Kalman filter based bias removal for global horizontal irradiance forecast.

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Outlook.

- KF bias removal method description.
- Test of KF bias removal for global horizontal irradiation forecast.
- Validation of results.
- Problems.
- Conclusion.

$b = x_f - x_o$

Method description.

$x_{f,k}$: any model variable

$x_{o,k}$: observation of the same variable

$b_k = x_{f,k} - x_{o,k}$

$B = E(b)$: bias

1. assumption : $b_k = \xi_{0,k} + \xi_{1,k} p_{1,k} + \ldots + \xi_{n,k} p_{n,k} + v_k \; , \; v_k \sim N(0, V_k)$

p_i : predictors – any subset of model variables.

In matrix form:

$$b = \mathbf{H}\xi + v \qquad \mathbf{H} = [1 \;\; \mathbf{p}_1 \;\; \mathbf{p}_2 \ldots \mathbf{p}_n]$$

Corrected forecast: $x_{c,k} = x_{f,k} - \hat{b}_k$

$$\hat{b}_k = \mathbf{H}_k \hat{\xi}_{k,k-1}$$

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Method description.

Forecast of coefficients:

$$\xi_{k,k-1} = \mathbf{F}_k \xi_{k-1,k-1} + \mathbf{w}_k \,,\ \mathbf{w}_k \sim N(0, \mathbf{W}_k)$$

2. assumption: $\mathbf{F}_k = \mathbf{I}$

$$\hat{\xi}_{k,k-1} = \hat{\xi}_{k-1,k-1}$$

Kalman filter: $\xi_{k,k} = \xi_{k,k-1} + \mathbf{K}_{k,k-1}(b_{o,k} - \hat{b}_k)$

$$\mathbf{K}_k = \mathbf{P}_{k,k-1}\mathbf{H}_k^T\left(\mathbf{H}\mathbf{P}_{k,k-1}\mathbf{H}^T + V_k\right)^{-1}$$

$$\mathbf{P}_{k,k-1} = \mathbf{P}_{k-1,k-1} + \mathbf{W}_k$$

$$\mathbf{P}_{k,k} = (\mathbf{I} - \mathbf{K}_k\mathbf{H}_k)\mathbf{P}_{k,k-1}$$

Method description.

Calculation of $\mathbf{W}_{k+1}$ and V_k.

$$\mathbf{w}_k = \hat{\xi}_{k,k} - \hat{\xi}_{k-1,k-1}$$

$$v_k = b_{o,k} - \mathbf{H}_k \hat{\xi}_{k,k-1}$$

We use sample of last *M* values of $\mathbf{w}_i$ $\mathbf{v}_i$,

$$i = k, k\text{-}1, \ldots, k\text{-}M + 1$$

Initial values:

$$\mathbf{P}_{0,0} = \mathbf{W}_0 = \varepsilon\mathbf{I} \quad \text{and} \quad V_k = \varepsilon$$

$$\xi_{0,0} = 0$$

k goes over consecutive forecasts starting each day at same time.

$x_{f,k}$ is a forecast of variable x at the same forecast horizon for each k.

Test of KF bias removal for global horizontal irradiation forecast.

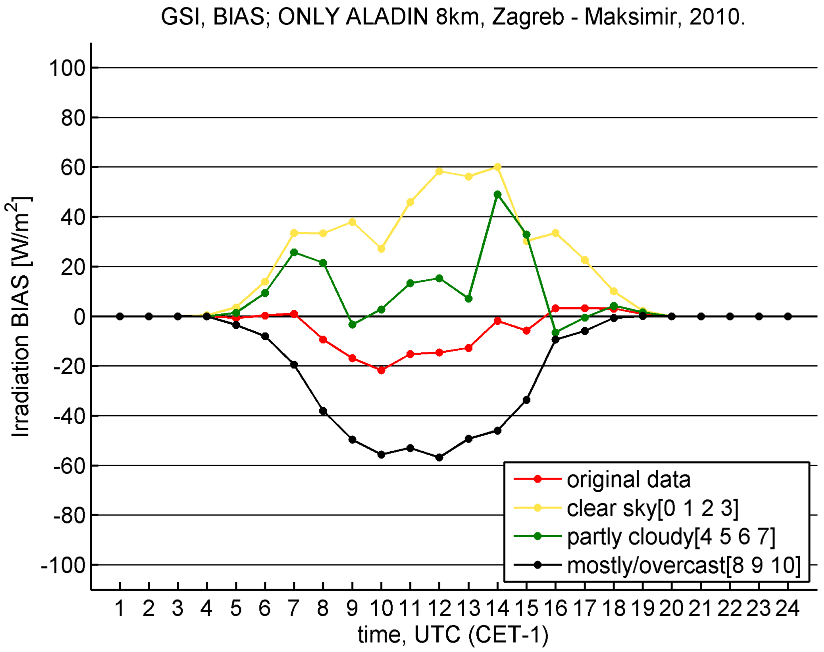
Model and data set.

Model: ALARO, CY32t1, 8 km, 37 levels

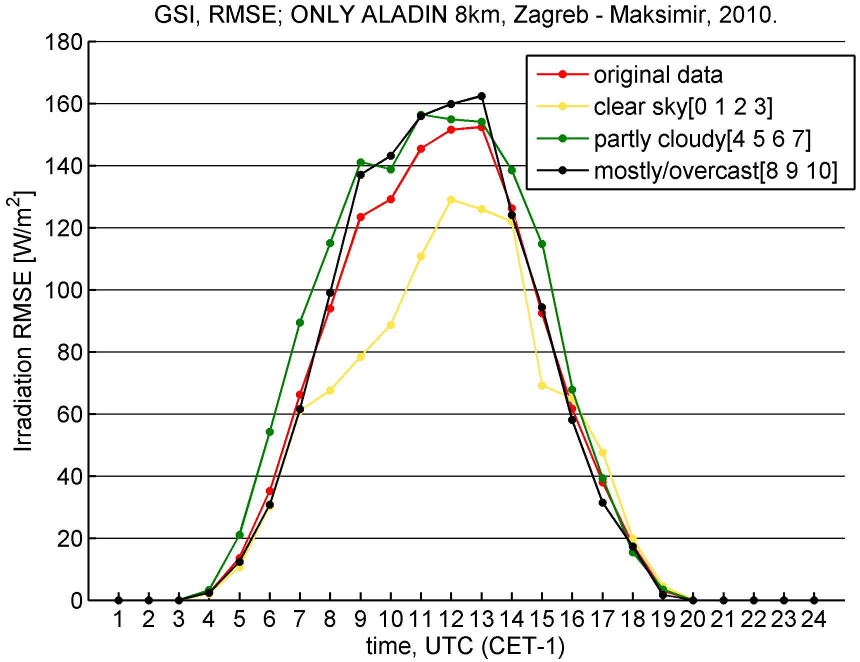
Data:
- 1 year of model foresacts, year 2010.
- 1 year of measurements of glogal horizontal irradiance (GHI) at station Zagreb_Maksimir.

GHI forecast verificatin.

BIAS



RMSE



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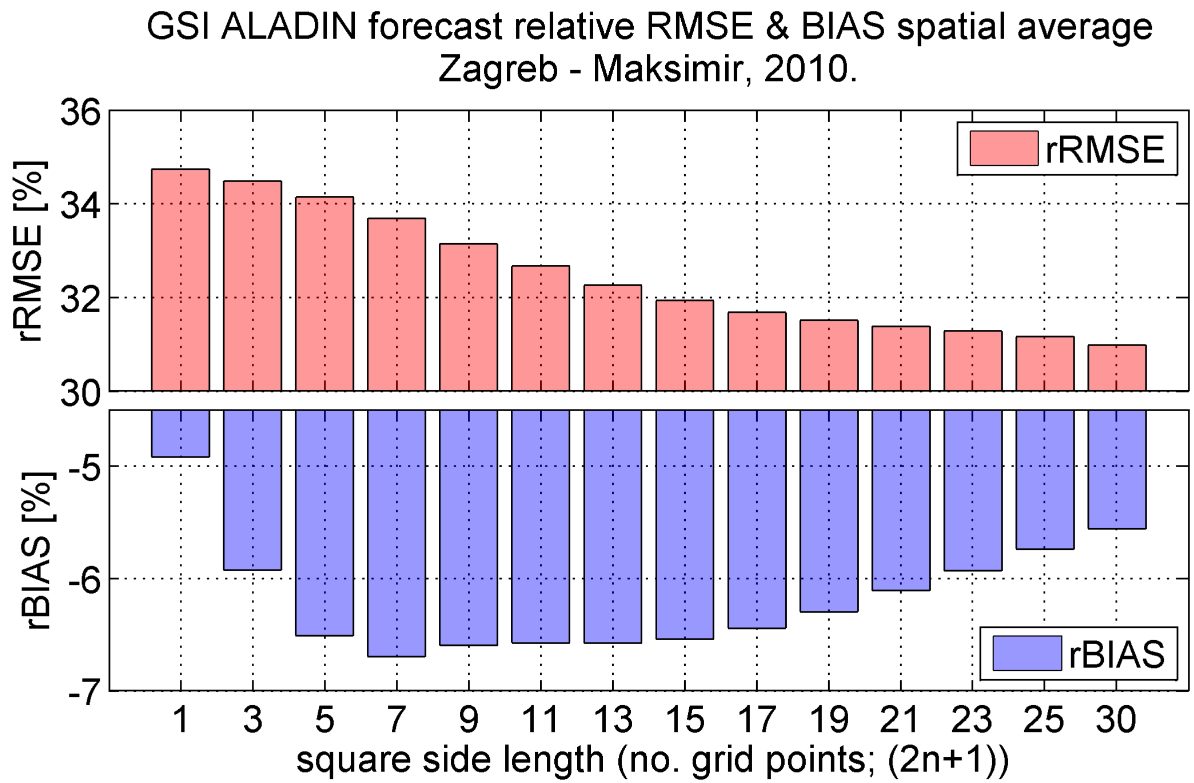
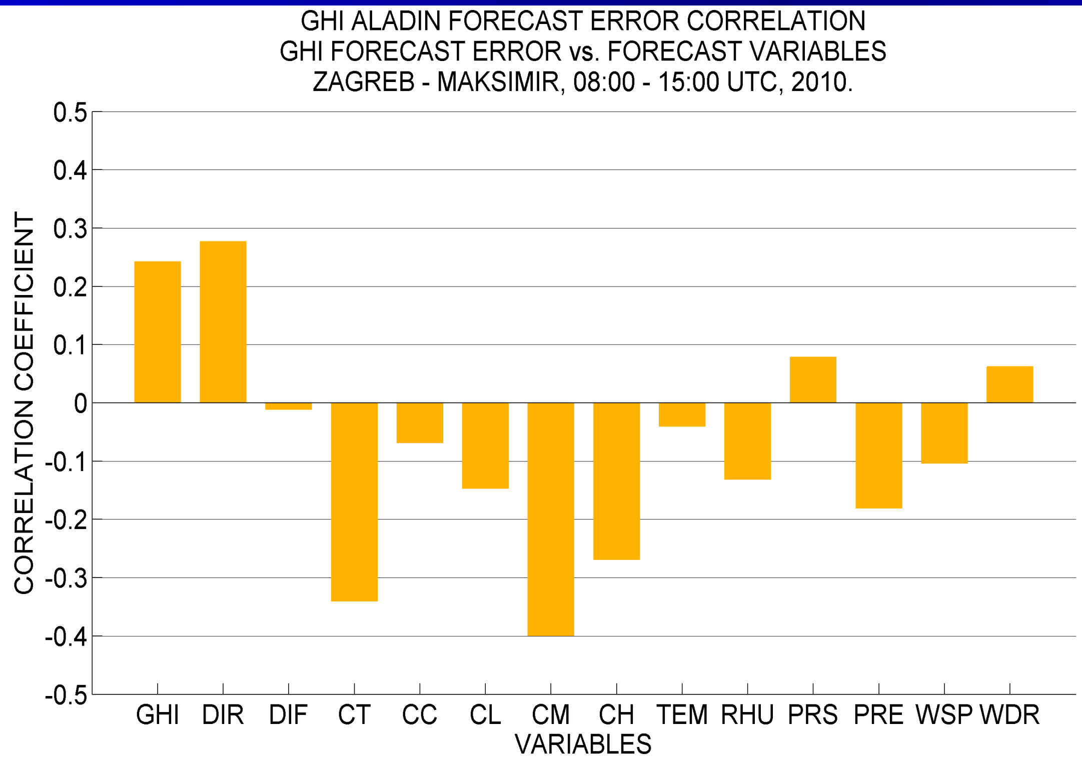
Spacial averaging.



Choosing predictors.

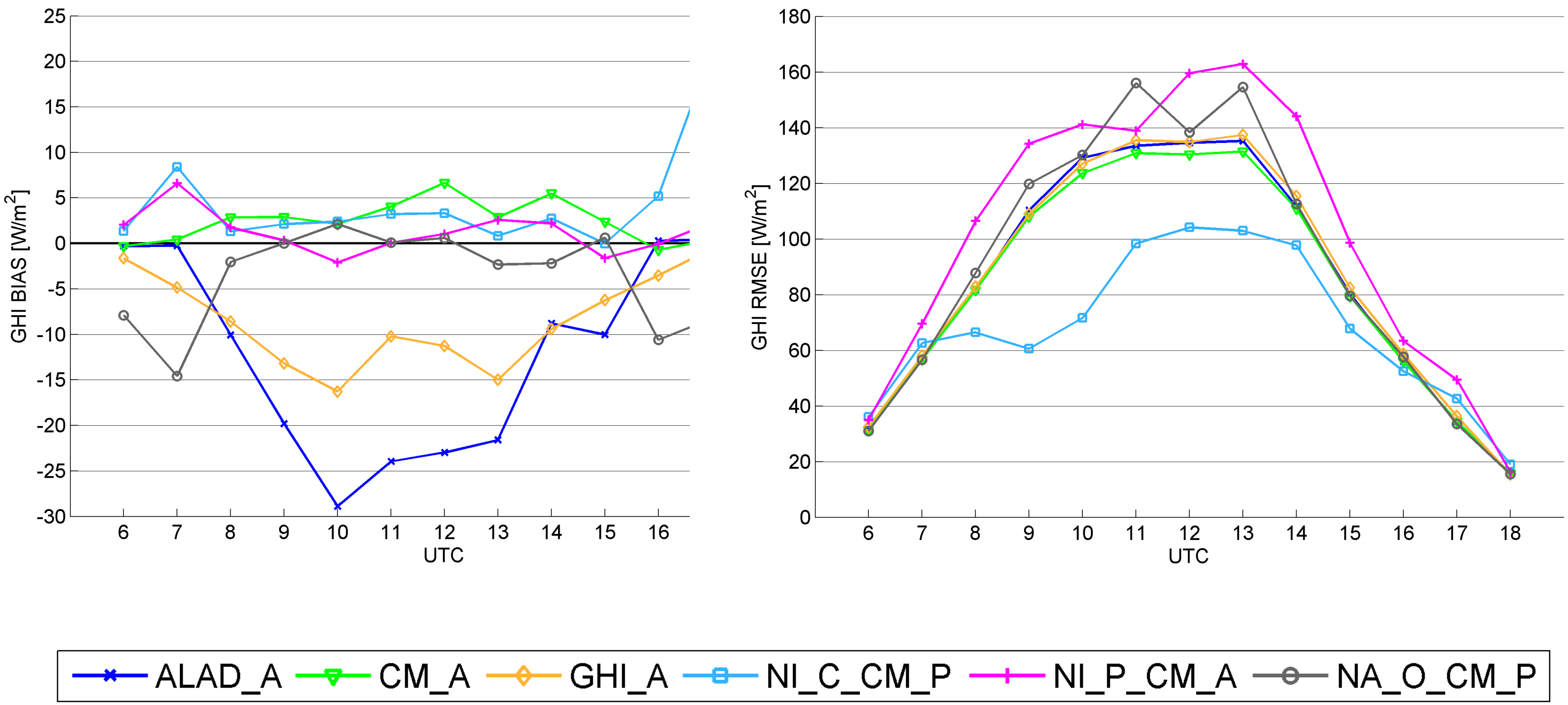


Predictors:
GHI
CM: mid level cloudiness

Results of KF bias removal method for GHI.

BIAS

RMSE



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Validation of results.

$$RMSE_f = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(x_{f,i}-x_{o,i}\right)^2} \qquad STDERR_f = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left[\left(x_{f,i}-x_{o,i}\right)-\left(\overline{x_{f,i}-x_{o,i}}\right)\right]^2}$$

$$RMSE_f^2 = STDERR_f^2 + BIAS_f^2$$

$$x_{c,i} = x_{f,i} - \hat{b}_i$$

$$RMSE_c^2 = STDERR_c^2 + BIAS_c^2$$

$$RMSE_c^2 - RMSE_f^2 = \left(STDERR_c^2 - STDERR_f^2\right) + \left(BIAS_c^2 - BIAS_f^2\right)$$

$$STDERR_c^2 - STDERR_f^2 = Var\left(\hat{b}\right) - 2Cov\left(b,\hat{b}\right)$$

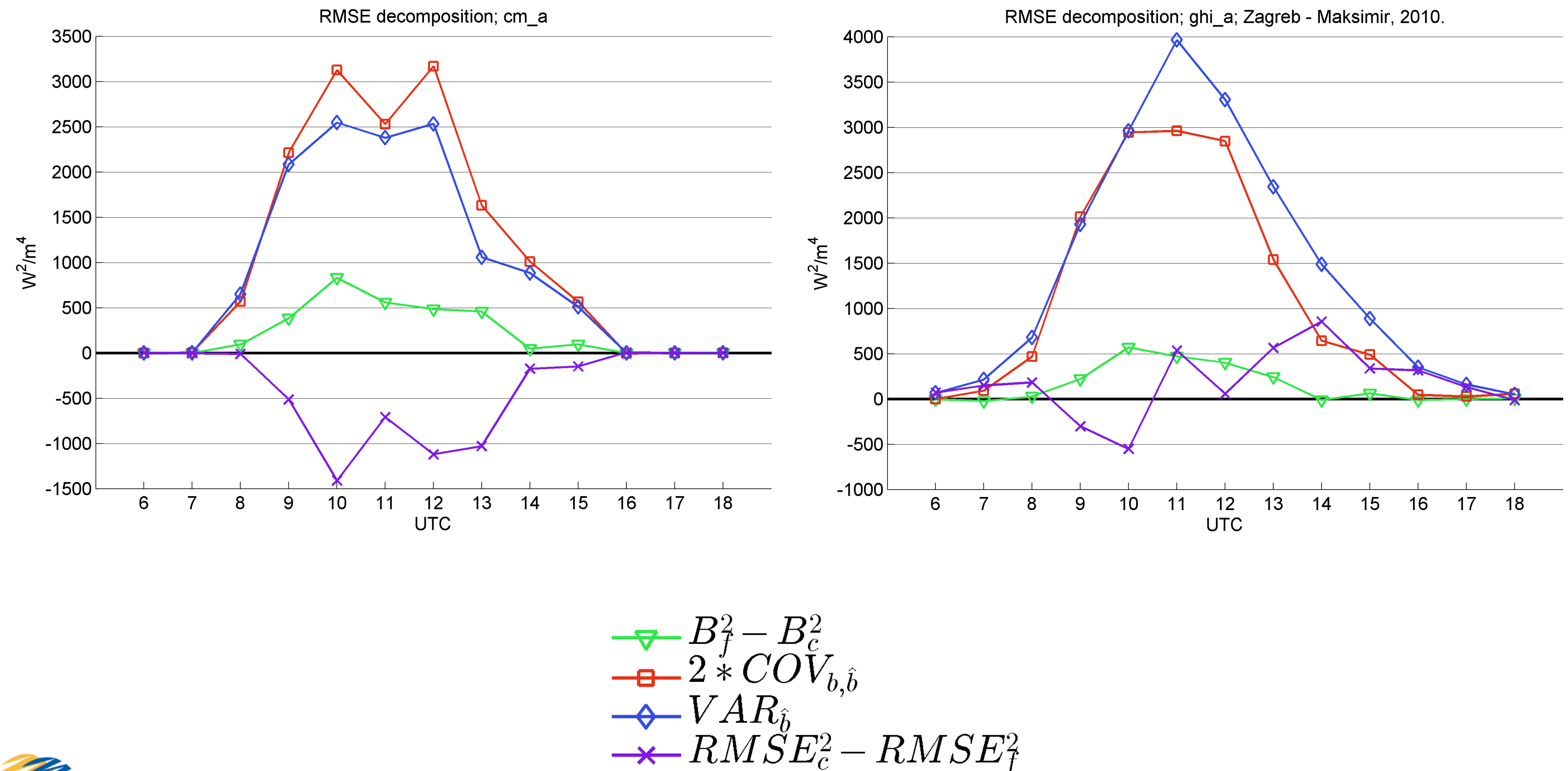
We want: $RMSE_c < RMSE_f$ and $BIAS_c < BIAS_f$,

but also: $STDERR_c < STDERR_f$,

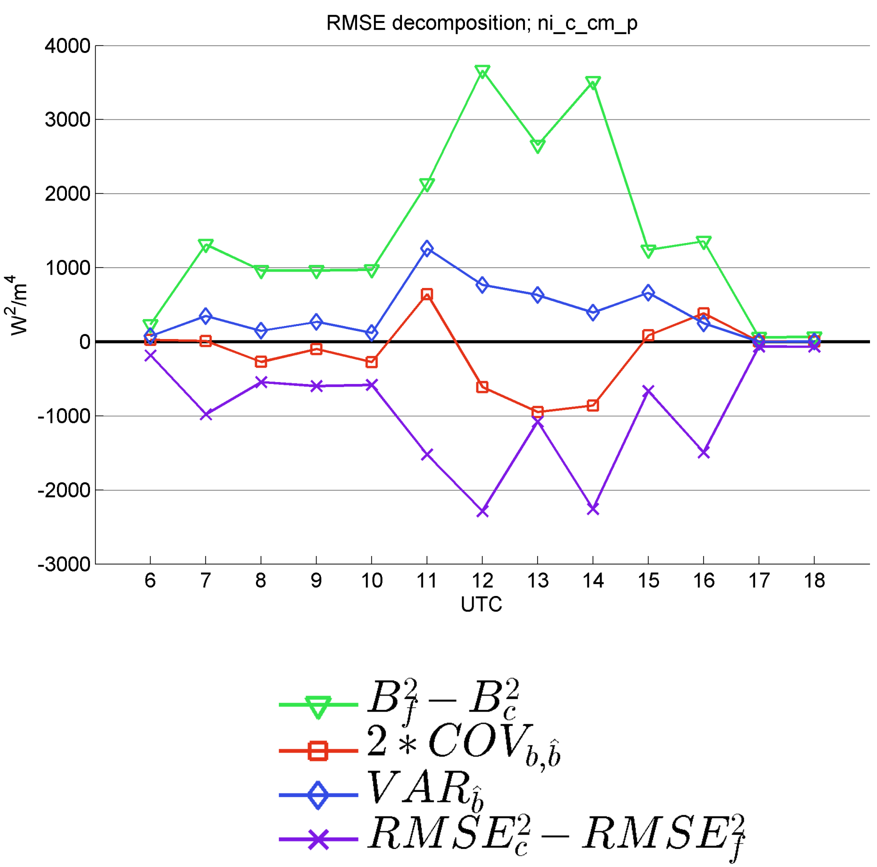
or

$B_c^2 - B_f^2 < 0$ and $Var\left(\hat{b}\right) - 2Cov\left(b,\hat{b}\right) < 0$

Validation of results.



Validation of results.



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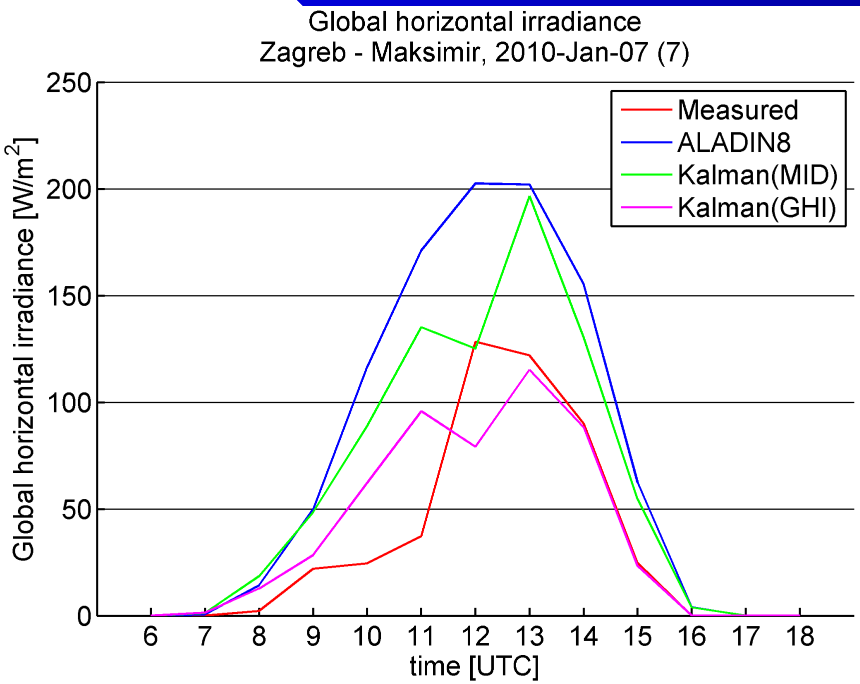
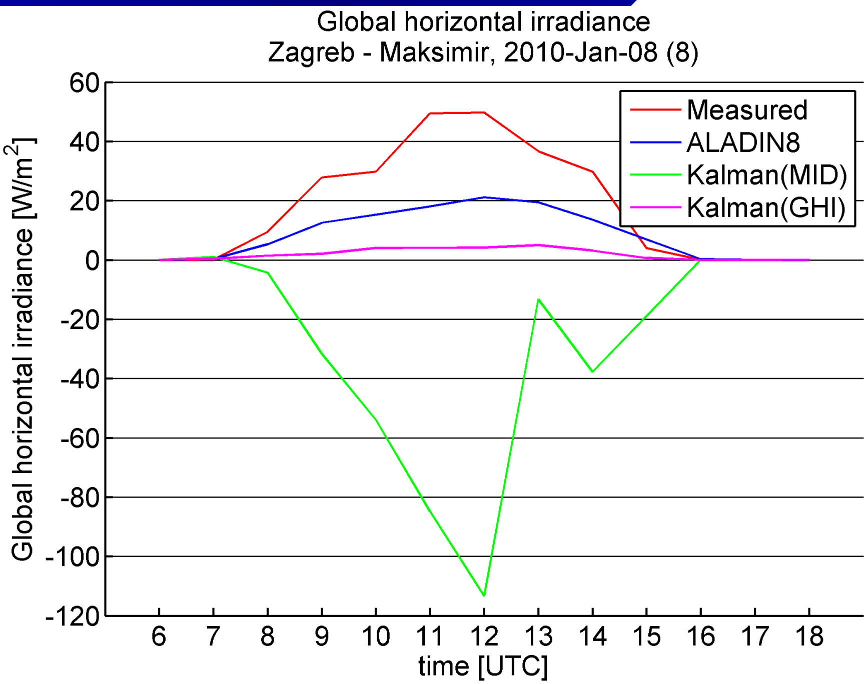
Problem.





$\alpha = \frac{Cov(b, \hat{b})}{Var(b)}$ must be closer to 1.

Analogy between KF bias removal and data assimilation

$$\hat{\mathbf{b}} = \mathbf{K}(\mathbf{y}_o - \mathbf{H}\mathbf{x}_b)$$

$$\mathbf{x}_a = \mathbf{x}_b + \mathbf{K}(\mathbf{y}_o - \mathbf{H}\mathbf{x}_b) \longrightarrow \boxed{\mathbf{x}_a = \mathbf{x}_b - \hat{\mathbf{b}} \longleftrightarrow x_{c,k} = x_{f,k} - \hat{b}_k}$$

$$\left.\begin{aligned} \mathbf{b} &= \mathbf{x}_b - \mathbf{y}_o \\ \mathbf{b}_a &= \mathbf{x}_a - \mathbf{y}_o \end{aligned}\right\} \longrightarrow \mathbf{b}_a = \mathbf{b} - \hat{\mathbf{b}} \quad \text{and} \quad \overline{\hat{\mathbf{b}}} = \overline{\mathbf{b}} - \overline{\mathbf{b}}_a$$

$$RMSE_f^2 = E\left[\left(\mathbf{x}_f - \mathbf{y}_o\right)^T\left(\mathbf{x}_f - \mathbf{y}_o\right)\right] = E\left(\mathbf{b}^T\hat{\mathbf{b}}\right)$$

$$RMSE_a^2 = E\left[\left(\mathbf{x}_a - \mathbf{y}_o\right)^T\left(\mathbf{x}_a - \mathbf{y}_o\right)\right] = E\left(\mathbf{b}_a^T\mathbf{b}_a\right)$$

$$RMSE_a^2 - RMSE_f^2 = E\left[\left(\hat{\mathbf{b}} - \overline{\hat{\mathbf{b}}}\right)^T\left(\hat{\mathbf{b}} - \overline{\hat{\mathbf{b}}}\right)\right] - 2E\left[\left(\mathbf{b} - \overline{\mathbf{b}}\right)^T\left(\hat{\mathbf{b}} - \overline{\hat{\mathbf{b}}}\right)\right] + \overline{\mathbf{b}}_a{}^T\overline{\mathbf{b}}_a - \overline{\mathbf{b}}^T\overline{\mathbf{b}}$$

$$\Updownarrow$$

$$RMSE_c^2 - RMSE_f^2 = Var\left(\hat{b}\right) - 2Cov\left(b, \hat{b}\right) + (B_c^2 - B_f^2)$$

Conclusion.

- Only one predictor was used for bias calculation.

- Two predictors were tested, GHI and CM.

- Bias is always decreased.

- RMSE is not decreased always.

- STDERR is nor decreased always too.

- Problem is that error is not forecasted well in all situations and persistence is not always fulfilled .

- Ratio $\alpha = \frac{Cov(b,\hat{b})}{Var(b)}$ must be high to have reliable bias removal.

Thank you for your attention!