Non-isothermal reference temperature profile in NH dynamics of model ALADIN

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1 Introduction

The iterative centered implicit (ICI) scheme with constant coefficient spectral solver is determined to be operationally exploited in the model AROME. It is a two time level scheme with semi-Lagrangian advection treatment. The scheme is based on the division of the dynamical terms into linear and nonlinear part. The linear part is averaged in time and the nonlinear part is centered (predictor step) and then iteratively averaged in time (corrector steps - iterations). The ICI scheme can be viewed as a generalization of the two-time level semi-implicit (2TL SI) scheme, which is used as a predictor step in ICI scheme.

The division of dynamical terms is done by means of the linear operator. It was proposed [1] to use as the linear operator the linear system such that it describes the infinite amplitude wave perturbations around the time fixed reference state. The reference state therefore play a crucial role in the time stepping scheme design. The linear model is obtained with the traditional linearization for small amplitude departures from the chosen reference state (second order perturbation terms are neglected).

The choice of reference state and consequently the choice of the linear operator influences the numerical stability and accuracy of the time stepping. The signals, described by the linear operator, are integrated implicitly and therefore unconditionally stable. Their phase speed is slowed down significantly when large time steps are used. But the part of the signal still remains to be treated explicitly (part described by the nonlinear part) and hence potentially unstable. The stability of the overall procedure is conditional and it depends on the choice of the reference state. The analyzes of this kind of instability (so called semi-implicit one) can be found for example in [2, 3].

It was shown that for hydrostatic model the reference state is sufficient to be isothermal, resting, hydrostatically balanced and horizontally independent [3]. The linear operator (derived by the traditional linearization from the primitive equations) than controls the fast moving gravity modes. They are of no interest for resolutions relevant for hydrostatic approximation and therefore they propagation can be distorted without significant impact on the results. The conditional stability of the 2TL SI scheme is achieved when the temperature of the reference state is chosen higher that the actual temperature. ICI scheme is of no interest for the hydrostatic models as it is less efficient that the 2TL SI scheme but providing the same stability properties.

To be complete, the conditional stability depends also on the choice of reference surface pressure in hybrid terrain following coordinate framework [4], but the pressure related instability is minor when compared to the temperature related one. This instability is completely eliminated in the sigma coordinate system.

The situation is different as far as the non-hydrostatic dynamics is considered. The two fast moving modes are present in the solved system, the gravity and the sound waves. The isothermal, resting, hydrostatically balanced reference state gives the linear operator which describes the both modes. But the stability of 2TL SI scheme can not be achieved, because the stability criterion determines that the reference temperature must be higher that the actual one for the gravity modes but colder for the acoustic modes [6]. This contradicting conditions can not be satisfied with the single isothermal profile. One has to replace the reference temperature in the linear operator in the terms responsible for vertical acoustic waves propagation [6]. In this way, we obtain the bi-isothermal linear operator, appropriate for the stable 2TL SI scheme. To be exact, it was proposed in [6] to replace the reference temperature by the new arbitrary temperature in all the terms where the reference temperature occurs in denominator.

It is known that the explicit schemes describes better the phase propagation of the waves. However, this schemes are not suitable for practical NWP due to their inefficiency. The 2TL implicit schemes (SI and ICI) suffer from the large phase speed errors when used with the large time steps. In practice this is always the case, because the operational NWP models are run with time steps providing CFL significantly greater than 1. Therefore it is important to eliminate the sources which influence the phase speed error in order to minimize it. The departure of reference state from the real atmospheric state is one of the elements that can be controlled. **The idea of work presented in this report is to replace the isothermal reference temperature profiles by the non-isothermal ones. This approach minimizes the departures between the reference and the real temperatures and consequently it is believed that the accuracy of the scheme could be improved together with the overall robustness of the scheme.**

We do not implement the "true" non-isothermal linear operator obtained by traditional linearization around the non-isothermal reference state. This approach has been found less stable that the simpler isothermal approach [2], because the conditional stability depends on the vertical static stability of reference state rather than on the temperature itself. Moreover the implementation of the "true" non-isothermal operator would lead to further complications in vertical discretization (see [7] and APPENDIX B of this report).

2 Non-isothermal reference state and the linear operator

We will describe the modified linear operator in this section. We start from the bi-isothermal linear operator introduced by Benard [6]. The equations are formulated in vertical hybrid mountain following coordinate η [8] and the prognostic variables are $q_s = log(\pi_s)$, temperature T, horizontal divergence D, vertical divergence $d_3 = -\frac{gp}{mRT}\frac{\partial w}{\partial \eta}$ and scaled pressure departure $\hat{q} = ln(\frac{p}{\pi})$.

Prognostic variable d_3 is employed for the sake of simplicity, although the prognostic variable $d_4 = d_3 + \frac{p}{mRT}\frac{\partial \vec{v}}{\partial \eta}\vec{\nabla}\phi$ is used in the practice when ALADIN is run. The linear operator is the same for the both prognostic variables d_3 and d_4 and hence the information provided here are valid for the system with d_4 as well.

The bi-isothermal solver is

$$
\begin{aligned}
\frac{\partial D}{\partial t} &= -R\mathbf{G}^*\triangle T + RT^*(\mathbf{G}^* - 1)\triangle \hat{q} - RT^*\triangle q_s - \triangle \phi_s \\
\frac{\partial T}{\partial t} &= -\frac{RT^*}{C_v}\left(\mathbf{m}_*{}^2 D + d_3\right) \\
\frac{\partial q_s}{\partial t} &= -\mathbf{N}^*\mathbf{m}_*{}^2 D \\
\frac{\partial d_3}{\partial t} &= -\frac{g^2}{RT_a{}^*}\mathbf{L}^*\hat{q} \\
\frac{\partial \hat{q}}{\partial t} &= -\frac{C_p}{C_v}(\mathbf{m}_*{}^2 D + d_3) + \mathbf{S}^*\mathbf{m}_*{}^2 D.
\end{aligned}
\tag{1}
$$

The system is written in the vertically discretized but time and horizontally continuous form. Therefore the variables D, T, d_3 and $\hat{q}$ represents vectors with length L (with L being the number of full vertical model levels) and the variable q_s is scalar.

The definition of linear operators $\mathbf{G}^*$, $\mathbf{S}^*$, $\mathbf{N}^*$ and $\mathbf{L}^*$ can be found in APPENDIX A and for even more information one could read internal documentation of NH ALADIN [9].

The two reference temperatures, the traditional one T^* and acoustic one $T_a{}^*$ were introduced in (1) to guarantee the stability of the acoustic and the gravity modes.

The linear set of equations (1) is derived for the infinitesimal perturbations around the reference state $X' = X - X^*$. Therefore one would expect the system to be formulated with the departures X' rather than with the model state X (X represents the vector of all prognostic variables of one atmospheric column). It can be easily verified that these two formulations are formally identical due to the properties of the reference state.

The aim of present work is to minimize the departures between real temperature and the reference one. To achieve this we replace the isothermal temperatures by the non-isothermal profiles $T^*(\eta)$ and $T_a{}^*(\eta)$ without the supplemental change in the system. We do not introduce any new terms into our linear system. It yields

$$
\begin{aligned}
\frac{\partial D}{\partial t} &= -R\mathbf{G}^*\triangle T + RT^*(\eta)(\mathbf{G}^* - 1)\triangle \hat{q} - RT^*(\eta)\triangle q_s - \triangle \phi_s \\
\frac{\partial T}{\partial t} &= -\frac{RT^*(\eta)}{C_v}\left(\mathbf{m}_*{}^2 D + d_3\right) \\
\frac{\partial q_s}{\partial t} &= -\mathbf{N}^*\mathbf{m}_*{}^2 D \\
\frac{\partial d_3}{\partial t} &= -\frac{g^2}{RT_a{}^*(\eta)}\mathbf{L}^*\hat{q} \\
\frac{\partial \hat{q}}{\partial t} &= -\frac{C_p}{C_v}(\mathbf{m}_*{}^2 D + d_3) + \mathbf{S}^*\mathbf{m}_*{}^2 D.
\end{aligned}
\tag{2}
$$

The definition of vertical operators $\mathbf{G}^*$, $\mathbf{S}^*$, $\mathbf{N}^*$ and $\mathbf{L}^*$ remained unchanged comparing to the bi-isothermal system. This is due to fact that the system is constructed in a way that the constrain $C1$ (see [7]) remain unaltered (shown later).

The vertically and time discretized linear system of equations (still horizontally continuous) is

$$
\begin{aligned}
D &= \tilde{D} + \tau\left[-R\mathbf{G}^*\triangle T + RT^*(\eta)(\mathbf{G}^*-1)\triangle\hat{q} - RT^*(\eta)\triangle q_s - \triangle\phi_s\right] \\
T &= \tilde{T} + \tau\left[-\tfrac{RT^*(\eta)}{C_v}\left({\mathbf{m}_*}^2 D + d_3\right)\right] \\
q_s &= \tilde{q_s} + \tau\left[-\mathbf{N}^*{\mathbf{m}_*}^2 D\right] \\
d_3 &= \tilde{d_3} + \tau\left[-\tfrac{g^2}{R{T_a}^*(\eta)}\mathbf{L}^*\hat{q}\right] \\
\hat{q} &= \tilde{\hat{q}} + \tau\left[-\tfrac{C_p}{C_v}({\mathbf{m}_*}^2 D + d_3) + \mathbf{S}^*{\mathbf{m}_*}^2 D\right].
\end{aligned}
\tag{3}
$$

This is general form of the solver valid for the 2TL and 3TL SI or ICI schemes. The differences between the schemes are in the definition of time step variable τ and in the way the variables with tilde are computed. The quantities with the tilde represent the explicit part of time stepping algorithm. They are computed in the grid point space and transformed into the spectral space by means of FFT. There is no change in computation of tilde variables comparing to the scheme with isothermal solver, only the explicit linear correction has to be evaluated using (2) instead of (1).

The elimination of system (2) is performed to acquire the equations for the vertical divergence d_3 and the horizontal divergence D. The temperatures T^* and ${T_a}^*$ are considered non-isothermal in the following text. This gives

$$
\begin{aligned}
&D - \tau^2\triangle\left[RT^*\left(\mathbf{G}^*-1\right)\left(\mathbf{S}^* - \tfrac{C_p}{C_v}\right) + R\mathbf{G}^*\left(\tfrac{RT^*}{C_v}\right) + RT^*\mathbf{N}^*\right]{\mathbf{m}_*}^2 D \\
&= D^\bullet + \tau^2\triangle\left[-RT^*\left(\mathbf{G}^*-1\right)\tfrac{C_p}{C_v} + R\mathbf{G}^*\left(\tfrac{RT^*}{C_v}\right)\right]d_3 \\
&d_3 - \tau^2\tfrac{g^2}{R{T_a}^*}\mathbf{L}^*\tfrac{C_p}{C_v}d_3 = {d_3}^\bullet - \tau^2\tfrac{g^2}{R{T_a}^*}\mathbf{L}^*\left(\mathbf{S}^* - \tfrac{C_p}{C_v}\right){\mathbf{m}_*}^2 D
\end{aligned}
\tag{4}
$$

Quantities with the bullet are computed as

$$
\begin{aligned}
D^\bullet &= \tilde{D} + \tau\triangle RT^*\left(\mathbf{G}^*-1\right)\tilde{\hat{q}} - \tau\triangle R\mathbf{G}^*\tilde{T} - \tau\triangle RT^*\tilde{q_s} \\
{d_3}^\bullet &= \tilde{d_3} - \tau\tfrac{g^2}{R{T_a}^*}\mathbf{L}^*\tilde{\hat{q}}.
\end{aligned}
\tag{5}
$$

The obtained equations are written in the more compact form if the new vertical operator $\mathbf{V}^*$ (ψ is a general vector of length L) is introduced

$$
\mathbf{V}^*\psi = \left(\mathbf{G}^* - \frac{1}{T^*}\mathbf{G}^*T^*\right)\psi, \tag{6}
$$

and the constrain C1

$$
C1 : (\mathbf{G}^*\mathbf{S}^* - \mathbf{G}^* - \mathbf{S}^* + \mathbf{N}^*)\psi = 0 \tag{7}
$$

is applied to the equation for horizontal divergence to further simplify the sytem (4). This is the same constrain as valid for the vertical discretization of the isothermal system. Thanks to this property, the system with pseudo solver is vertically discretized in the same way as the system isothermal. The system of the two equations then stands

$$\begin{aligned}
\left[I-\tau^2 c^{*2}\triangle \mathbf{m}_*{}^2-\tau^2\triangle \mathbf{m}_*{}^2\left(gH^*-c^{*2}\right)\mathbf{V}^*\right]D &= D^\bullet+\tau^2\triangle\left[\left(-RT^*\mathbf{G}^*+c^{*2}\right)+\left(gH^*-c^{*2}\right)\mathbf{V}^*\right]d_3\\
\left(1-\tau^2\frac{c^{*2}}{H^*H_a{}^*}\mathbf{L}^*\right)d_3 &= d_3{}^\bullet+\tau^2\frac{1}{H_a{}^*}\mathbf{L}^*\frac{1}{H^*}\left(-RT^*\mathbf{S}^*+c^{*2}\right)\mathbf{m}_*{}^2 D
\end{aligned} \tag{8}$$

The vertical scaling factors are considered to be vertically dependent and their definition on the model layer with index l is

$$c^{*2}_l = \frac{C_p}{C_v}RT^*_l \tag{9}$$

$$H^*{}_l = \frac{RT^*_l}{g} \tag{10}$$

$$H_a{}^*{}_l = \frac{RT_a{}^*{}_l}{g} \tag{11}$$

.

Further elimination is not feasible as the operators on LHS of equations (8) are not commutative. The system can be written in the matrix form and solved using the apparatus of linear algebra. It leads to

$$\mathbf{M_A}D = D^\bullet+\mathbf{M_B}d_3 \tag{12}$$

$$\mathbf{M_E}d_3 = d^\bullet+\mathbf{M_F}D. \tag{13}$$

The previous system consists from the two decoupled subsystems, each with dimension L. The clearest approach to work out the problem is to write it in form with dimension $2L$. The $2Lx2L$ system writes

$$\begin{pmatrix} \mathbf{M_A} & -\mathbf{M_B}\\ -\mathbf{M_F} & \mathbf{M_E}\end{pmatrix}\begin{pmatrix} D\\ d_3\end{pmatrix}=\begin{pmatrix} D^\bullet\\ d_3{}^\bullet\end{pmatrix} \tag{14}$$

To improve the efficiency of the solver we will pre-compute the following matrix in the model setup and store it during the model integration (such matrix is stored for each spectral space wavenumber (n,m).

$$SIEHEL=\begin{pmatrix} \mathbf{M_A} & -\mathbf{M_B}\\ -\mathbf{M_F} & \mathbf{M_E}\end{pmatrix}^{-1} \tag{15}$$

The map factor is considered to be constant throughout this document. But it was evidenced that for the big domains where the variation of the map factor is large, the assumption of constant map factor in the linear system leads to instability. This can be avoided by the approximation of map factor with the linear function on the horizontal plane (key LSIDG in the model). However, this further complicates the solver. This possibility has not been yet considered, but once the pseudo non-isothermal solver will come out to have positive impact on properties of model time stepping, this feature will have to be reexamined.

3 Implementation

3.1 Initialization of non-isothermal reference profiles

The reference profiles have to be statically stable in order to provide the time stepping scheme which controls effectively the gravity waves. The pseudo non-isothermal solver doesn't describe the infinite amplitude signal in physically correct way (as it is not derived by the exact linearization approach). The signals are artificially distorted, but there is no addition signal when compared to the correct non-isothermal linear system. Therefore there would be no gravity modes in our pseudo system if the profiles were statically unstable or neutral.

The static stability is automatically satisfied in the case of isothermal profiles. In the case of the non-isothermal profiles this conditions has to be carefully checked. The profiles is checked to be statically stable in SUNHSI routine.

The stability condition related to non-isothermal temperature profiles has not been theoretically studied. Such study could be (probably) performed in the vertically discretized framework only and therefore it is not possible to obtain the general condition as it is in the isothermal case (${T_a}^* < T < T^*$). Some insight into the theory of such analysis can be found in [2].

The experiments revealed that the top model layer of the non-isothermal profiles has to be isothermal in order to maintain stability of the time stepping procedure. The reason of instability is probably a large jump in temperature associated with the zero pressure assumption at the model top (the depth of the model top layer can be significantly greater than of others ones).

The way the reference temperature profiles are constructed is controlled by the namelist variable NSITR_METHOD.

When first method is applied, the non-isothermal profiles are build as a constant static stability profiles (see namelist variable NSITR_METHOD). We defined the two values of constant brunt vaisala frequency, one N^* for the profile T^* and one N_a^* for the acoustic profile ${T_a}^*$ (namelist variables SITR_BVF and SITRA_BVF). The frequencies shall satisfy the relation

$$N^* < N_a^*.$$

In the isothermal context this condition is equivalent to ${T_a}^* < T^*$.

Additional to brunt vaisala frequencies, we define the surface value of reference temperature profile T^* (namelist variable SITR), and the model top value of the acoustic temperature profile (namelist variable SITRA). The values of SITR and SITRA have to be from the interval $SITR \in (0, \frac{g^2}{C_p N^*} >$ resp. $SITRA \in (0, \frac{g^2}{C_p N_a^*} >$. This procedure doesn't guarantee that the profiles of T^* and ${T_a}^*$ specify sufficiently large temperature interval for real atmospheric tests.

The second method of profiles construction was implemented for the purpose of practical NWP. The profiles are constructed in a way that they specify a sufficiently wide belt around the standard atmosphere temperature profile (namelist variables SITR_SHIFT and SITRA_SHIFT).

Last method of profiles construction is their determination via namelist directly. Still the condition of static stability has to be fulfilled.

The appropriate method to define the reference temperature profiles has to be further studied.

3.2 Definition of spectral space matrices

For better understanding we write down here the matrices $\mathbf{M_A}, \mathbf{M_B}, \mathbf{M_E}$ and $\mathbf{M_F}$ (from (15)) in the form as they are implemented in the code of the model ALADIN. Here the operators $\mathbf{G}^*$, $\mathbf{V}^*$,$\mathbf{S}^*$ and $\mathbf{L}^*$ are represented as matrices with dimension LxL (for more information see APPENDIX A).

Comment:
Matrix $M(m,n)$ has m rows and n columns. The multiplication of matrix with vector is defined as $y_i = \sum_{k=1}^{n} M_{i,k} x_k$, and the multiplication of matrix A with the matrix B is defined as $C_{i,j} = \sum_{k=1}^{n} A_{i,k} B_{k,j}$.

The spectral matrices can be then written

$$\mathbf{M_A}_{i,j} = \delta_{i,j} \left[1 - \tau^2 c^{*2}_i \triangle \mathbf{m}_*{}^2 \right] - \tau^2 \triangle \mathbf{m}_*{}^2 \left[g H^*{}_i - c^{*2}_i \right] \mathbf{V}^*_{i,j} \tag{16}$$

$$\mathbf{M_B}_{i,j} = \triangle \left(\tau^2 \left[-R T^*_i \mathbf{G}^*_{i,j} + c^{*2}_i \delta_{i,j} \right] + \tau^2 \left[g H^*{}_i - c^{*2}_i \right] \mathbf{V}^*_{i,j} \right) \tag{17}$$

$$\mathbf{M_E}_{i,j} = \left[\delta_{i,j} - \tau^2 \frac{c^{*2}_i}{H_a{}^*{}_i H^*{}_i} \mathbf{L}^*_{i,j} \right] \tag{18}$$

$$\mathbf{M_F}_{i,j} = \sum_{k=1}^{L} \tau^2 \mathbf{m}_*{}^2 \frac{1}{H_a{}^*{}_i} \mathbf{L}^*_{i,k} \frac{1}{H^*{}_k} \left[-R T^*_k \mathbf{S}^*_{k,j} + c^{*2}_i \delta_{i,k} \right] \tag{19}$$

The symbol $\delta_{i,k}$ represents the identity matrix. The horizontal laplacian operator $\triangle$ is in the spectral space replaced by the sum of square of horizontal wavenumber $\triangle X = (-k_x^2 - k_y^2)X$.

3.3 Code aspects

All development has been made with CY29T4 and the sources are available in pack mrpm622@tora:pack/cy29t4_pseudo_solver.

YOMDYNA

The fact that we use pseudo non-isothermal solver has to be known before matrix SIEHEL is allocated in subroutine SUELLO. Therefore the key to switch on or off the usage of solver was introduced into YOMDYNA module (YOMDYN is called after SUELLO).

NAMDYNA `LSI_NONISOTHERMAL` .T. - turn on the pseudo non-isothermal solver

YOMDYN

The following variables has been introduced or they changed their meaning in the case of `LSI_NONISOTHERMAL=.T.` in module YOMDYN

NAMDYN	`SITR_N(1:NFLEVG)`	reference non-isothermal temperature T^* profile
NAMDYN	`SITRA_N(1:NFLEVG)`	reference non-isothermal temperature ${T_a}^*$ profile
NAMDYN	`NSITR_METHOD`	choose method to initialize the profiles T^* and ${T_a}^*$
		=1 - constant static stability profiles
		=2 - profiles with constant departure from standard atmosphere
		=3 - profiles are set explicitly in NAMDYN
NAMDYN	`SITR_BVF`	brunt vaisala frequency of T^* profile (`NSITR_METHOD=1`)
NAMDYN	`SITRA_BVF`	brunt vaisala frequency of ${T_a}^*$ profile (`NSITR_METHOD=1`)
NAMDYN	`SITR`	surface temperature of T^* profile (`NSITR_METHOD=1`)
NAMDYN	`SITRA`	top model layer temperature of ${T_a}^*$ profile (`NSITR_METHOD=1`)
NAMDYN	`SITR_SHIFT`	shift of T^* profile from standard temperature (`NSITR_METHOD=2`)
NAMDYN	`SITRA_SHIFT`	shift of T^* profile from standard temperature (`NSITR_METHOD=2`)
	`SIRPRG_N(NFLEVG)`	profile of quantity RT^*

The conditions `SITR_SHIFT>0` and `SITRA_SHIFT<0` have to be satisfied.

SUDYNA (NAMDYNA)

Set default value of `LSI_NONISOTHERMAL=.F.` and read its value from the namelist NAMDYNA.

SUELLO

Allocate SIEHEL matrix as $2Lx2L$ matrix in the case of `LSI_NONISOTHERMAL=.T.`.

SUDYN

Set default values of NAMDYN variables

NAMDYN	`SITR_N(:)`	-999.
NAMDYN	`SITRA_N(:)`	-999.
NAMDYN	`NSITR_METHOD`	1
NAMDYN	`SITR_BVF`	$\sqrt{\frac{g^2}{C_p 350.}}$
NAMDYN	`SITRA_BVF`	$\sqrt{\frac{g^2}{C_p 250.}}$
NAMDYN	`SITR`	330.
NAMDYN	`SITRA`	120.
NAMDYN	`SITR_SHIFT`	50.
NAMDYN	`SITRA_SHIFT`	-50.

If `NSITR_METHOD=2`, the profiles `SITR_N(:)` and `SITRA_N(:)` are computed as a departures from standard temperature profile `STTEM` as

$$
\begin{aligned}
SITR\_N(:) &= STTEM(:) + SITR\_SHIFT \\
SITRA\_N(:) &= STTEM(:) + SITRA\_SHIFT \qquad (20)
\end{aligned}
$$
(21)

SUNHSI

If `NSITR_METHOD=1`, the profiles `SITR_N` and `SITRA_N` are computed as constant stability profiles using the values of SITR, SITRA, SITR_BVF and SITRA_BVF.

The static stability of temperature profiles SITR_N and SITRA_N is controlled. If profiles are unstable the execution is aborted.

Isothermal top layer is set as $SITR\_N(1) = SITR\_N(2)$ and $SITRA\_N(1) = SITRA\_N(2)$.

SIGAM, SIPTP, SIDD, SITNU

Routines have been adjusted to take into account the non-isothermal variables $SITR\_N$, $SITRA\_N$ and $SIRPRG\_N$. The old parts of code for isothermal solver are protected.

SUEHEL

Computation of $2Lx2L$ matrix SIEHEL for the case of pseudo non-isothermal solver.

ESEIMPLS, ESPNHSI, LACDYN, LANHSI, LATTEX, CPEULDYN

Adjusted to take into account the pseudo non-isothermal solver.

Removal of dummy arguments PSITR and PSITRA from LANHSI routine. They are read from the module directly.

4 Tests

4.1 Explicit Convection Experiments

The introduction of pseudo non-isothermal solver has aimed to improve the phase speed propagation of internal gravity waves, mainly for large time steps. Explicit convection is the non-linear process where the short gravity waves play an important role, we decided to test the new solver on the idealize explicit convection test.

The dry atmosphere has a constat value of potential temperature $\theta_e = 300K$. The air with positive potential departures is located near the ground. It has the positive buoyancy and the shape of gaussian circle with its center located at the ground (see figuren 1). The model mesh size is 100m and the approximate depth of vertical layers is 100m as well. The modeled area has size 12km in horizontal and vertical direction. There is no orography present and the domain is horizontally periodic.

We exploited the 2TL time stepping with the semi-Lagrangian treatment. The key namelist options are listed in the following table

key	value
LSLAG	TRUE
LTWOTL	TRUE
LPC_FULL	TRUE
LPC_NESC	TRUE
NSITER	1
LGWADV	TRUE
NVDVAR	4

The tests were run without any additional dissipative process (no physics, no horizontal diffusion, no sponge layer, no decentering).

All the tests were run with the following values of crucial namelist variables

key	isothermal solver	pseudo solver
LSI_NONISOTHERMAL	FALSE	TRUE
SITR	320	320
SITRA	120	120
SITR_BVF		$\frac{g^2}{C_p 350}$
SITRA_BVF		$\frac{g^2}{C_p 250}$

First tests was dedicated to test the correctness of the pseudo isothermal profile installation. The model was run with short time step $\Delta t = 0.5s$ with isothermal profile (test EC31) and with the pseudo

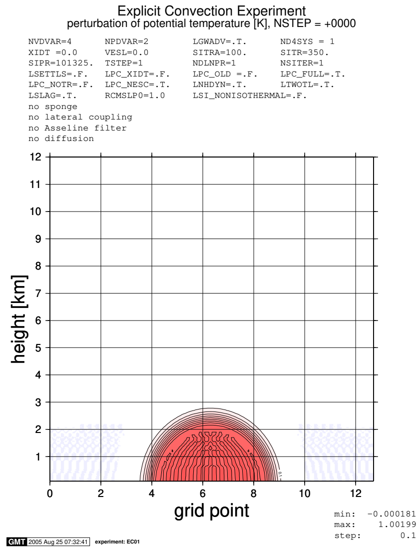

Figure 1: The experiments initial state. The departure of potential temperature from its background value is shown with contour step $0.1K$. The horizontal and vertical resolution is approx. 100m in both directions.

solver (test EC33). The obtained results, after 2000s of integration, were almost identical and the are shown on the figure 2.

In further tests we check the distortion of rising bubbles when the time step was enlarged to value $\Delta t = 5s$ (tests EC11 and EC13 on figure 3) and to value $\Delta t = 20s$ (tests EC21 and EC23 on figure 4). The difference brought by pseudo non-isothermal is negligible, although the standard deviation, when compared to the reference results obtained with $\Delta t = 0.5s$, is slightly smaller for pseudo non-isothermal solver.

4.2 Stability Experiment (Nonlinear flow over obstacles)

To be done.

4.3 Real Case Experiment - Tatra's wind storm 2004

To be done.

5 Conclusion

The pseudo non-isothermal solver turn out to have neutral effect on the explicit convection when run with large time steps. The improvement can be considered to be negligible.

There is still aspect of improved robustness of time stepping procedure when pseudo non-isothermal solver is exploited. Due to lack of time we did not succeed to test this aspect and this work still have

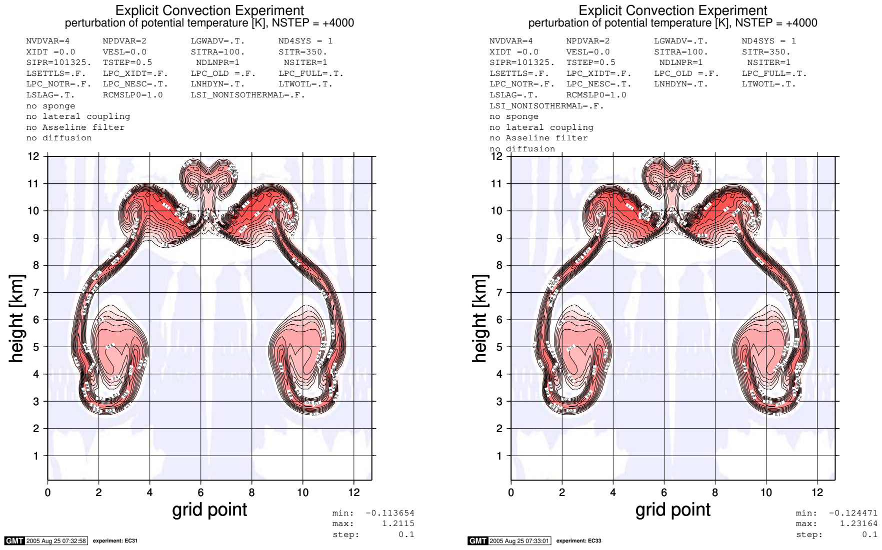

Figure 2: The experiments with very short time step $\Delta t = 0.5s$. The departure of potential temperature from its background value is shown with contour step $0.1K$ after 2000s of integration. The scheme with the isothermal solver (left) against the scheme with pseudo non-isothermal solver (right).

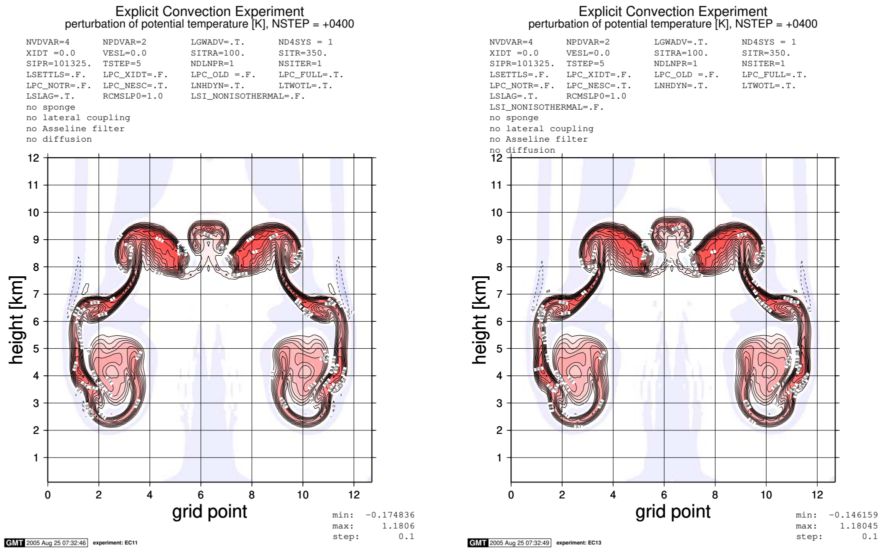

Figure 3: As on the figure 2 but with the time step $\Delta t = 5s$.

to be done.

The pseudo non-isothermal solver is designed in a way that the constrain $C1$ is identical to the

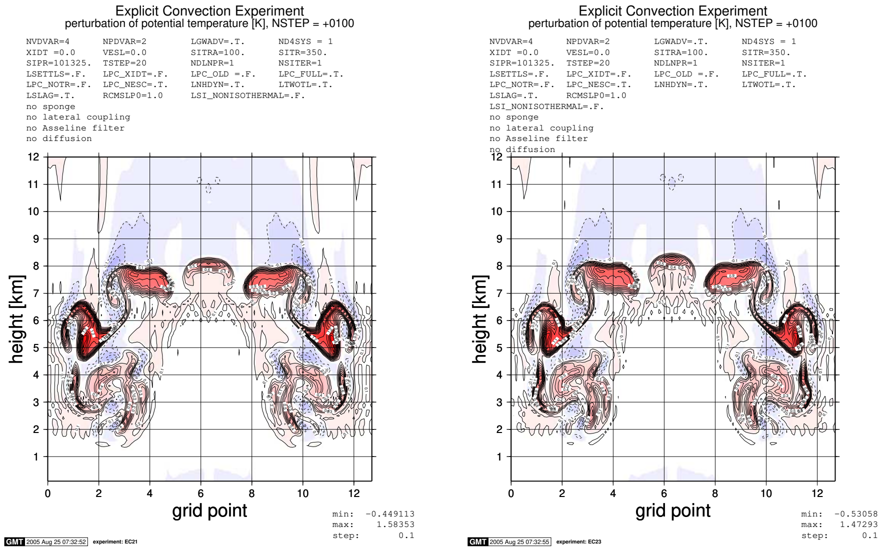

Figure 4: As on the figure 2 but with the time step $\Delta t = 20s$.

isothermal case one and there is no additional constrain $C2$ equivalent to the one in isothermal case. Hence, the vertical discretization could be kept unchanged and it is justified.

The disadvantage of pseudo non-isothermal solver is the need to solve $2Lx2L$ correction problem in the spectral space. It brings the 15% increase in memory consumption and 2.5% in CPU consumption (measured with the domain 120x90 points with 37 levels and 3 hour long integration).

6 APPENDIX A: The vertical operators

The vertical operators appears in our system. Their origin is from the computation of the geopotential and the computation of the vertical advective velocity and the tendency of the surface pressure related prognostic variable.

The vertical operators are defined as

$$\begin{aligned}
\left(\mathbf{G}^{*}\psi\right)_{l} &= \sum_{k=l+1}^{L} \delta_{k}^{*}\psi_{k} + \alpha_{l}^{*}\psi_{l} \\
\left(\mathbf{S}^{*}\psi\right)_{l} &= \frac{1}{\pi_{l}^{*}} \sum_{k=1}^{l-1} \delta\pi_{k}^{*}\psi_{k} + \beta_{l}^{*}\psi_{l} \\
\left(\mathbf{N}^{*}\psi\right)_{l} &= \frac{1}{\pi_{s}{}^{*}} \sum_{k=1}^{L} \delta\pi_{k}^{*}\psi_{k} \\
\left(\partial^{*}\psi\right)_{l} &= \frac{\pi_{l}^{*}}{\delta\pi_{l}^{*}} \left(\psi_{\tilde{l}} - \psi_{\tilde{l}-1}\right) \\
\left(\mathbf{L}^{*}\psi\right)_{l} &= \partial^{*}\left(\partial^{*}\psi + 1\right)_{l} \\
\left(\mathbf{V}^{*}(\psi)\right)_{l} &= \sum_{k=l+1}^{L} \delta_{k}^{*} \left(1 - \frac{T_{k}^{*}}{T_{l}^{*}}\right) \psi_{k}.
\end{aligned} \tag{22}$$

The linear vertical operator $\mathbf{L}^*$ is discretized in the form

$$(\mathbf{L}^*\psi)_l = \left[\frac{\pi^*}{m^*}\frac{\partial}{\partial\eta}\left(\frac{1}{m^*}\frac{\partial\pi^*\psi}{\partial\eta}\right)\right]_l. \tag{23}$$

It can be easily verified that this form is equivalent to

$$(\mathbf{L}^*\psi)_l = \left[\frac{\pi^*}{m^*}\frac{\partial}{\partial\eta}\left(\frac{\pi^*}{m^*}\frac{\partial}{\partial\eta}+1\right)\psi\right]_l. \tag{24}$$

The definition of vertically dependent variables δ^*, α^* and the exact form of $\mathbf{L}^*$ is out of scope of this report and it can be found in Bubnova, 1995 (resp. in more detailed documentation written by Benard).

7 APPENDIX B: The traditional "true" non-isothermal linear operator

When the traditional linearization is carried out using the non-isothermal profile $T^*(\eta)$ and after the linearization the profiles in denominators are replaced by $T_a{}^*(\eta)$ then following linear system is received.

$$\begin{aligned}
\frac{\partial D}{\partial t} &= -R\mathbf{G}^*\triangle T + R\mathbf{G}^*(T^*(\eta)\triangle\hat{q}) - RT^*(\eta)\triangle\hat{q} - RT^*(\eta)_s\triangle q_s + R\mathbf{G}^*(\partial^* T^*(\eta)\triangle q) - \triangle\phi_s \\
\frac{\partial \xi}{\partial t} &= 0 \\
\frac{\partial T}{\partial t} &= -\frac{RT^*(\eta)}{C_v}(D+d_3) - \dot{\eta}\frac{\partial T^*(\eta)}{\partial\eta} \\
\frac{\partial q_s}{\partial t} &= -\mathbf{N}^* D \\
\frac{\partial d_3}{\partial t} &= -\frac{g^2}{RT_a{}^*(\eta)}\mathbf{L}^*\hat{q} \\
\frac{\partial \hat{q}}{\partial t} &= -\frac{C_p}{C_v}(D+d_3) + \mathbf{S}^* D.
\end{aligned} \tag{25}$$

It was presented in [2] that the SI (2TL and 3TL) scheme build with the previous system is conditionally stable and the stability is limited by the difference between the reference profile static stability and the static stability in the modeled atmosphere. Such stability condition is more restrictive one than for the scheme with purely isothermal profile.

The constrain $C1$ (equivalent to equation (42) in [7]) obtained during the elimination of the previous continuous system yields

$$A_1^* : -\mathbf{G}^*(\partial^* T^*(\eta)\mathbf{S}^* X) \quad = \quad -T_s{}^*\mathbf{N}^* X + T^*(\eta)\mathbf{S}^* X + \mathbf{G}^*(T^*(\eta)X) - \mathbf{G}^*(T^*(\eta)\mathbf{S}^* X) \tag{26}$$

The equivalent constrain plays a key role in the determination of vertical discretization in [7]. We did not succeed to solve (26) satisfactorily.

Comparing to the isothermal case the elimination of the linear system for the one prognostic variable is not possible in a way as it is done in [7], because the operators are not anymore commutative. Hence the vertical operators are not constrained by the relation equivalent to $C2$ in [7] (see equation 44 in that article).

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