Consistency of Arome equations with Aladin dynamics (and physics) assumptions and hypotheses

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Contents

1 Introduction

Model Arome is based upon Aladin dynamics and Meso-NH physics. A set of equations for the Arome model has been developed by J. Stein, S. Malardel and P. Benard, including the application of these equations to the Meso-NH model (ref. Arome et. al.). Aladin model is based on a few (considerably crude) assumptions. But it also contains interesting options concerning mass conservation. The question to be answered was how the current Aladin system could work inside the Arome framework and if all the available options will remain available in the future system of Arome. The current Aladin equations can be derived from the Arome equations. The second section will present Arome equations listed as in Arome & al. The third section will show how Aladin assumptions can be introduced into Arome and how Aladin equations are obtained.

2 Arome equations

Arome model equations and the assumptions are listed here. For more detail, please refer to Arome et. al. (ref).

2.1 Assumptions

1. All the different components in the atmosphere are at the same temperature T.
2. The different gasses are ideally mixed ...
3. Local equilibrium is valid at the size of the computational grid.
4. The specific heat capacities and the gas constants are temperature independant.
5. Under and super saturation do not exist.
6. The dependancy of specific enthalpy upon pressure for the condensed phases and their specific volumes are neglected.
7. Turbulent fluxes may be source and sink or subgrid scale transport for the different conservation laws (?).
8. Kinetic energy of diffusion is a part of the internal energy.

The system of equations is listed below.

2.2 Continuity equation

Respective conservation of dry air an total water is assumed. The equation for the total density of the mixture is

$$\frac{\partial \rho}{\partial t} = -\vec{\nabla} \cdot (\rho \vec{u}) <==> \frac{d\rho}{dt} = -\rho \nabla \cdot \vec{u} \tag{1}$$

ρ is the total density of the mixture

$$\rho = \sum_{k=0}^{N} \rho_k \tag{2}$$

where $k = 0$ for dry air, $k = 1$ for water vapour, $k = 2$ for cloud water, $k = 3$ for rain, $k = 4$ for cloud ice, $k = 5$ for snow and $k = 6$ for graupel. $\vec{u}$ is the barycentric velocity is defined as

$$\vec{u} = \frac{1}{\rho} \sum_{k=0}^{N} \rho_k \vec{u}_k \tag{3}$$

It is allowed for different species in the atmosphere to have different velocities. However, we will use a simplified version of this possibility. Only the vertical component of the precipitable species is different. All the non-precipitating species have the same velocity.

2.3 Conservation of species

In the atmosphere, model recognises 6 species: dry air, water vapour, cloud water, ran, ice, snow and graupel.

$$\frac{dq_k}{dt} = -\frac{1}{\rho} \vec{\nabla} \cdot (\rho q_k (\vec{u}_k - \vec{u})) \; + \frac{\dot{\rho}_k}{\rho} \tag{4}$$

where $q_k = \frac{\rho_k}{\rho}$ is the specific mass content in an unit volume of the species k. The system, written as here is not closed, since each species k might have different velocity. It is assumed that the velocities differ only in the vertical due to the falling of precipitation. Each precipitating species has its own terminal velocity V_{fk} calculated in the physics parameterisation (E. Richard, personal communication). All the non-precipitating species have the same velocity. Therefore, the velocity of the non-precipitating species is:

$$\vec{u}_k = \vec{u} + \frac{1}{\rho}(\rho_3 V_{f3} + \rho_5 V_{f5} + \rho_6 V_{f6})\vec{k} \tag{5}$$

the velocity of the precipitating species is:

$$\vec{u}_k = \vec{u} + \frac{1}{\rho}(\rho_3 V_{f3} + \rho_5 V_{f5} + \rho_6 V_{f6})\vec{k} - V_{fk}\vec{k} \tag{6}$$

This means that the terminal velocity is defined relative to the velocity of the non-precipitating species, not relative to the barycentric velocity.

2.4 Velocity equation

Velocity equation has been erived from the conservation of the total momentum where each species is accounted for, so the interaction force betweem the different species and the effect of the stress tensor have to be considered. From the momentum consevarion equations of the atmospheric species total momentum conservation equation is derived and the evolution equation for the barycentric velocity is obtained as

$$\frac{d\vec{u}}{dt} = -\frac{1}{\rho}\vec{\nabla}p + \frac{1}{\rho}\frac{\partial \sigma_{\alpha\beta}}{\partial x_\beta} + \vec{g} \tag{7}$$

Where $\sigma_{\alpha\beta}$ is viscous part of the stress. It is a consequence of an interaction force between the different species of the atmosphere.

2.5 Enthalpy equation

Conservation of the total energy of the mixture is assumed.

$$\frac{dh}{dt} = \frac{1}{\rho}\frac{dp}{dt} + \dot{Q}_i + \frac{1}{\rho}\sigma_{\alpha\beta}\frac{\partial u_\beta}{\partial x_\alpha} - \frac{1}{\rho}\nabla \cdot \sum_{k=0}^{N} h_k \rho_k (\vec{u}_k - \vec{u}) \tag{8}$$

where $h = \sum_{k=0}^{N} h_k q_k = e_{int} + \frac{p}{\rho}$ is the enthalpy.
$\dot{Q}_i$ represents diabatic cooling/heating term from physics parameterisations. $\frac{1}{\rho}\sigma_{\alpha\beta}\frac{\partial u_\beta}{\partial x_\alpha}$ is the contribution of the work of the stress tensor. $\frac{1}{\rho}\nabla \cdot \sum_{k=0}^{N} h_k \rho_k (\vec{u}_k - \vec{u})$ is the heat diffusion flux due to the difference in the specific enthalpies and different velocities of the components of the atmospheric mixture.

2.6 Temperature equation

$$C_p\frac{dT}{dt} = \frac{1}{\rho}\frac{dp}{dt} + \dot{Q}_i + \frac{1}{\rho}\sigma_{\alpha\beta}\frac{\partial u_\beta}{\partial x_\alpha} - \frac{1}{\rho}\sum_{k=0}^{N} \rho_k (\vec{u}_k - \vec{u}) \cdot \vec{\nabla} h_k - \frac{1}{\rho}\sum_{k=0}^{N} h_k \dot{\rho}_k \tag{9}$$

where $C_p = \sum_{k=0}^{N} q_k C_{pk}$ and $h_k = C_{pk}T$.

The term $\frac{1}{\rho}\sum_{k=0}^{N} \rho_k (\vec{u}_k - \vec{u}) \cdot \vec{\nabla} h_k$ can be written differently when the closure assumption with terminal velocities is included. Also, the term $\frac{1}{\rho}\sum_{k=0}^{N} h_k \dot{\rho}_k$ can be written in more detail.

$$C_p\frac{dT}{dt} = \frac{1}{\rho}\frac{dp}{dt} + \dot{Q}_i + \frac{1}{\rho}\sigma_{\alpha\beta}\frac{\partial u_\beta}{\partial x_\alpha} - \left(C_p V_f - \frac{1}{\rho}\sum_{k=3,5,6} \rho_k V_{fk} \right) \frac{\partial h_k}{\partial z} - \frac{1}{\rho}\left(L_{21}(\dot{\rho}_2 + \dot{\rho}_3) + L_{41}(\dot{\rho}_4 + \dot{\rho}_5 + \dot{\rho}_6) \right) \tag{10}$$

L_{21} and L_{41} are specific heat of evaporation/condensation of water and sublimation of ice. Please note that L_{41} is used for $\dot{\rho}_6$ (graupel).

3 Aladin equations

In this section, Aladin assumptions are introduced to the Arome system of equations and the system of equations of the Aladin model is obtained. Equations are first derived in z coordinate and afterwards transformed in p coordinate. In the Arome system precipitating species have a finite mass and a finite terminal velocity. In the current Aladin framework, mass of precipitations is neglected, and the precipitation is removed from a column of the atmosphere in a single time-step. In Aladin, precipitation is handled as a precipitation flux P throuht the level interfaces. However, Arome considers the fall of precipitation using terminal velocities. Since we don't want to have a time-step dependent terminal velocity, we assume infinite terminal velocity for precipitation. However, we assume that the mass-flux due to precipitation has a finite value P. This total precipitation flux actually consists of rain P_l, snow P_i and graupel P_g.

3.1 Assumptions and simplifications

(a) all the non-precipitating species move with the same speed $\vec{u}_a$
(b) mass of precipitations is neglected
(c) velocity of precipitations is infinite (to allow removal during one time-step)
(d) We define the product of the mass to the velocity of precipitations as:

$$\rho_{prec}\vec{u}_{prec} = -P\vec{k} \tag{11}$$

where P is the total precipitation flux (taken positive downwards).
(e) no graupel $\dot{\rho}_6 = 0$, only cloud water, cloud ice, rain and snow exist.

3.2 Conservation of mass

Currently there are two options existing in Aladin:

(1) $\delta_m = 0$ The total mass is conserved by adding/removing dry air to compensate for the loss/gain of the mass of water due to precipitation/evaporation of precipitation. The evaporation from ground is treated by the turbulence scheme (that is mass conserving itself). At surface the vertical velocity of the mixture is zero. Bottom boundary condition for the velocity is then $\vec{u}\cdot\vec{n} = 0$ where n is the vector perpendicular to the surface.

(2) $\delta_m = 1$ The mass of dry air is conserved. The total mass changes due to the changes in the budget of the moist species. The evaporation of E mass of water from ground surface is treated as a vertical advection of qE of mass of water vapour +(1-q)E mass of dry air = total advection of E mass of moist air with specific humidity q. The rest of the water vapour, (1-q)E enters the atmosphere through the turbulent flux. Therefore, at surface, vertical velocity of the mixture is non-zero. Bottom boundary condition for the velocity is then $\vec{u}\cdot\vec{n} = \frac{(E-P)}{\rho}\vec{k}\cdot\vec{n}$.

Using assumptions (a), (b) and (c) with the definition (d), we reach the following expression for the barycentric velocity

$$\vec{u} = \frac{1}{\rho}\sum_{k=0}^{N}\rho_k\vec{u}_k = \vec{u}_a - \frac{P}{\rho}\vec{k} \tag{12}$$

$\vec{u}_a$ is the common velocity of the non precipitating atmospheric species. Instead of introducing artificial sources and sinks of dry air to compensate for the loss of mass in the case $\delta_m = 0$ we can introduce a mass flux of dry air $\rho_{air}\vec{u}_{air} = P\vec{k}$ (in the opposite direction to the precipitation mass flux). Therefore the barycentric velocity becomes:

$$\vec{u} = \vec{u}_a - \frac{P}{\rho}\vec{k} + (1-\delta_m)\frac{P}{\rho}\vec{k} = \vec{u}_a - \delta_m\frac{P}{\rho}\vec{k} \tag{13}$$

Remark: To introduce the conservation of total mass into the barycentric velocity, we have modified the velocity of the dry air. The only purpose of the additional term $\frac{P}{\rho}\vec{k}$ is to compensate for the loss of mass, it does not advect the air properties. In the $\delta_m = 0$ case, velocity of the dry air is not equal to the velocity of the moist species (the non-precipitating - vapour, cloud water and ice). All the properties of the air mixture are advected with the barycentric velocity (kinetic energy, momentum, enthalpy), but the species k in the mixture is advected with the velocity $\vec{u}_k$. The Lagrangian derivative becomes

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \vec{u}_a \cdot \nabla - \delta_m \frac{P}{\rho}\frac{\partial}{\partial z} \tag{14}$$

the last part represents the vertical pseudo-advection because of the precipitation flux. We have the advection due to dynamics (forces acting on the mixture) with $\vec{u}_a \cdot \nabla$ and advection term due to physics $\delta_m \frac{P}{\rho}\frac{\partial}{\partial z}$ that may or may not have an influence, depending on δ_m.

The continuity equation becomes

$$\frac{\partial \rho}{\partial t} = -\vec{\nabla} \cdot (\rho \vec{u}) \tag{15}$$

$$\frac{\partial \rho}{\partial t} = -\vec{\nabla} \cdot (\rho \vec{u}_a) + \delta_m \frac{\partial P}{\partial z} \tag{16}$$

or

$$\frac{d\rho}{dt} = -\rho \vec{\nabla} \cdot \vec{u} \tag{17}$$

$$\frac{d\rho}{dt} = -\rho \vec{\nabla} \cdot \vec{u}_a + \delta_m \rho \frac{\partial}{\partial z}\frac{P}{\rho} \tag{18}$$

3.3 Conservation of species

3.3.1 Conservation of moist species

We start from the Arome conservation equations:

$$\frac{dq_k}{dt} = -\frac{1}{\rho}\vec{\nabla} \cdot (\rho q_k(\vec{u}_k - \vec{u})) + \frac{\dot{\rho}_k}{\rho} \tag{19}$$

where $q_k = \frac{\rho_k}{\rho}$. $k = 1$ for water vapour, $k = 2$ for the cloud water, $k = 3$ for rain, $k = 4$ for cloud ice and $k = 5$ for snow. Introducing the assumed $\vec{u}_k = \vec{u}_a$ for each k and the Lagrangian derivative formulation gives

$$\frac{\partial q_k}{\partial t} + \vec{u}_a \cdot \nabla q_k - \delta_m \frac{P}{\rho}\frac{\partial q_k}{\partial z} = -\frac{1}{\rho}\vec{\nabla} \cdot (\rho q_k \delta_m \frac{P}{\rho}\vec{k}) + \frac{\dot{\rho}_k}{\rho} \tag{20}$$

$$\frac{\partial q_k}{\partial t} + \vec{u}_a \cdot \nabla q_k - \delta_m \frac{P}{\rho}\frac{\partial q_k}{\partial z} = -\delta_m \frac{P}{\rho}\frac{\partial q_k}{\partial z} - \delta_m \frac{q_k}{\rho}\frac{\partial P}{\partial z} + \frac{\dot{\rho}_k}{\rho} \tag{21}$$

$$\frac{\partial q_k}{\partial t} + \vec{u}_a \cdot \nabla q_k = \frac{\dot{\rho}_k}{\rho} - \delta_m \frac{q_k}{\rho}\frac{\partial P}{\partial z} \tag{22}$$

The last term on the right side is the consequence of the total mass not being conserved. Because $q_k = \frac{\rho_k}{\rho}$, q_k changes if ρ changes.

3.3.2 Conservation of dry air

Since dry air is used to compensate for the loss of mass, the conservation of dry air is

$$\frac{\partial q_0}{\partial t} + \vec{u}_a \cdot \nabla q_{0k} = \frac{\dot{\rho}_0}{\rho} - \delta_m \frac{q_0}{\rho}\frac{\partial P}{\partial z} - (1-\delta_m)\frac{1}{\rho}\frac{\partial P}{\partial z} \tag{23}$$

In this equation, the last term ensures conservation of the total mass in the atmosphere, the middle term on the right-hand-side represents the change in q_0 due to the change of ρ. The first term represents a source or sink of the dry air. This term could be zero.
$\delta_m = 0$ The total mass of the atmosphere is conserved, the mass flux due to precipitation is compensated by the mass flux of the dry air.
$\delta_m = 1$ The mass of the dry air is conserved, so $\dot{\rho} = 0$. In Aladin, the mass exchange due to the evaporation from the ground is treated by the turbulence scheme that is mass conserving itself. But if the turbulence scheme is not mass-conserving (the part for the exchange with the surface) then this term might be different than zero.

3.4 Velocity equation

We start from the Arome equation for the evolution of the barycentric velocity

$$\frac{d\vec{u}}{dt} = -\frac{1}{\rho}\vec{\nabla}p + \frac{1}{\rho}\frac{\partial \sigma_{\alpha\beta}}{\partial x_\beta} + \rho\vec{g} \tag{24}$$

And introduce the formulation of the Lagrangian derivative

$$\frac{\partial \vec{u}}{\partial t} + \vec{u}_a \cdot \nabla \vec{u} - \delta_m \frac{P}{\rho}\frac{\partial \vec{u}}{\partial z} = -\frac{1}{\rho}\vec{\nabla}p + \frac{1}{\rho}\frac{\partial \sigma_{\alpha\beta}}{\partial x_\beta} + \vec{g} \tag{25}$$

$$\frac{\partial \vec{u}}{\partial t} + \vec{u}_a \cdot \nabla \vec{u} = -\frac{1}{\rho}\vec{\nabla}p + \frac{1}{\rho}\frac{\partial \sigma_{\alpha\beta}}{\partial x_\beta} + \vec{g} + \delta_m \frac{P}{\rho}\frac{\partial \vec{u}}{\partial z} \tag{26}$$

Remark: The barycentric velocity in Aladin is $\vec{u} = \vec{u}_a - \delta_m \frac{P}{\rho}\vec{k}$. Therefore the horisontal component $\vec{u}_h = \vec{u}_{ah}$ is the same. Vertical velocity is diagnosed from the continuity equation. $\delta_m = 1$ option is not yet coded for the nonhydrostatic variables.

3.5 Enthalpy equation

$$\frac{dh}{dt} = \frac{1}{\rho}\frac{dp}{dt} + \dot{Q}_i + \frac{1}{\rho}\sigma_{\alpha\beta}\frac{\partial u_\beta}{\partial x_\alpha} - \frac{\delta_m}{\rho}\sum_{k=0}^{N} P\frac{\partial h_k}{\partial z} - \frac{1}{\rho}\sum_{k=0}^{N} h_k\dot{\rho}_k \tag{27}$$

where $h = \sum_{k=0}^{N} h_k q_k = e_{int} + \frac{p}{\rho}$ is the enthalpy. $\dot{Q}_i$ represents diabatic cooling/heating term from physics parameterisations.

3.6 Temperature equation

We start from the Arome temperature equation

$$C_p\frac{dT}{dt} = \frac{1}{\rho}\frac{dp}{dt} + \dot{Q}_i + \frac{1}{\rho}\sigma_{\alpha\beta}\frac{\partial u_\beta}{\partial x_\alpha} - \frac{1}{\rho}\sum_{k=0}^{N} C_{pk}\rho_k(\vec{u}_k - \vec{u})\cdot\vec{\nabla}h_k - \frac{1}{\rho}(L_{21}(\dot{\rho}_2 + \dot{\rho}_3) + L_{41}(\dot{\rho}_4 + \dot{\rho}_5 + \dot{\rho}_6)) \tag{28}$$

where $h_k = CpkT$. $\rho_6 = 0$ in Aladin. $\dot{Q}_i$ represents diabatic cooling/heating term from physics parameterisations. Introduce

$$\frac{dT}{dt} = \frac{\partial T}{\partial t} + \vec{u}_a \cdot \nabla T - \delta_m \frac{P}{\rho}\frac{\partial T}{\partial z} \tag{29}$$

$$\frac{dp}{dt} = \frac{\partial p}{\partial t} + \vec{u}_a \cdot \nabla p - \delta_m \frac{P}{\rho}\frac{\partial p}{\partial z} \tag{30}$$

for the $\frac{d}{dt}$ terms, and for the different components in the summation term, we have

$$C_{pd}\rho_0(\vec{u}_0 - \vec{u})\cdot\nabla T = C_{pd}(\rho_0\delta_m\frac{P}{\rho}\frac{\partial T}{\partial z} + (1-\delta_m)P\frac{\partial T}{\partial z}) \tag{31}$$

for the dry air

$$C_{pk}\rho_k(\vec{u}_k - \vec{u})\cdot\nabla T = C_{pk}\rho_k\delta_m\frac{P}{\rho}\frac{\partial T}{\partial z} \tag{32}$$

for the non-precipitable species of water

$$C_{pk}\rho_k(\vec{u}_k - \vec{u})\cdot\nabla T = -C_{pk}P\frac{\partial T}{\partial z} \tag{33}$$

for the precipitation species.
Therefore, the temperature equation becomes

$$\frac{\partial T}{\partial t}+\vec{u}_a\cdot\nabla T = \frac{1}{\rho C_p}\frac{dp}{dt}+\frac{1}{C_p}\dot{Q}_i+\frac{1}{C_p\rho}\sigma_{\alpha\beta}\frac{\partial u_\beta}{\partial x_\alpha} \tag{34}$$

$$- \delta_m\frac{P}{C_p\rho^2}\frac{\partial p}{\partial z}-\frac{1}{C_p}((1-\delta_m)C_{pd}\frac{P}{\rho}\frac{\partial T}{\partial z}- C_{prec}\frac{P}{\rho}\frac{\partial T}{\partial z}) \tag{35}$$

$$- \frac{1}{\rho C_p}(L_{21}(\dot{\rho}_2+\dot{\rho}_3)+L_{41}(\dot{\rho}_4+\dot{\rho}_5)) \tag{36}$$

where C_{prec} is specific heat of precipitation (any species). $\frac{\dot{Q}_i}{C_p}$ term represents diabatic heating and cooling.
$\frac{1}{C_p}(C_{prec}\frac{P}{\rho}\frac{\partial T}{\partial z})$ term represents the heating of the faling precipitations.
$\delta_m\frac{P}{C_p\rho^2}\frac{\partial p}{\partial z}$ term represents the conversion of the potential energy of precipitations.
$\frac{1}{C_p}(1-\delta_m)C_{pd}\frac{P}{\rho}\frac{\partial T}{\partial z}$ represents the heat flux associated to the compensation flux of dry air.
$\frac{1}{\rho}\left[L_{21}(\dot{\rho}_2+\dot{\rho}_3)+L_{41}(\dot{\rho}_4+\dot{\rho}_5)\right]$ term represents evaporation, condensation, freezing and sublimation processes.
The last term can be rewritten to show the contributions from the different phisics processes as condensation, evaporation, freezing, sublimation, conversion between cloud water and rain and cloud ice and snow.
The second line shows the effect of different assumptions for the conservation law. The first term is the conversion of the potential energy of the precipitations. The second term is a term contributing to the temperature tendency due to the difference of specific heats of dry air and precipitaton in the case $\delta_m = 0$, or the temperature tendency due to the heating of the precipitations (since we have assumed that the precipitation has the same temperature as the surrounding air, the precipitation heat while falling through the warmer layers).

4 The pressure coordinate

If pressure is used as a vertical co-ordinate, the continuity equation becomes

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial \omega}{\partial p}=-\delta_m g\frac{\partial P}{\partial p} \tag{37}$$

with the bottom boundary condition $\omega=\delta_m(E-P)$. Velocity equation is

$$\frac{\partial \vec{u}}{\partial t}+\vec{u}_a\cdot\nabla\vec{u}= -\vec{\nabla}\phi-\frac{p}{\rho}\vec{\nabla}\ln p+\frac{1}{\rho}\frac{\partial\sigma_{\alpha\beta}}{\partial x_\beta}+\vec{g} -\delta_m gP\frac{\partial\vec{u}}{\partial p} \tag{38}$$

The conservation of mass for moist variables:

$$\frac{\partial q_k}{\partial t}+\vec{u}_a\cdot\nabla q_k=\frac{\dot{\rho}_k}{\rho}+\delta_m g q_k\frac{\partial P}{\partial p} \tag{39}$$

and the conservation of the dry air

$$\frac{\partial q_0}{\partial t}+\vec{u}_a\cdot\nabla q_{0k}=\frac{\dot{\rho}_0}{\rho}+\delta_m g q_0\frac{\partial P}{\partial p}+(1-\delta_m)g\frac{\partial P}{\partial p} \tag{40}$$

Enthalpy equation

$$\frac{dh}{dt}=RT\frac{\omega}{p}+\dot{Q}_i+\frac{1}{\rho}\sigma_{\alpha\beta}\frac{\partial u_\beta}{\partial x_\alpha}+\delta_m g\sum_{k=0}^{N}P\frac{\partial h_k}{\partial p}+g\sum_{k=0}^{N}h_k\dot{\rho}_k \tag{41}$$

where $h=\sum_{k=0}^{N}h_kq_k = e_{int}+\frac{p}{\rho}$ is the enthalpy. $\dot{Q}_i$ represents diabatic cooling/heating term from physics parameterisations. I expect that the term $\frac{1}{\rho}\sigma_{\alpha\beta}\frac{\partial u_\beta}{\partial x_\alpha}$ might have a different shape in p coordinate than in z, but this is a general expression for it that does not require rewriting.
The temperature equation is

$$\frac{dT}{dt}=\frac{RT}{C_p}\frac{\omega}{p}+\frac{\dot{Q}_i}{C_p}+\frac{\sigma_{\alpha\beta}}{\rho C_p}\frac{\partial u_\beta}{\partial x_\alpha}-\delta_m\frac{gP}{C_p}\frac{\partial\phi}{\partial p}+\frac{g}{C_p}\left[(1-\delta_m)\,C_{pd}P\frac{\partial T}{\partial p}-C_{prec}P\frac{\partial T}{\partial p}\right]-\frac{1}{\rho}\left[L_{21}(\dot{\rho}_2+\dot{\rho}_3)+L_{41}(\dot{\rho}_4+\dot{\rho}_5)\right] \tag{42}$$

$\frac{\dot{Q}_i}{C_p}$ term represents diabatic heating and cooling.
$\frac{g}{C_p} C_{prec} P \frac{\partial T}{\partial p}$ term represents the heating of the faling precipitations.
$\delta_m \frac{gP}{C_p} \frac{\partial \phi}{\partial p}$ term represents the conversion of the potential energy of precipitations.
$\frac{g}{C_p} (1 - \delta_m) C_{pd} P \frac{\partial T}{\partial p}$ represents the heat flux associated to the compensation flux of dry air.
$\frac{1}{\rho} [L_{21}(\dot{\rho}_2 + \dot{\rho}_3) + L_{41}(\dot{\rho}_4 + \dot{\rho}_5)]$ term represents evaporation, condensation and sublimation processes.
The last term can be rewritten to show the contributions from the different phisics processes as condensation, evaporation, freezing, sublimation, conversion between cloud water and rain and cloud ice and snow.

5 Conclusion

Except for the $\sigma_{\alpha\beta}$ term, these equations correspond to the currently used set of equations of Aladin model. Both options for the conservation law for mass may be kept.
This report concerns a very special case that the terminal velocity of precipitations is infinite and the mass of precipitations is zero. Please refer to the manuscript of Bart Catry for the case when the mass of the precipitation is not neglected and the terminal velocity has some finite value.

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