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Timestepping
etc.

(report from stay)

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1 Introduction

We want to solve an equation

$$\frac{\partial\psi}{\partial t} + u\frac{\partial\psi}{\partial x} = L + N + P \tag{1}$$

where L is the linear part of dynamics, N is the non-linear part of dynamics, and P is physics tendency.

2 Two-time-level Semi-implicit Semi-Lagrangian

To discrteize this in time and space in a semi-implicit semi-lagrangian way we can do the following

$$\psi_A^{t+\Delta t} - \psi_D^t = \frac{\Delta t}{2}(L_A^{t+\Delta t} + L_D^t) + N_M^{t+\frac{\Delta t}{2}} + P_M^{t+\frac{\Delta t}{2}} \tag{2}$$

We have to estimate the positions of the origin point D and the middle point M and to find appropriate approximations for $N_M^{t+\frac{\Delta t}{2}}$ and $P_M^{t+\frac{\Delta t}{2}}$.

Truly semi-implicit semi-lagrangian way of discretizing this equation would be

$$\psi_A^{t+\Delta t} - \psi_D^t = \frac{\Delta t}{2}(L_A^{t+\Delta t} + L_D^t) + \frac{\Delta t}{2}(N_A^{t+\Delta t} + N_D^t) + \frac{\Delta t}{2}(P_A^{t+\Delta t} + P_D^t) \tag{3}$$

Therefore, we split the contribution of physics from the contribution of dynamics and calculate them separately and subsequently join them together.

REMARK: we could split the physics contribution into the linear and the non linear parts

$$P = PL + PN \tag{4}$$

and use the same treatement as for the linear and non-linear contributions from dynamics.

$$\psi_A^{t+\Delta t} - \psi_D^t = \frac{\Delta t}{2}(L_A^{t+\Delta t} + L_D^t) + \frac{\Delta t}{2}(N_A^{t+\Delta t} + N_D^t) + \frac{\Delta t}{2}(PL_A^{t+\Delta t} + PL_D^t) + \frac{\Delta t}{2}(PN_A^{t+\Delta t} + PN_D^t) \tag{5}$$

However, this is would be very difficult to solve due to

- $N_A^{t+\Delta t}$
- $PN_A^{t+\Delta t}$
- some physics parameterisations need dynamical tendencies as input

So the only physics tendency used here is the total tendency P (linear and non-linear parts of physics are not solved separately).

3 When to calculate physics?

This section attempts to give a short overview of possibilities when to calculate physics tendencies during a time-step. The calculation of physics tendency is split from the calculation of the dynamical one. But when to calculate it? There are several possibilities listed below. In this section we do not discuss where to calculate the physics tendency (arrival point, origin point or some mix of the two). But there are some suggestions listed in the equations.

- physics after the semi-implicit adjustment and after the semi-lagrangian interpolation

$$\psi_A^{t+\Delta t*} - \psi_O^t = \frac{\Delta t}{2}(L_A^{t+\Delta t*} + L_O^t) + \Delta t N_M^{t+\frac{\Delta t}{2}} \tag{6}$$

$$\left(\frac{\partial \psi}{\partial t}\right)_{dyn} = \frac{\psi_A^{t+\Delta t*} - \psi_O^t}{\Delta t} \tag{7}$$

is used as the dynamical tendency for input to physics. The final value is obtained as

$$\psi_A^{t+\Delta t} = \psi_A^{t+\Delta t*} + \Delta t P^{t+\Delta t*} \tag{8}$$

we could use P^t instead of $P^{t+\Delta t*}$.

- physics before the semi-implicit adjustment and after the semi-lagrangian interpolation

$$\psi_A^{t+\Delta t*} - \psi_O^t = \frac{\Delta t}{2}(L_A^t + L_O^t) + \Delta t N_M^{t+\frac{\Delta t}{2}} \tag{9}$$

where

$$N_M^{t+\frac{\Delta t}{2}} = \frac{1}{2}\left(N_A^{t+\frac{\Delta t}{2}} + N_D^{t+\frac{\Delta t}{2}}\right) \tag{10}$$

$$\left(\frac{\partial \psi}{\partial t}\right)_{dyn} = \frac{\psi_A^{t+\Delta t*} - \psi_O^t}{\Delta t} \tag{11}$$

is used as the dynamical tendency for input to physics. The final value is obtained as

$$\psi_A^{t+\Delta t} = \psi_A^{t+\Delta t*} + \frac{\Delta t}{2}\left(L_A^{t+\Delta t} - L_A^t\right) + \Delta t P^t \tag{12}$$

we could use $P^{t+\Delta t*}$ instead of P^t ?

- physics before the semi-implicit adjustment and before the semi-lagrangian interpolation

$$\psi_A^{t+\Delta t} - \psi_O^t = \frac{\Delta t}{2}\left(L_A^{t+\Delta t} + L_O^t\right) + \Delta t N_M^{t+\frac{\Delta t}{2}} + \frac{\Delta t}{2}\left(P_A^t + P_D^t\right) \tag{13}$$

where

$$N_M^{t+\frac{\Delta t}{2}} = \frac{1}{2}\left(N_A^{t+\frac{\Delta t}{2}} + N_D^{t+\frac{\Delta t}{2}}\right) \tag{14}$$

The dynamical tendency $\left(\frac{\partial \psi}{\partial t}\right)_{dyn}$ for input to physics in ALADIN is computed explicitly at the beginning of each time-step. Also, in ALADIN we use $\Delta t P_D^t$ instead $\frac{\Delta t}{2}\left(P_A^t + P_D^t\right)$.

- physics before the semi-implicit adjustment and before the semi-lagrangian interpolation, with storing the physics tendency from the previous time-step

$$\psi_A^{t+\Delta t*} - \psi_O^t = \frac{\Delta t}{2}\left(L_A^t + L_O^t\right) + \Delta t N_M^{t+\frac{\Delta t}{2}} + \frac{\Delta t}{2}\left(P_D^{t-\Delta t} - P_A^{t-\Delta t}\right) \tag{15}$$

where

$$N_M^{t+\frac{\Delta t}{2}} = \frac{1}{2}\left(N_A^{t+\frac{\Delta t}{2}} + N_D^{t+\frac{\Delta t}{2}}\right) \tag{16}$$

$$\left(\frac{\partial \psi}{\partial t}\right)_{dyn} = \frac{\psi_A^{t+\Delta t*} - \psi_O^t}{\Delta t} \tag{17}$$

is used as the dynamical tendency for input to physics. The final value is obtained as

$$\psi_A^{t+\Delta t} = \psi_A^{t+\Delta t*} + \frac{\Delta t}{2}\left(L_A^{t+\Delta t} - L_A^t\right) + \frac{\Delta t}{2}\left(P^t + P^{t-\Delta t}\right) \tag{18}$$

- physics after the semi-implicit adjustment and before the semi-lagrangian interpolation

$$\psi_A^{t+\Delta t*} - \psi_O^t = \frac{\Delta t}{2}\left(L_A^{t+\Delta t*} + L_O^t\right) + \frac{\Delta t}{2} N_M^{t+\frac{\Delta t}{2}} \tag{19}$$

$$\left(\frac{\partial \psi}{\partial t}\right)_{dyn} = \frac{\psi_A^{t+\Delta t*} - \psi_O^t}{\Delta t} \tag{20}$$

is used as the dynamical tendency for input to physics. The final value is obtained as

$$\psi_A^{t+\Delta t} = \psi_D^{t+\Delta t*} + \frac{\Delta t}{2} N_A^{t+\frac{\Delta t}{2}} + \Delta t P^{t+\Delta t*} \tag{21}$$

- SLAVEPP - semi-lagrangian averaging of physics parameterisations (used at ECMWF)

$$\psi_{Adyn}^{t+\Delta t} - \psi_D^t = \frac{\Delta t}{2}(L_A^{t+\Delta t} + L_D^t) + \Delta t N_M^{t+\frac{\Delta t}{2}} \tag{22}$$

$$\left(\frac{\partial \psi}{\partial t}\right)_{dyn} = \frac{\psi_{Adyn}^{t+\Delta t*} - \psi_A^t}{\Delta t} \tag{23}$$

is used as the dynamical tendency for input to physics. The final value is obtained as

$$\psi_A^{t+\Delta t} = \psi_D^t + \frac{\Delta t}{2}(L_A^{t+\Delta t} + L_D^t) + \Delta t N_M^{t+\frac{\Delta t}{2}} + + \frac{\Delta t}{2}(P_A^{t+\Delta t} + P_D^t) \tag{24}$$

actually not all physics tendencies are averaged along trajectory. Radiation, convection and cloudiness contributions are averaged along trajectory, while only arrival point contributions are taken from the vertical diffusion and gravity wave drag parameterisation schemes.

$$P_A^{t+\Delta t} = P_{Arad}^{t+\Delta t} + P_{Acnv}^{t+\Delta t} + P_{Acld}^{t+\Delta t} + 2(P_{Avdf}^{t+\Delta t} + P_{Agwd}^{t+\Delta t}) \tag{25}$$

$$P_D^t = P_{Drad}^t + P_{Dcnv}^t + P_{Dcld}^t \tag{26}$$

Some of the parameterisation schemes need tendencies from the dynamics or other physics schemes as input, therefore $P_A^{t+\Delta t}$ is calculated in the following way:

$$P_A^{t+\Delta t} = P_{Arad}^{t+\Delta t}(\psi_A^t) + P_{Acnv}^{t+\Delta t}(\psi_{Apred}^{t+\Delta t}) + P_{Acld}^{t+\Delta t}(\psi_{Apred}^{t+\Delta t}, cnv) + P_{Avdf}^{t+\Delta t}(\psi_A^t, dyn, P_{Arad}^{t+\Delta t}) + P_{Agwd}^{t+\Delta t}(\psi_A^t, dyn) \tag{27}$$

where $\psi_{Apredictor}^{t+\Delta t}$ is a first-guess predictor of the model variables at the arrival point at ime $t+\Delta t$ computed from the dynamic tendency, tendencies of radiation, convection and cloudiness from time t and tendencies of vertical diffusion and gravity wave drag from time $t + \Delta t$.

$$\psi_{Apredictor}^{t+\Delta t} = \psi_{Adyn}^{t+\Delta t} + \Delta t P_{Drad}^t + \Delta t P_{Dcnv}^t + \Delta t P_{Dcld}^t + \Delta t P_{Avdf}^{t+\Delta t} + \Delta t P_{Agwd}^{t+\Delta t} \tag{28}$$

4 How to approximate $N_M^{t+\frac{\Delta t}{2}}$ (and $\vec{V}_M$, $\vec{r}_M$, $\vec{r}_O$) ?

$N_M^{t+\frac{\Delta t}{2}}$ is the non-linear contribution from dynamics in the middle point of the trajectory at a time $t + \frac{\Delta t}{2}$.

4.1 Extrapolate in time and average between O and A

The non-linear terms are first extrapolated in time

$$N^{t+\frac{\Delta t}{2}} = \frac{3}{2}N^t - \frac{1}{2}N^{t-\Delta t} \tag{29}$$

and the middle point value of the non-linear term is obtained by interpolation

$$N_M^{t+\frac{\Delta t}{2}} = \frac{N_A^{t+\frac{\Delta t}{2}} + N_D^{t+\frac{\Delta t}{2}}}{2} \tag{30}$$

The procedure is the same for the wind speed

$$\vec{V}^{t+\frac{\Delta t}{2}} = \frac{3}{2}\vec{V}^t - \frac{1}{2}\vec{V}^{t-\Delta t} \tag{31}$$

which is used to calculate the origin point

$$\vec{r}_A^{t+\Delta t} = \vec{r}_D^t + \frac{\Delta t}{2}(\vec{V}_A^{t+\frac{\Delta t}{2}} + \vec{V}_D^{t+\frac{\Delta t}{2}}) \tag{32}$$

These extrapolations and interpolations are done in time and space separately, disregarding the fact that the parcel changes its position in space during time. This kind of evaluation of $N_M^{t+\frac{\Delta t}{2}}$ can be unstable. Obtained in ALADIN when LSETTLS=.F. and LPC_NESC=.F.

4.2 Average along trajectory

$$N_M^{t+\frac{\Delta t}{2}} = N_D^t + \frac{\Delta t}{2}\left(\frac{dN}{dt}\right)_{AV} \tag{33}$$

where

$$\left(\frac{dN}{dt}\right)_{AV} = \frac{N_A^t - N_D^{t-\Delta t}}{\Delta t} \tag{34}$$

so the non-linear term in the middle point of the trajectory is approximated by

$$N_M^{t+\frac{\Delta t}{2}} = N_D^t + \frac{1}{2}\left(N_A^t - N_D^{t-\Delta t}\right) \tag{35}$$

if we do the same with the wind speed, the origin point is determined in the following way

$$\vec{r}_A^{t+\Delta t} = \vec{r}_D^t + \Delta t \vec{V}_D^t + \frac{\Delta t}{2}(\vec{V}_A^t + \vec{V}_D^{t-\Delta t}) \tag{36}$$

so this scheme extrapolates in time 'along trajectory'. This is stable extrapolation for two-time-level scheme (SETTLS) and it is used in ALADIN when LSETTLS=.T.

4.3 No extrapolation

This approach can be used only with the predictor-corrector timestepping. When the predictor - corrector scheme is used, the non-linear terms are reiterated during corrector(s) so we can use the following scheme

$$N_M^{t+\frac{\Delta t}{2}} = \frac{1}{2}\left(N_A^t + N_D^t\right) \tag{37}$$

During corrector steps, the wind components for trajectories and non-linear residuals are centered in time using results from the predictor or the previous corrector.

$$\begin{aligned}
\text{predictor} \quad \psi_A^{t+\Delta t|0} - \psi_D^t &= N_M^{t+\frac{\Delta t}{2}} + \frac{1}{2}\left(L_A^{t+\Delta t|0} + L_D^t\right) && (38)\\
\text{corrector} \quad \psi_A^{t+\Delta t|1} - \psi_D^t &= \frac{1}{2}\left(N_A^{t+\Delta t|0} + N_D^t\right) + \frac{1}{2}\left(L_A^{t+\Delta t|1} + L_D^t\right) && (39)\\
&\ldots && (40)
\end{aligned}$$

However, $N_M^{t+\frac{\Delta t}{2}}$ in the predictor can be obtained using either (29 and 30), (35) or (37).

5 Bibliography

[**1**] Cullen, M.J.P., 2001: Alternative implementations of the semi-Lagrangian semi-implicit schemes in the ECMWF model *Q.J.R.M.S.*, **127**, 2787-2802.

[**2**] Cullen, M.J.P. and D.J. Salmond, 2002: On the use of a predictor-corrector scheme to couple the dynamics with the physical parameterisations in the ECMWF model *ECMWF Tech. Memo.*, **357**, 20pp.

[**3**] Cullen, M.J.P., D.J. Salmond and N.P. Wedi, 2002: Interaction of parameterised processes with resolved dynamics *ECMWF Seminar proceedings on 'Key issues in the parameterisation of subgrid physical processes'*, 127-149.

[**4**] Grabowski, W.W. and P.K. Smolarkiewicz, 1996: Two-time-level semi-Lagrangian modelling of precipitating clouds *Mon. Wea. Rev.*, **124**, 487-497

[**5**] Hortal, M. 1998: Aspects of the numerics of the ECMWF model *ECMWF Workshop proceedings on 'Recent developements in numerical methods for atmospheric modelling'*, 127-143.

[**6**] Martinez Marco, I. 2004: Tests prior to the operational implementation of the physics-dynamics coupling *HIRLAM Newsletter*, **45**, 184-198.

[**6**] McDonald, A. 1998: The orogin of noise in semi-Lagrangian integrations *ECMWF Workshop proceedings on 'Recent developements in numerical methods for atmospheric modelling'*, 308-334.

[**7**] Staniforth, A., N. Wood and J. Cote, 2002: A simple comparison of four physics-dynamics coupling schemes *Mon. Wea. Rev.*, **130**, 3129-3135.

[**8**] Staniforth, A., N. Wood and J. Cote, 2002: Analysis of the numerics of physics-dynamics coupling *QJRMS*, **128**, 2779-2799.

[**9**] Wedi, N.P. 1999: The numerical coupling of the physical parameterisations to the "dynamical" equations in a forecast model, *ECMWF Tech. Memo.*, **274**, 37pp.