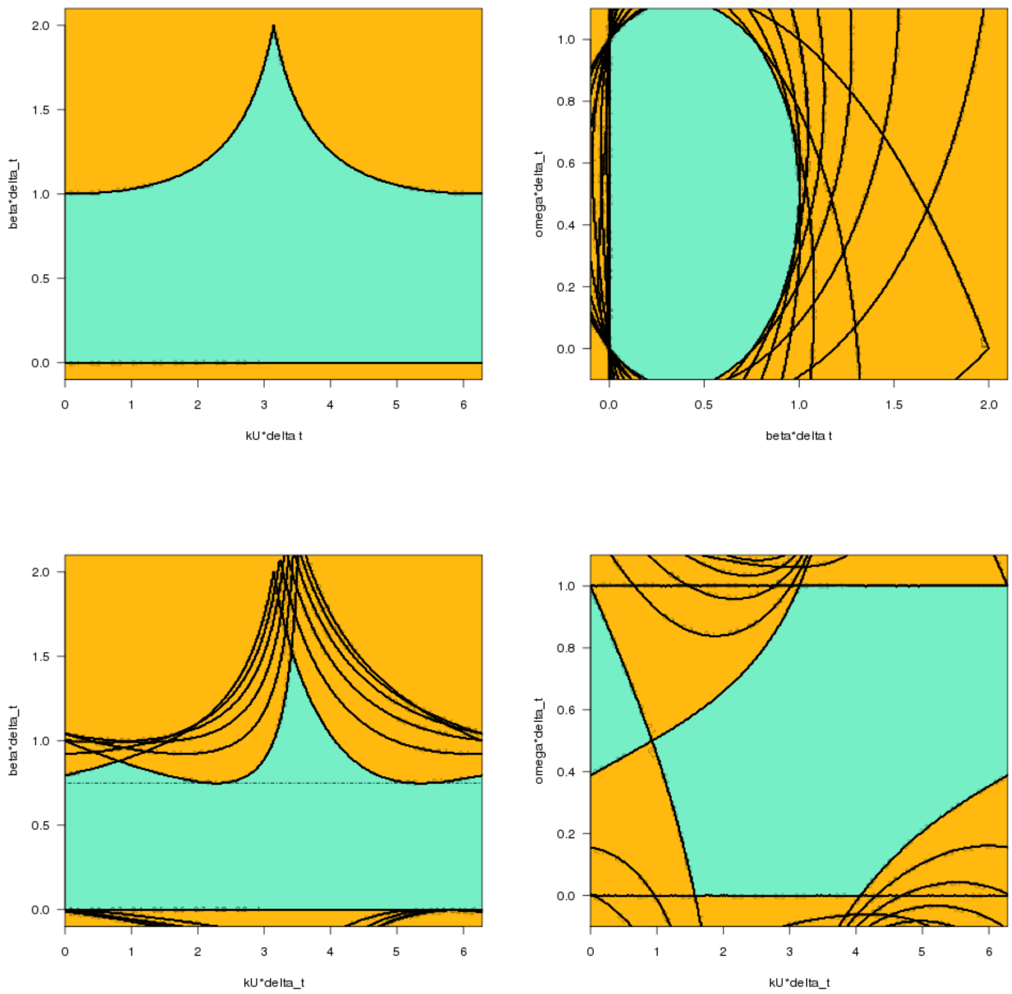

Figure 9: *SETTLS type coupling. The explanation is the same as for Fig.8.*