

# Projection of Heat on Temperature and Pressure in Fully Compressible Case

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RC LACE Note, May 2010

## **Introduction**

Following the development of fully non-hydrostatic interface between physics and dynamics (Petra Smolíková, 2008) there is an option allowing to project heat simultaneously on temperature and pressure. However when this option is activated, results of the model are changed quite a lot, although there is no real physical reason to support such a change. The aim of present study was to investigate plausible causes of the observed results' change and to pose correctly the problem of physics and dynamics interplay which is behind.

## **Description of the Problem**

In order to understand better what makes results so different, we simplified the experimental environment. To exclude suspicion that something was forgotten among moist processes, we kept only radiation active among the entire parameterization package. Like that all other fluxes but enthalpy remained zero.

This choice has many advantages. As already mentioned, one excludes any consideration on phase changes, dependency of  $R$ ,  $c_p$ ,  $c_v$ , etc. We consider only a source of heat, but important source. We also do not care about any detail how the parameterization works, for us it provides source or sink of heat.

Even in such quite simplified conditions difference in experiments remain quite large, as shown by DDH output on total temperature tendency computed over inner lat/lon zoom of LACE domain after 6h of integration (Figure 1).

First attempt was to search for a possible bug in the code, however so far nothing wrong was found. Next step was to check the set of equations, as published in Catry at al, 2007, namely thermodynamic Equation (38) and pressure tendency Equation (39) for fully compressible case. Derivation of all equations is of course correct in the continuous case; however the implementation in the model, including time-step organization and discretization must be kept consistent

Normally, whatever form of equations is used, resulting temperature field should not be noticeably different between the case we shall denote as “anelastic” (projection of heat is done on temperature only) and the “elastic” case (projection of heat is done simultaneously on temperature and pressure). This was not verified in the experiments, where this projection was the only difference (all tests were using non-hydrostatic dynamical kernel). The problematic step is linked to the use of  $c_v T$  form instead of  $c_p T$  form of the thermodynamic equation.

# TEMPERATURE : TENDANCE MOYENNE

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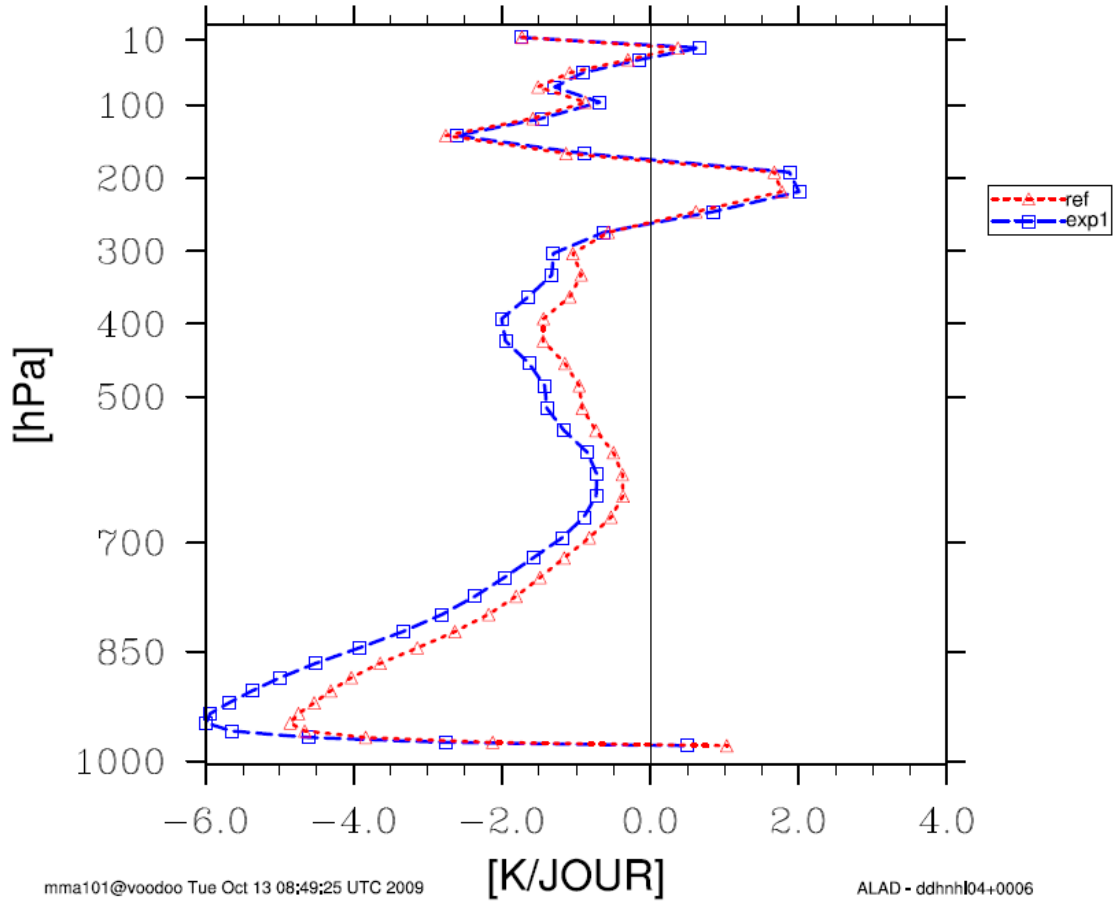


Fig. 1: Total temperature tendency after 6h of integration. Red color – reference experiment (heat is projected on temperature). Blue color – test with projection of heat on both temperature and pressure.

## Analysis of the problem

To explain more, we shall start with the set of four equations (thermodynamic, mass continuity equation, mass continuity equation for the hydrostatic pressure coordinate and state equation):

$$c_p \frac{dT}{dt} - RT \frac{d \ln p}{dt} = Q \quad (1)$$

$$\frac{d \ln \rho}{dt} + D_3 = 0 \quad (2)$$

$$\frac{d \ln \pi}{dt} = \frac{\omega}{\pi}; \frac{\partial \omega}{\partial \pi} + D_2 = 0 \quad (3)$$

$$p = \rho RT \quad (4)$$

In Equation (1) change in temperature (at constant pressure) is due to work of gas (pressure tendency term) and by the contribution of the heat source. This thermodynamic equation may get its other known form by combining it with the state equation of gas (4):

$$c_p \frac{dT}{dt} - RT \left( \frac{d \ln R}{dt} + \frac{d \ln \rho}{dt} + \frac{d \ln T}{dt} \right) = Q \quad (5)$$

Where terms in parenthesis are:

- 1) Time evolution of gas constant – in our simple experiment this term is not relevant;
- 2) Time evolution of density – we get it from continuity equation as  $D_3$  divergence term;
- 3) Temperature tendency – it is combined with the first term and it leads to express thermodynamic equation in  $cv dT$  form:

$$c_v \frac{dT}{dt} + RT D_3 = Q \quad (6)$$

We have thus two possible forms of the mechanical conversion term, giving temperature tendency due to work of gas:

1. Conversion term is expressed as tendency of pressure; in this case temperature change due to heat happens at constant pressure;
2. Conversion term is expressed as tendency of density, in fact specific volume; in this case temperature change due to heat happens at constant volume.

Recombination of continuity Equation 2 with state Equation 4 provides the tendency of pressure:

$$c_v \frac{d \ln p}{dt} + c_p D_3 = \frac{Q}{T} \quad (7)$$

Equations (6) and (7) are the same like Equation (38) and (39) in the paper Catry et al., 2007, where we consider for our simple case constant  $R$ ,  $c_p$  and  $c_v$ .

Now, the total time tendency within the time step of the model is split in the two steps – physics and dynamics.

The time evolution due to the physics is a local change and the current choice is that heat does not touch the conversion term, which would indeed mean to transform heat directly to some mechanical energy. In one case, starting from Equation 1, we get the so-called “anelastic” projection of heat at constant pressure:

$$c_p \frac{\partial T}{\partial t_{phys}} = Q \quad (8)$$

Consistently, we get zero time tendency of pressure due to physics.

What was prepared as new option, called “elastic” projection, was to start from Equation (6) and (7), where local changes of temperature and pressure due to heat were computed as it follows:

$$c_v \frac{\partial T}{\partial t_{phys}} = Q ; c_v \frac{\partial \ln p}{\partial t_{phys}} = \frac{Q}{T} \quad (9)$$

We know, however, that using Equations 9, where all perturbation in pressure is projected on the non-hydrostatic pressure departure, yield quite different and also wrong model results.

The odd thing is that also in this case the conversion term (in its divergence form) has no direct contribution due to heat. But, as paradox, allowing any contribution of heat to pressure actually means that the conversion term feels the heat, because otherwise there is no heat source but in thermodynamic equation.

Therefore let's try to examine more the thermodynamic equation differentiated just with partial time derivatives – we do it both for the Equation 1 and for the developed form of Equation 5 using state equation:

$$c_p \frac{\partial T}{\partial t} - RT \left( \frac{\partial \ln \pi}{\partial t} + \frac{\partial \hat{q}}{\partial t} \right) = Q \quad (10)$$

Where we separated true pressure tendency to the tendency of hydrostatic pressure  $\pi$  and logarithmic non-hydrostatic pressure departure  $\hat{q}$ ; alternatively we may write also:

$$c_p \frac{\partial T}{\partial t} - RT \left( \frac{\partial \ln R}{\partial t} + \frac{\partial \ln \rho}{\partial t} + \frac{\partial \ln T}{\partial t} \right) = Q \quad (11)$$

Now the interesting thing is how to obtain the NH pressure departure tendency due to physics. In fact we have four variables at stake: temperature, density, hydrostatic pressure and NH pressure departure and we need four equations as well – thermodynamics, state law and the two kinds of continuity equations. But at the same time we have a “hidden” degree of freedom in the way we separate the heating contributions – we have various links of equations and variables.

To get the result (9), we must do some arbitrary decisions which parts of local tendencies we neglect when differentiating state equation – it is hydrostatic pressure and density. By doing so we get straight:

$$\frac{\partial \ln T}{\partial t_{phys}} = \frac{\partial \hat{q}}{\partial t_{phys}} \quad (12)$$

The funny thing is that the same result we may obtain by completely forgetting mechanical conversion term and simply differentiate temperature tendency due to heat with the help of state law – it is in fact a logical result and it explains the above mentioned small paradox.

What is even more funny is that the result (12) is the same regardless whether the tendency of temperature due to physics is computed as  $Q/c_p$  or  $Q/c_v$  – it is enough to suppose that the rest of tendencies due to conversion term and the prognostic Equations 2 and 3 are treated along the trajectory by the dynamics. This is another demonstration on the key role of the model step discretization approach in separation of tendencies due to physics and dynamics.

## **Outlook**

According to literature on adjustment (c.f. Sotack and Bannon, 1999), the result we obtain by applying the anelastic projection is close to the final state of the adjustment. But this result is got without description of any transition stages; we may say it is got straight by “filtering”. When we try the elastic projection, it corresponds to what happens at the very beginning of the adjustment process, where the entropy perturbation is just pressure perturbation at constant density.

However with the discretization of the time step made as it is now, we do not obtain convergence to the correct final state. It looks that the practical application (9) of the continuous Equations 6 and 7 is derived too hastily, using arbitrary assumptions of neglecting at the same time part of contribution from hydrostatic pressure change and from density change. As we have also shown above, the separation of the tendencies due to physics and due to dynamics is the key issue. Their arbitrary combinations may lead to solutions which do not work properly.

Maybe we can derive correctly what the NH pressure tendency due to heat could be, but not without using correctly and consistently all terms in the equations. In order to guide us in this endeavor, we should keep in mind the idea that we wish to simulate a transition from the elastic situation at the beginning of the forcing to the anelastic situation after a certain time.