A tuning of the humidity background error profile in ALD/HU assimilation system

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1. INTRODUCTION

Background error statistics matrix (**B** error matrix) is the key element in the variational assimilation system. Its structure defines the magnitude and the structure of the analysis increments, both in horizontal and vertical directions.
In recent years, ALD/HU 3D-var assimilation system used the standard NMC type of **B** error matrix for its operational setup. This type of background error statistics includes multivariate couplings between different variables, as defined in Berre (2000). However, in discussions on the study of vertical analysis increments of the single observation experiments done by Horvath and Bölöni (2003), it was noticed that the **B** error matrix shows a certain asymmetry in the multivariate propagation of the temperature and humidity analysis increments. In other words, qualitatively speaking, the influence of temperature on humidity analysis increments seemed to be greater then vice versa. The goal of this study is to investigate this asymmetry theoretically and experimentally in a series of single-observation experiments. Using this information, as a second step, an independent tuning of the humidity standard deviations in **B** error matrix will be performed using the Lönnberg-Hollingsworth type of statistics. In addition, the tuning is going to involve experiments with a modified observational error matrix definition. All these modifications are to be systematically proposed theorethically and then calucalted and verified in a series of single and full observation assimilation experiments.

2. THEORETICAL CONSIDERATIONS

Let us first consider a virtual single observation experiments with any variable innovation. According to the multivariate formulation of the background error statistics used in ALD/HU, this innovation will produce increments of vorticity, divergence temperature and humidity (Berre, 2000). In other words, taking into account temperature and humidity variables only, temperature innovation will induce both temperature and humidity analysis increments and humidity innovation will induce both humidity and temperature analysis increments. More mathematically, for a *q* single observation experiment with a grid point *q* innovation, the temperature analysis increment looks like:

$$dT = \frac{\mathrm{cov}(eb(T), eb(q))}{sb(q)^2 + so(q)^2} \Delta q \tag{1}$$

where *dT* is the temperature analysis increment, *Δq* is the specific humidity innovation, *cov(eb(T),eb(q)* is the cross-covariance of background errors of temperature and specific humidity and *sb(x)* and *so(x)* are the standard deviations of the background and observations errors of a variable *x*.
Equation (1) can be decomposed to a following set of equations:

$$dq = \frac{sb(q)^2}{sb(q)^2 + so(q)^2} * \Delta q \tag{2}$$

$$dT = \frac{\mathrm{cov}(eb(T), eb(q))}{sb(q)^2} dq \tag{3}$$

a transformation of equation (3) yields:

$$dT = cor(eb(T), eb(q)) * \frac{sb(T)}{sb(q)} * dq \tag{4}$$

Following the discussion in Hollingsworth (1987) and Daley (1991,p125) equation (2) can be seen as a filtering step of humidity innovation, which produces the increment *dq* (at the observation point), depending simply on the *so(q)/sb(q)* ratio (and innovation amplitude). Equation (4) can be seen as a multivariate propagation step in which the increment *dq* is converted into an increment *dT*, and the resulting amplitude of *dT* depends not only on the cross-correlation *cor(eb(T),eb(q))* and on the increment *dq*, but also on the standard-deviation ratio *sb(T)/sb(q)*.

Indeed, it is similarly possible to write the two increment equations in the case of a *T* single observation experiment:

$$dT = \frac{sb(T)^2}{sb(T)^2 + so(T)^2} * \Delta T \tag{5}$$

$$dq = cor(eb(q), eb(T)) * \frac{sb(q)}{sb(T)} * \Delta T \tag{6}$$

With these simple single-obs equations, it is even possible to be more precise on the discussion about the specified statistics. Namely, it can be derived that *S(A/B)*, the ratio between the multivariate propagation factors that are involved in equations (4) and (6), is equal to the ratio between the humidity and temperature background error variances:

$$S(A/B) = \frac{\left(\frac{dq}{dT}\right)_{Tinn}}{\left(\frac{dT}{dq}\right)_{Qinn}} = \left(\frac{sb(q)}{sb(T)}\right)^2 \tag{7}$$

Similarly, it can be shown that *P(A*B)*, the product between the two output/input increment ratios of the equations (4) and (6), is equal to the squared cross-correlation between *eb(q)* and *eb(T)*:

$$P(A*B) = \left(\frac{dq}{dT}\right)_{Tinn} * \left(\frac{dT}{dq}\right)_{Qinn} = cor(eb(q), eb(T))^2 \tag{8}$$

3. EXPERIMENTAL EVIDENCES

In the recent years a number of experiments was performed in the so-called standard NMC ALD/HU data assimilation system, that showed indications that there is an asymmetric propagation between temperature and humidity variables analysis increments. First, some assimilation experiments showed better results with the univariate, then multivariate treatment of humidity (Fig. 1), indicating the the multivariate humidity coupling is not tuned at best in the system.

Evidence about the asymmetric propagation came after a series of a single observation experiments (Horvath and Bölöni, 2003; Horvath et al., 2004). These type of simple single observation experiments provided a platform for application of the theoretical equations derived above. First, influence of 2m temperature innovation to the relative humidity vertical structure shows the intensive (seemingly unrealistic) propagation of humidity increments to the upper levels (Fig. 2). This indications of the unrealistic humidity tuning are analysed quantitatively and in more detail by investigating temperature and humidity analysis increments from the T and q innovation experiments at 500hPa (Fig. 3a-d). Using single-observation equations derived earlier and applying numbers to these experiments calling A the T single-obs experiment, and B the q single-obs experiment, yields that experimental value of Eq. 7 at 500 hPa is:

$$\sqrt{S(A/B)} = \frac{sb(RH)}{sb(T)} = 60.57\frac{\%}{K} \tag{9}$$

Thus, if $sb(T)$=1K, then $sb(RH)$=60%. This value seems to be too high, because it indicates that if an average model error in temperature is 1K, the associated error in relative humidity is 60%.

For instance, on Figure 1. the RH innovation root mean square error values at 250 hPa are around 25%. If we consider the example where sb(q)~so(q), this suggests that *sb(q)*=25%/*sqrt*(2) ~ 18%. This suggests that sb(q) may be overestimated by a factor 2 or 3.

Experimental value of *P(A*B)* derived from single observation experiments shown on Fig. 2 is:

$$\sqrt{P(A*B)} = cor(eb(T), eb(RH)) = 0.0537 \qquad (10)$$

Indeed, this cross-correlation value looks very small. If we notice that $cor^2(eb(T),eb(RH))$ can be seen as the percentage of the temperature variance explained by humidity variance, the value of $0.0537^2=0.0029$ looks considerably too small.

Overestimation of the *S(A/B)* ratio in Eq. 7 and Eq. 9 implies that in our assimilation experiments influence of temperature on humidity is stronger then influence of humidity on temperature. In other words, the influence seem to be “asymmetric” and if we consider *sb(T)* well chosen, this overestimation comes from the overestimation of *sb(q).*
Secondly, we can consider that *so(T)* and *cor(eb(T),eb(q))* are well estimated, or concerning the latter one, that it is at least not too big (as shown by the experimental value of the Eq. 9). Thus, in regard to theoretical single observation equations, from Eq. 6 it can be deduced that in order for overestimation of *q* increments in *T* innovation experiment to take place *sb(q)* has to be overestimated.
Finally, from Eq. 4 we can see that in order for underestimation of *T* increments in *q* innovation experiment to take place, either *dq* increment must be too small or *sb(q)* must be too big. The latter is consistent with the former conclusions, but the first possibility deserves attention as well. Namely, in order for *dq* to be too small, either *sb(q)* must be too small or *so(q)* must be too big. Since the first is less consistent with the conclusions derived from the value of *S(A/B)* ratio and *T* innovation

experiment, we will keep in mind the possibility of the overestimation of *so(q)*, in addition to the overestimation of *sb(q)*. Indeed, from Eq.2 it can be seen that reduction of the *so(q)* with the same factor like *sb(q)* keeps the filtering step the same, and focuses the changes in the system only on the multivatiate propagation steps. This line of approach is going to be taken in this study.

In conclusion, former experiments seem to imply that influence of *T* innovation on *q* increments is greater then influence of *q* innovation on *T* increments. Thus, there is an asymmetry in mutual influence that seems to exist due to overestimation of *sb(q)*. Moreover, in order to change only the propagation steps (Eq. 4, Eq. 6), experiments will be performed with a modified *so(q)* as well, keeping the *sb(q)/so(q)* ratio and the filtering step in *q* innovation experiment unchanged.

4. METHOD OF TUNING *sb(q)*

4.1 Theory

Beside model estimates of the background error variances (NMC, ensemble, EKF, etc.), there exists an independent approach to the approximation of the forecast error covariances, so-called Lönnberg-Hollingsworth (LH) method (Hollingsworth and Lönnberg, 1986). These authors analysed the statistical structure of the forecast errors, by verifying the forecasts against the radiosonde data, the principle adopted in our study. In short, the procedure and assumptions are (refer to Hollingsworth and Lönnberg, 1986 and Horvath et al. 2004 for more information):

1. evaluate in observation space statistics of the innovation vector *y-H(xb)* assuming that:
    - i) errors are unbiased
    - j) observation and background errors are uncorrelated
2. make additional assumptions out of which the most prominent is that observation errors are not spatially correlated

3. sort in distance bins
4. do the function fitting to separate the observation and background error covariances at zero distance (observation location)

As indicated in 4., beside estimates of the background error covariances in physical space, as a by-product, this method provides estimates of the observation error covariance for the observation type analysed.

4.2 Experimental design

The period of calculation was summer, May 02 – Sept 09 2004, using the net of 74 TEMP stations in an approximate area of roughly 800 km radius, centred in Hungary. A typical specific humidity horizontal covariance function is shown on Fig. 4. Bins were chosen to be equidistant and 200 km wide, starting at 100 km distance from the observation location. Points *(x=200km, sb(q(x=200km))* and *(x=400km, sb(q(x=400km))* were used for linear function fitting, using line equation. The intersection of this line and the ordinate was a separator of the $sb^2(q)$ and $so^2(q)$ that sum to the value of LH statistics at the origin point. The aim was to calculate *sb(q)* at all model levels in order to incorporate this data to the assimilation system. Two approaches were tested:

i) an interpolation of first guess departs to model levels followed by covariance calculation
j) a covariance calculation at standard TEMP levels followed by interpolation to model levels

These two approaches provided similar results what is shown on Figure 5. Vertical profile shown on the figure is a *sb(q)_LH/sb(q)_NMC* ratio and indicates that LH statistics gives smaller values of *sb(q)* in the whole troposphere, and higher values above the tropopause level. This profile will be used in tuning of the *sb(q)* in the subsequent experiments.

5. RESULTS

Thus, having the result of the LH statistics, modification of *sb(q)* profile in the assimilation system included the following ingredients:

1. no modifications done below the 820 hPa level
2. middle and upper troposphere profile (up to approx. 250 hPa level) was modified in accordance with LH statistics results
3. in some experiments, profile was heavily reduced around and above tropopause (by a factor of 0.005) in accordance with the assimilation setup in ECMWF (Anderson et al., 1998), to reduce the propagation of humidity increments to the lower stratosphere

In general, experiments will also include the modification of the *so(q)* profile scaled with the same scaling (i.e. reduced by the same factor) as *sb(q)* in order to keep the humidity univariate (filtering) step the same. The (non)existence of those modifiactions will be indicated in the each experiment description

5.1 Single observation experiments

First, single observation experiments were performed in order to test the new *sb(q)* and *so(q)* profiles and verify the expected asymmetry reduction. Results are summarised in Table 1.

EXP	1	2	3	4
sb(q)	NMC	NMC	LH	LH
so(q)	orig	orig*0.5	orig	orig*0.5
S(A/B)^0.5 [%/K]	60.57	60.1	35.54	33.9
P(A*B)^0.5	0.0537	0.0542	0.085	0.0893

Table 1. Asymmetry ratios and cross-correlation coefficients in 4 single observation experiments.

Experiment 1 shows values for the control experiment i.e. simulation with original *sb(q)* and *so(q)* values. Setup of

experiment 2 was designed to increase the filtering step in humidity innovation experiment and thus, increase the humidity to temperature propagation step (this also corresponds to a possibility discussed in the Ch. 3 that the *sb(q)* is well chosen and *so(q)* is overestimated, what was shown very probably not to be the case). The asymmetry ratio was weakly reduced and showed the need to reduce *sb(q)*. This has been done in experiment 3 using the values provided by the LH statistics (roughly half of the value in control experiment), which showed the strong reduction of the asymmetry ratio and a bigger cross-correlation, more in line with expected and desired values. The final choice of *sb(q)* and *so(q)* values were tested in experiment 4, with reduced *so(q)* in order to keep the filtering step in *q* innovation experiment similar to the one in the control experiment.
Thus, these experiments yielded results in accordance to theoretical predictions in former chapters.

5.2 Full observation experiments

Since the LH tuning of the *sb(q)* showed the desired reduction of the asymmetry ratio in single observation experiments, the calculated LH statistics *sb(q)* vertical profile was used to tune the NMC statistics *sb(q)* vertical profile in the full observation assimilation setup.
The full observation assimilation system used for the experiments was made to resemble the operational setup as close as possible. Observation vector included SYNOP, TEMP, AMDAR and ATOVS data. Observations of the 2004 were examined in order to isolate very dry and very wet 2 week periods and the choice was the following:
1. wet: May 31 – Jun 13
2. dry: Aug 29 – Sept 11

Up to now, experiments were performed only on the dry period. Three experiments were designed and compared with the control run:
A. EXP1

i) only *sb(q)* profile was modified including ingredients 1., 2. and 3. (see the beginning of Chapter 5)

B. EXP2
   i) both *sb(q)* and *so(q)* profiles were modified including ingredients 1., 2. and 3. (see the beginning of Chapter 5)

C. EXP3
   i) both *sb(q)* and *so(q)* profiles were modified including ingredient 1. and 2. only (see the beginning of Chapter 5)

In a standard assimilation experiment procedure, first assimilation cycle was ran followed by the 48 hours forecasts. The verification package was used to calculate the verification scores i.e. biases and root mean square errors (RMSE).
Since the forecast score differences diminish with forecast range, we will constrain discussion to the +06 forecast only. Also, the greatest differences in scores exist for humidity and temperature, which will therefore be the focus of our analysis.

Figure 6.a-f shows full-observation experiments EXP1 and EXP2 for the dry 2 weeks period, compared to the reference run of the quasi-oparational ALD/HU setup through BIAS and RMSE (root mean square error). Since the results of EXP1 and EXP2 are quite similar, we will refer to them together, except where specially emphasised. Starting from the stratosphere, there are no significant differences in scores at 100 hPa level (not shown). In contrast, near tropopause levels (250 hPa) the changes already do exist. The most notable effect of the tuning is a greater hunidity BIAS and RMSE, while tuning of *sq(o)* does not seem to have an effect on this feature at all. This result implies that ECMWF type of tuning (severe reduction of *sb(q)*) does not show a positive result at high levels in ALD/HU. However, at these levels there is very few humidity in the atmosphere which does not play a strong role in the dynamics around tropopause levels. Thus, for a possibly good reason (e.g. in troposphere) this tuning might be kept as it is, without

strong drawbacks. Changes in temperature BIAS and RMSE at 250 hPa levels are small.
At 500 hPa level (Fig. 6.c-d), the effect of tuning on the humidity scores seems to induce a consistent improvement in scores, both in BIAS and RMSE. The effect on temperature is variable, but it seems to be in general neutral on both BIAS and RMSE. Although not of a primary interest, it is interesting to note that a tuning of humidity can have a strong influence on wind, where an individual difference in direction increment reached more then 10°.
At 700 hPa, the effect of tuning on temperature scores is slightly positive in BIAS and neutral in RMSE. Consistent with the results on 500 hPa level, the effect on humidity scores is positive both in BIAS and RMSE.
The overall results seem to imply that the difference between expoeriments EXP1 and EXP2 is very small. However, compared to the reference experiment, tuning has a notable effect. The worst and most promonent ingredient of the tuning is the worsening of the humidity scores at high levels, around tropopause. The effect of tuning at the middle tropospheric levels is overall positive regarding humidity and neutral regarding the temperature forecast scores.

For the reason of the bad results around and above the tropopause levels, EXP3 was performed. This experiment included no modification of *sb(q)* at levels above tropopause. Experiment results are shown on Fig 7.a-e. The most notable effect is a lack of a bad humidity BIAS and RMSE result at high levels (Fig. 7.a-b). Indeed, humidity RMSE scores seem to be very slightly improved. With this experiment setup, similarily as in EXP1 and EXP2 temperature scores are not strongly affected and equal to the reference experiment.
At 500 hPa the overall influence of this tuning to humidity and temperature BIAS seems to be rather neutral. Comparison of EXP2 and EXP3 (not shown) shows that while neutral temperature BIAS stayed similar in both experiments, humidity BIAS is in EXP3 slighly worse then in

EXP2 experiment, thus positive impact of tuning on humidity BIAS scores is mitigated in the EXP3. As regards to the RMSE of temperature and humidity they stayed roughly the same like in EXP3 i.e. temperature neutral and humidity slighly better then in the reference experiment.
At 700 hPa a positive impact of the tuning on temperature and humidity BIAS, as well as on humidity RMSE is preserved and differences between EXP3 and EXP2 are small. Temperature BIAS stayed rather neutral, as it was in EXP2.

Conclusions

A tuning of the *sb(q)* and *so(q)* in the ALD/HU background error matrix was performed by calculating humidity standard deviations by Lönnberg-Hollingsworth method and scaling the NMC humidity standard deviations. The resulting modification (decrease in *sb(q)*) was theorethically justified in advance and explored by the single observation experiments and the associated assymetry ratios.
For the verification of the new tuning, first single observation experiments were done and the improvement in the assymetry ratio was confirmed. At this point, an additional constraint on the final *sb(q)* profile was done by strong reduction (by a factor 0.005) above tropopause, inspired by the results at ECMWF.
Full observation experiments were done with a quasi-operational ALD/HU assimilation system for a 2 weeks long assimilation period. The reduction of *sb(q)* above trpopopause had a distinguished negative effect on humidity scores at high altitudes and is not useful in ALD/HU system. Reduction of *so(q)* and maintenance of the filtering step amplitude showed not to have a significant impact on the results.
However, reduction of standard deviation humidity profile in the middle atmophere showed an overall positive impact

on both humidity BIAS and RMSE at those levels. Overall effect on temperature scores is mostly neutral.
At this point, the results of tuning done in EXP3 (without a modification of the standard deviation humidity profile above tropopause) show a potential for further inverstigation, with an eye on potentional operational implementation. Before that and in addition, it is recommended to run more experiments, specially for the moist and wet periods.

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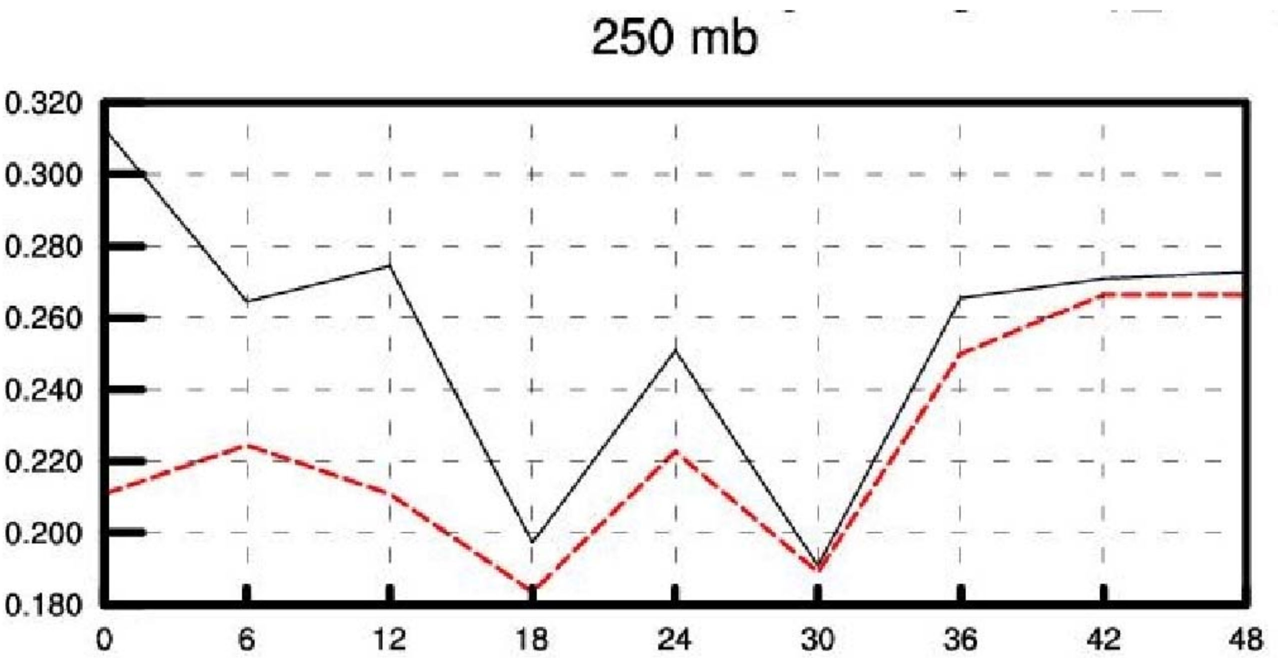

Fig.1: Relative humidity RMSE scores at 250 hPa level for univariate (red line) and multivariate (black line) treatment of humidity in background error matrix statistics (courtesy of Roger R.).

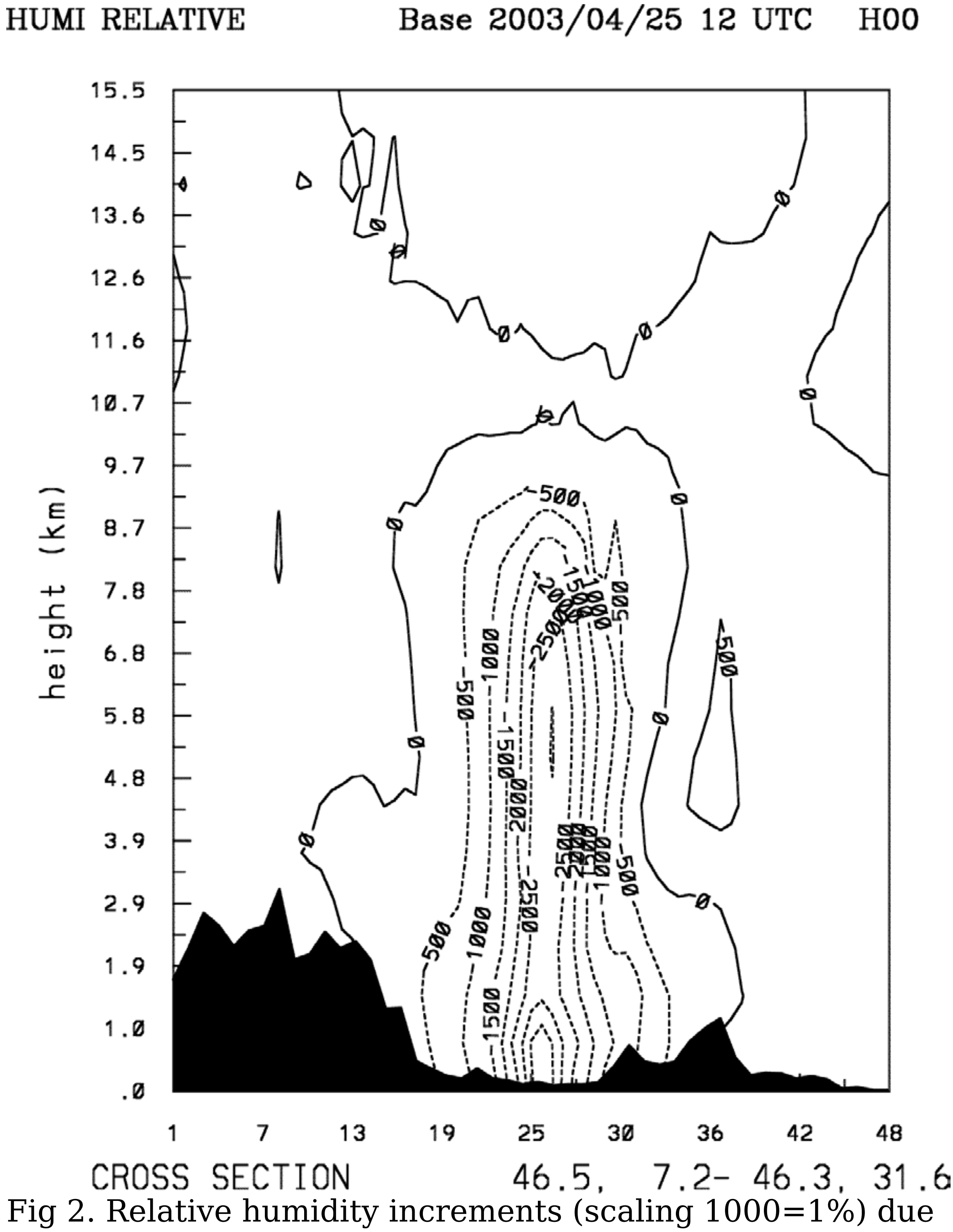

Fig 2. Relative humidity increments (scaling 1000=1%) due to 1K 2m temperature innovation.

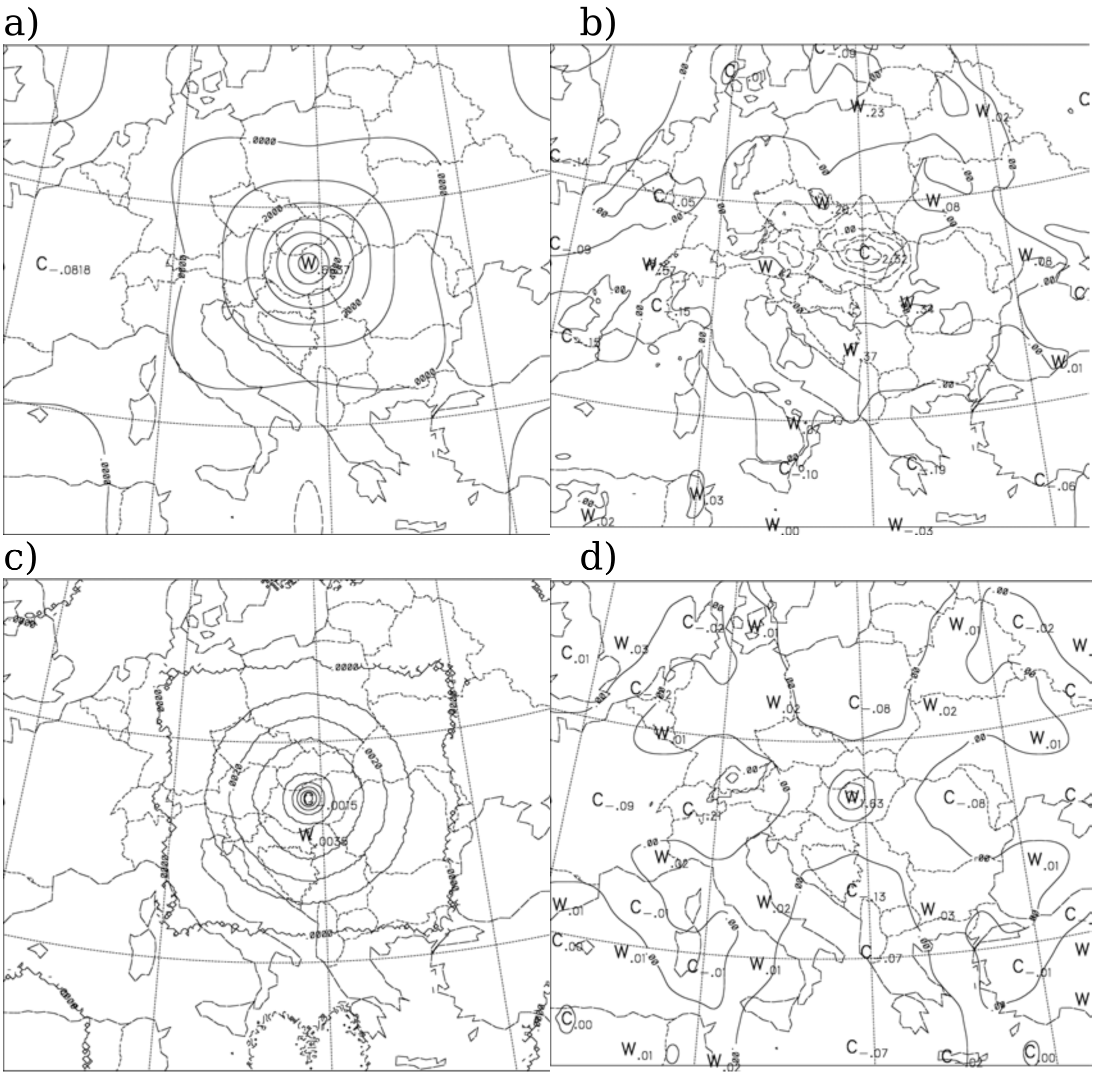

Fig 3. Single observation T (a) and RH (b) increments due to T innovation and and T (c) and RH (d) increments due to Q innovation. Innovations and increments are at approx. 500 hPa level. Experimental setup used standard ALD/HU operational NMC statistics.

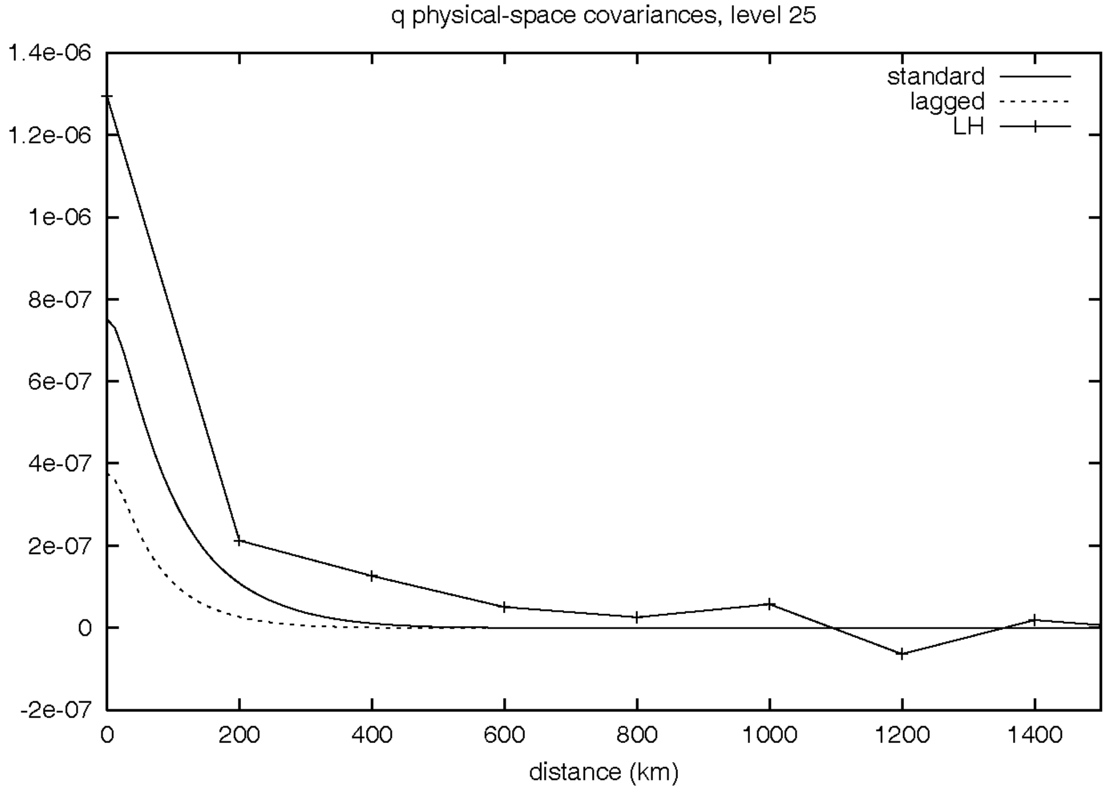

Fig 4. A comparison of a typical specific humidity horizontal covariance functions of so-called standard and lagged NMC and LH statistics in physical space. Value at zero distance is a sum of background error and observation variances.

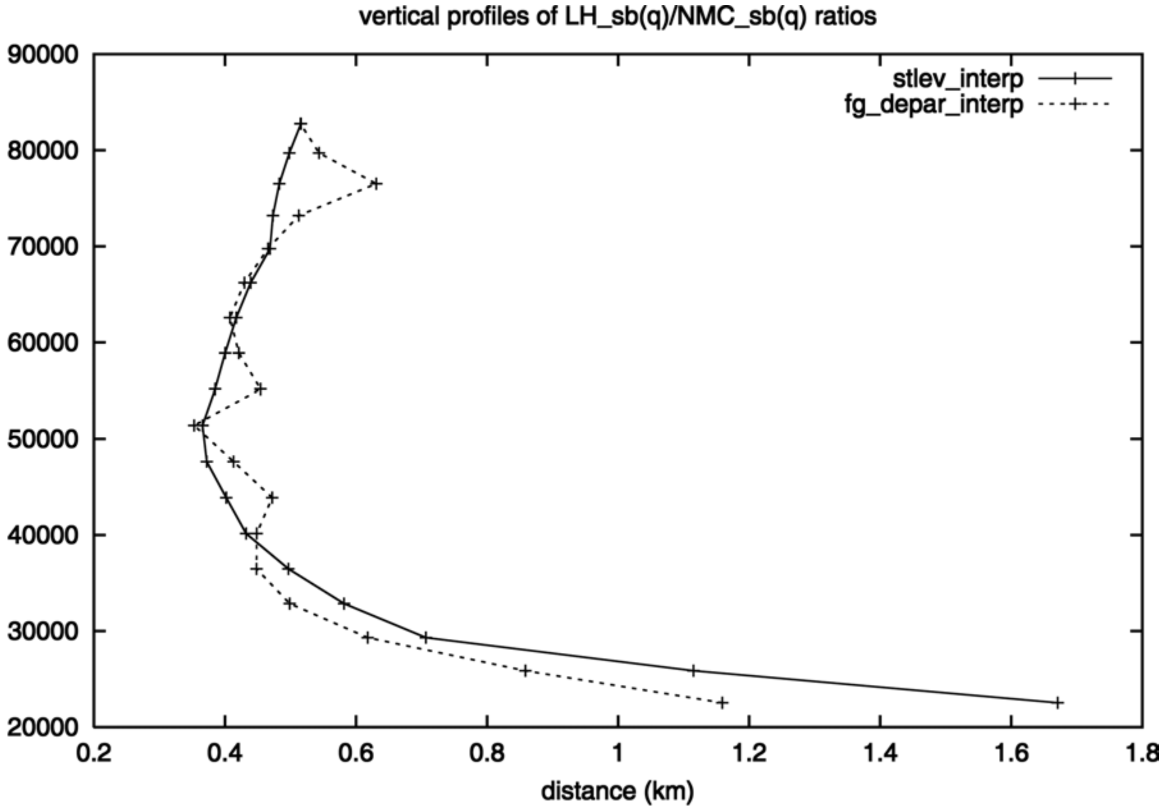

Fig. 5: Vertical profile of a *sb(q)_LH/sb(q)_NMC* ratio used for tuning the *sb(q)* values in the ALD/HU.

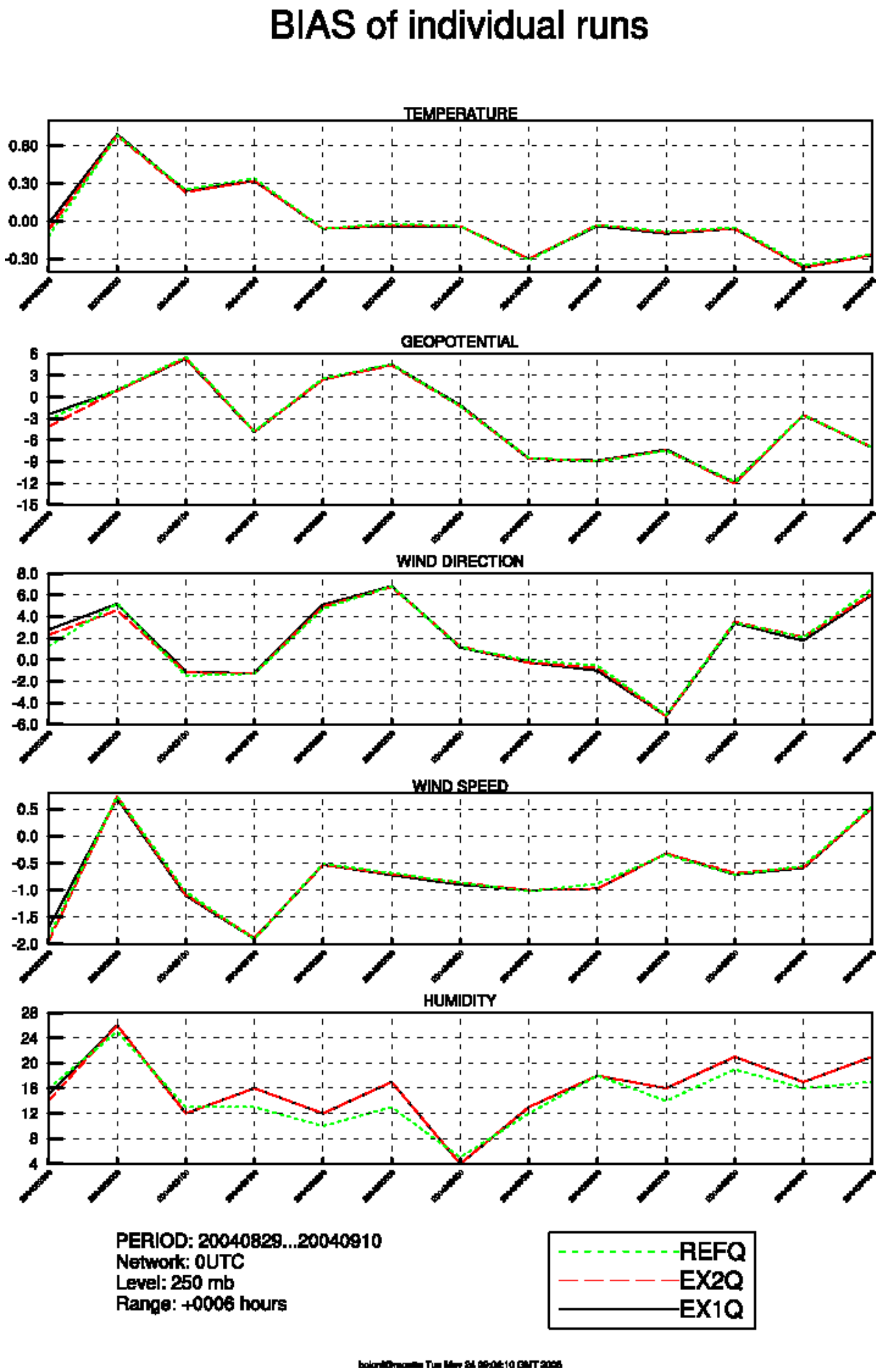

Fig 6.a: BIAS of the individual runs at 250 hPa. REFQ is the reference experiment (ALD/HU oparational setup), EXP1 experiment with only *sb(q)* and EXP2 experiment with both *sb(q)* and *so(q)* tuned.

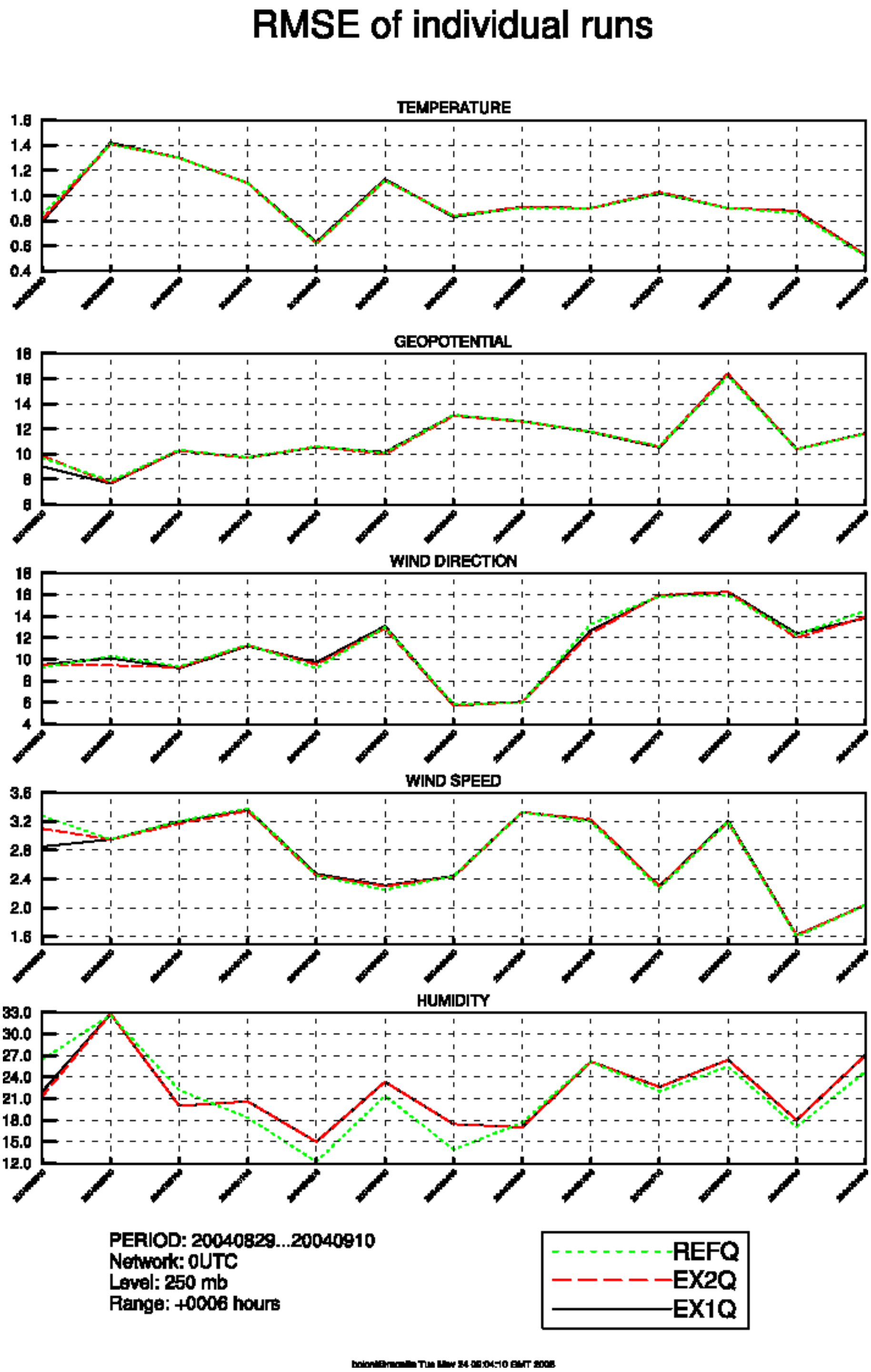

Fig 6.b: RMSE of the individual runs at 250 hPa. Experiment denotations are like in Fig 6.a.

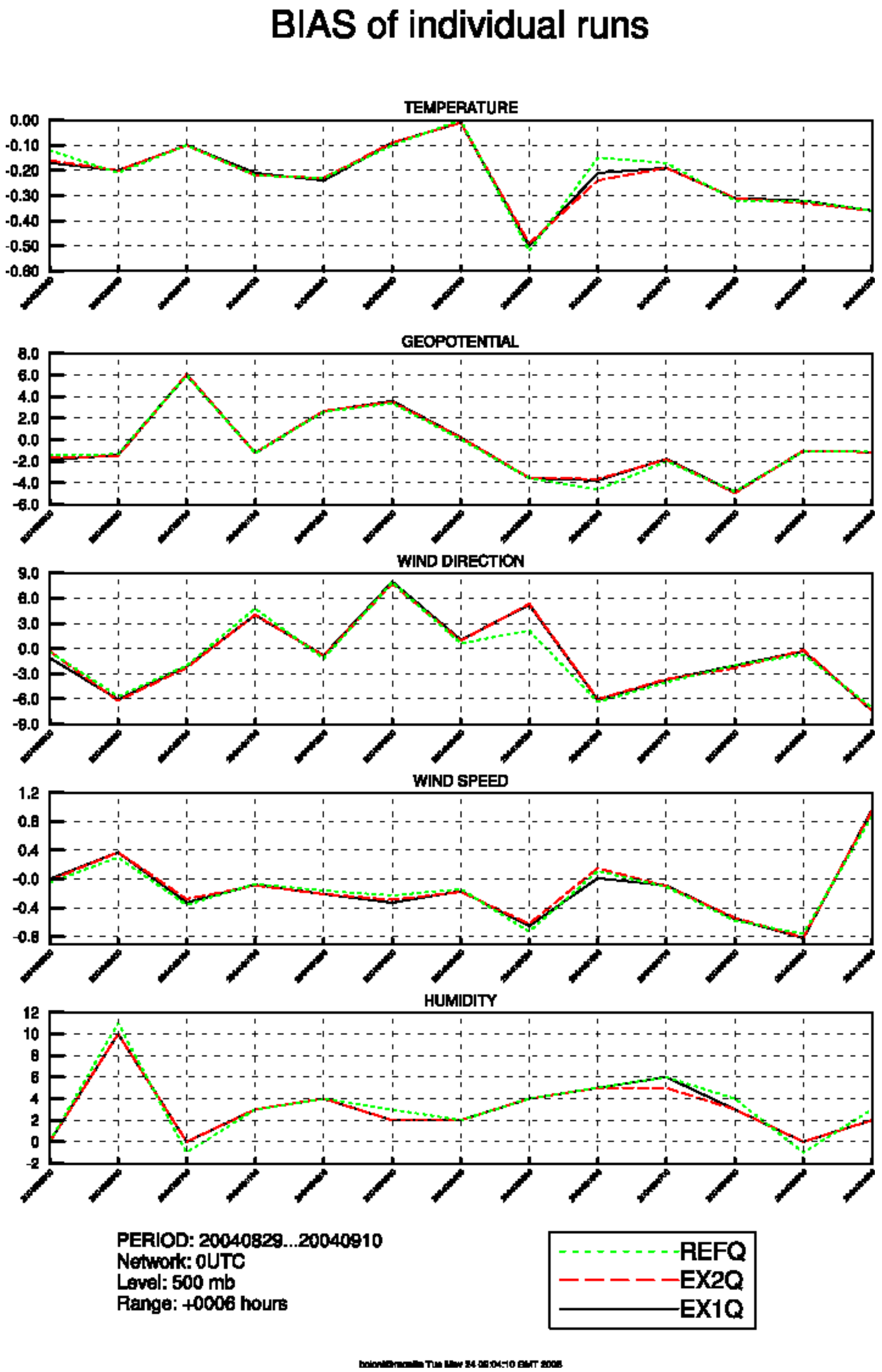

Fig 6.c: BIAS of the individual runs at 500 hPa. Experiment denotations are like in Fig 6.a.

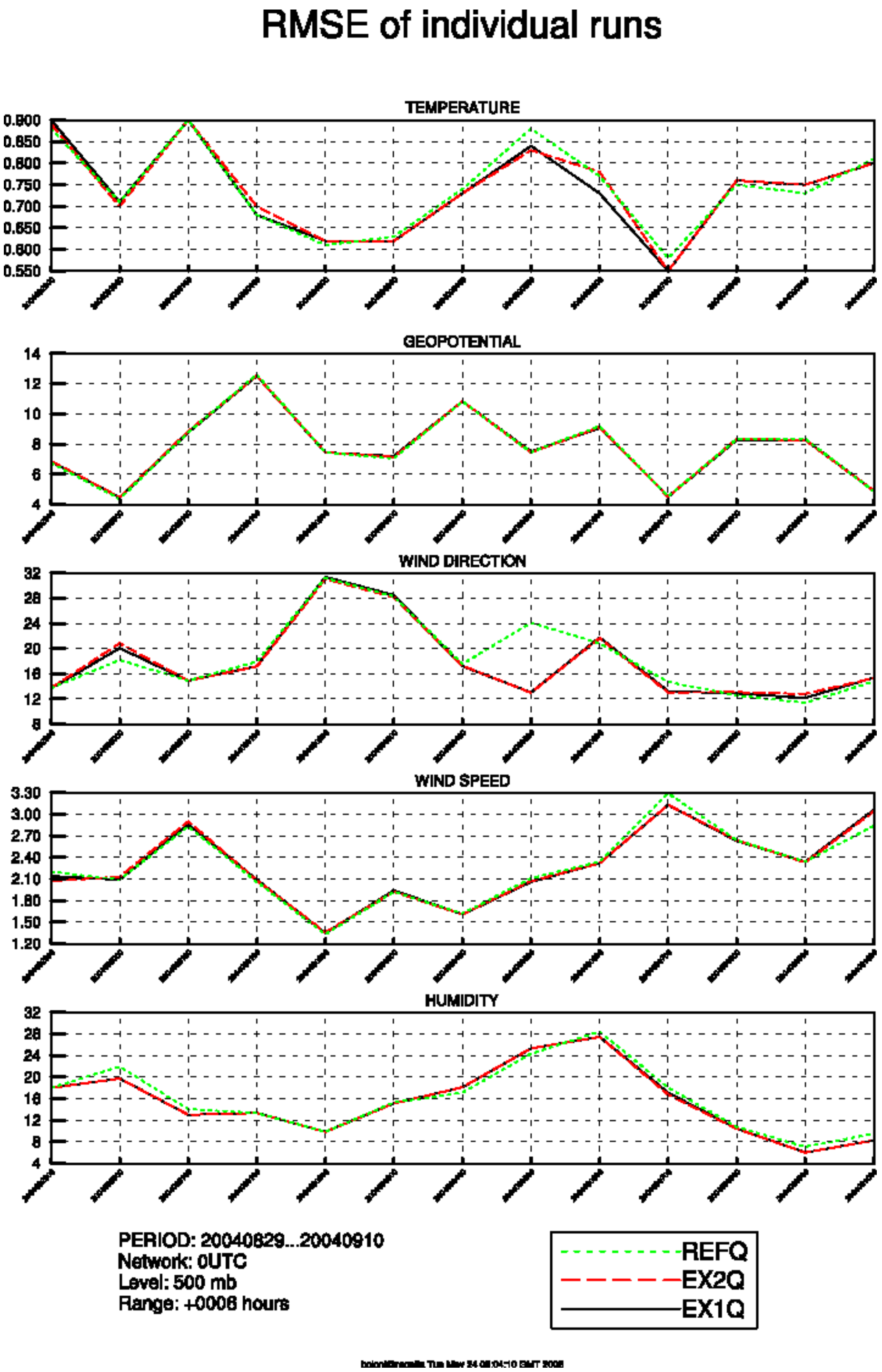

Fig 6.d: RMSE of the individual runs at 500 hPa. Experiment denotations are like in Fig 6.a.

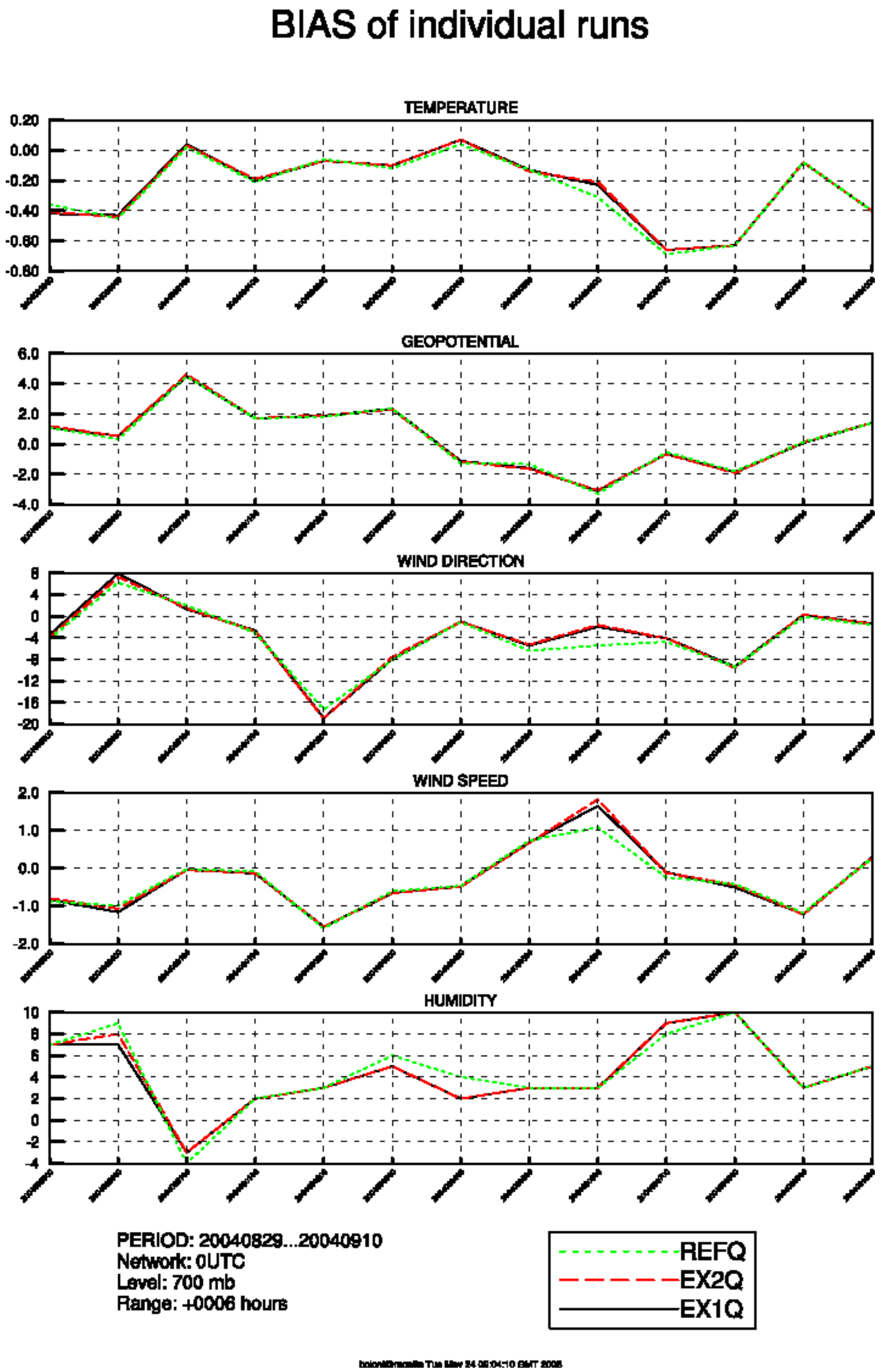

Fig 6.e: BIAS of the individual runs at 700 hPa. Experiment denotations are like in Fig 6.a.

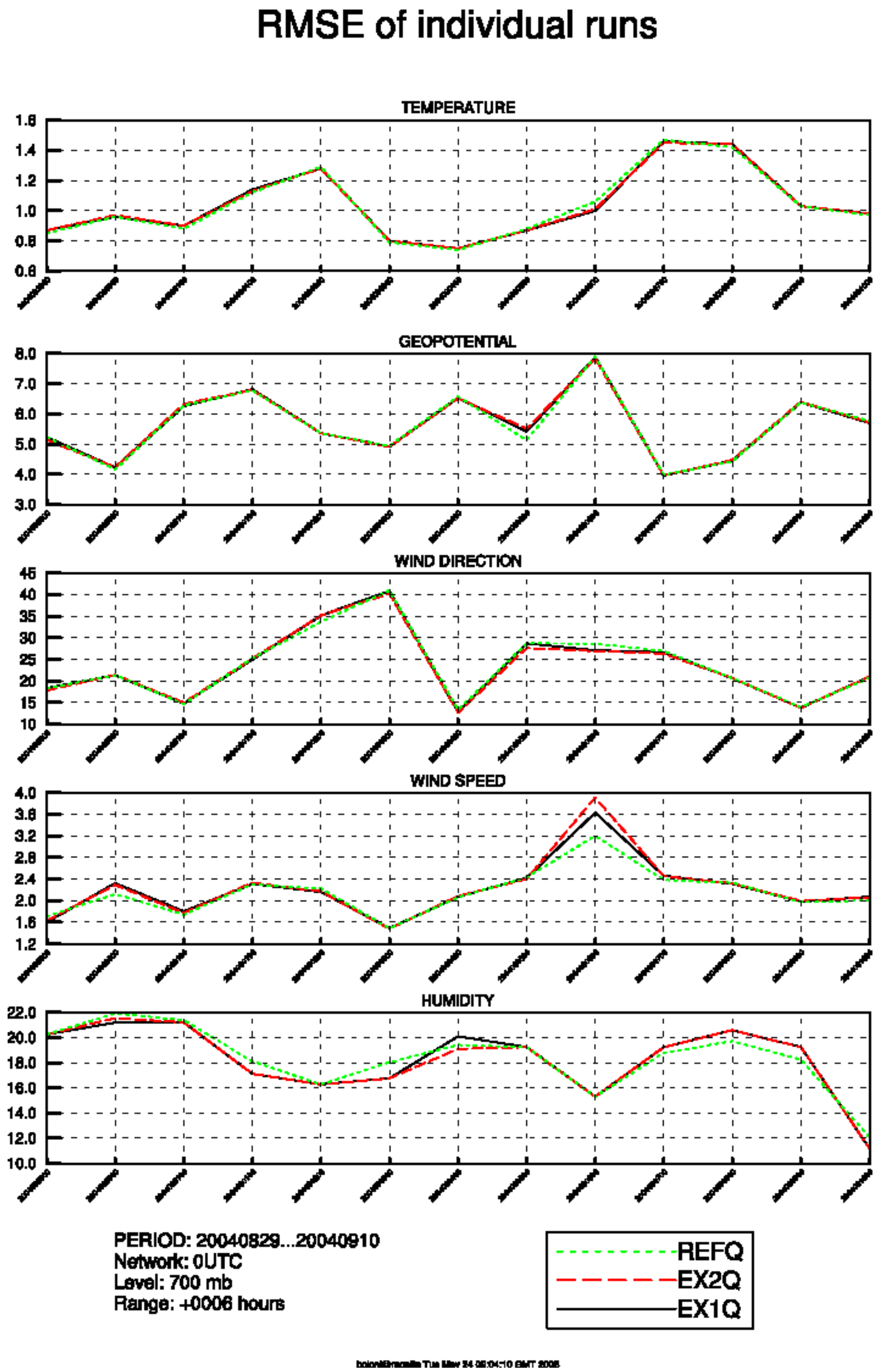

Fig 6.f: RMSE of the individual runs at 700 hPa. Experiment denotations are like in Fig 6.a.

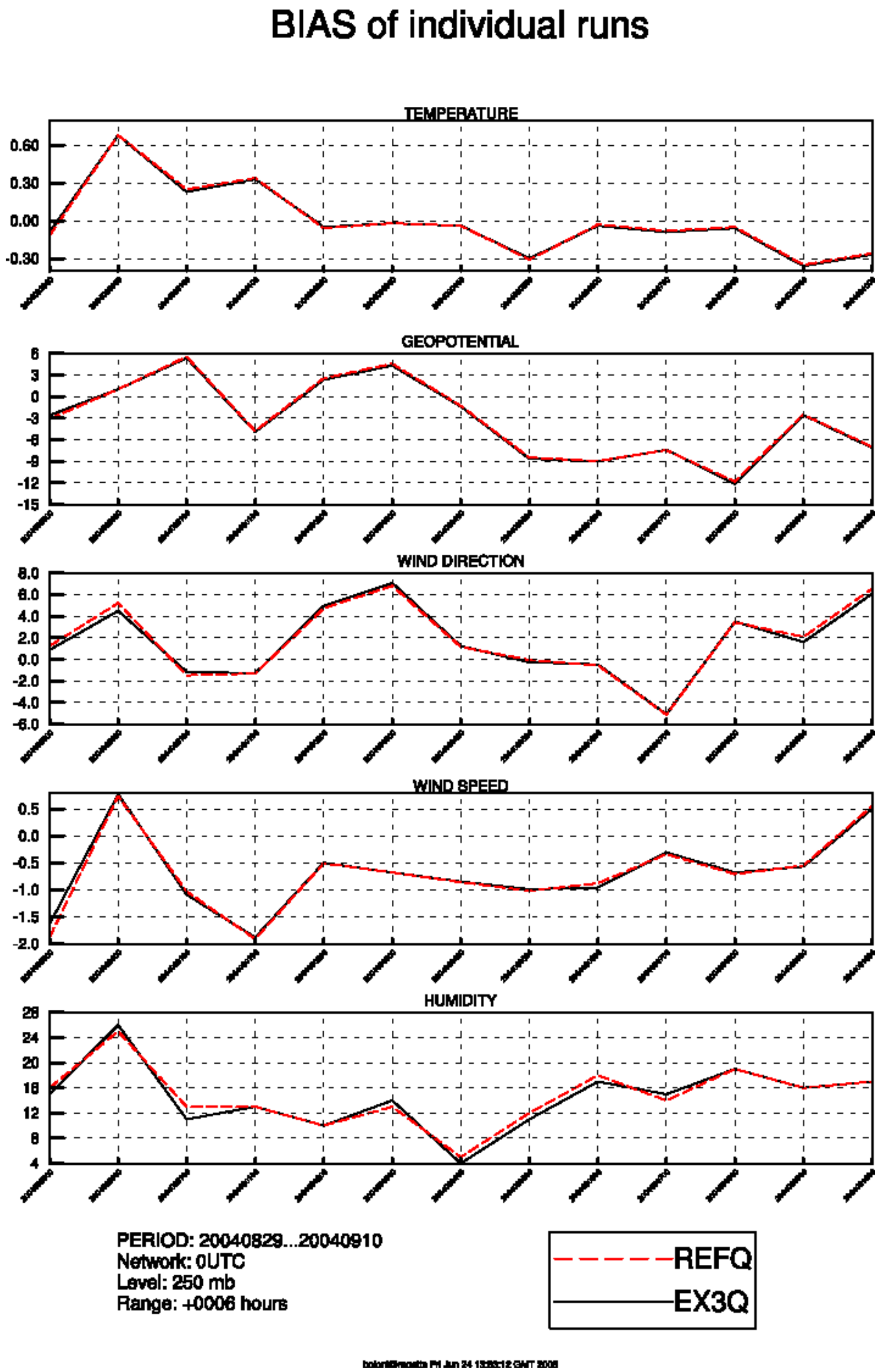

Fig 7.a: BIAS of the individual runs at 250 hPa. REFQ is the reference experiment (ALD/HU operational setup) and EXP3 experiment with no *sb(q)* tuning in the stratosphere.

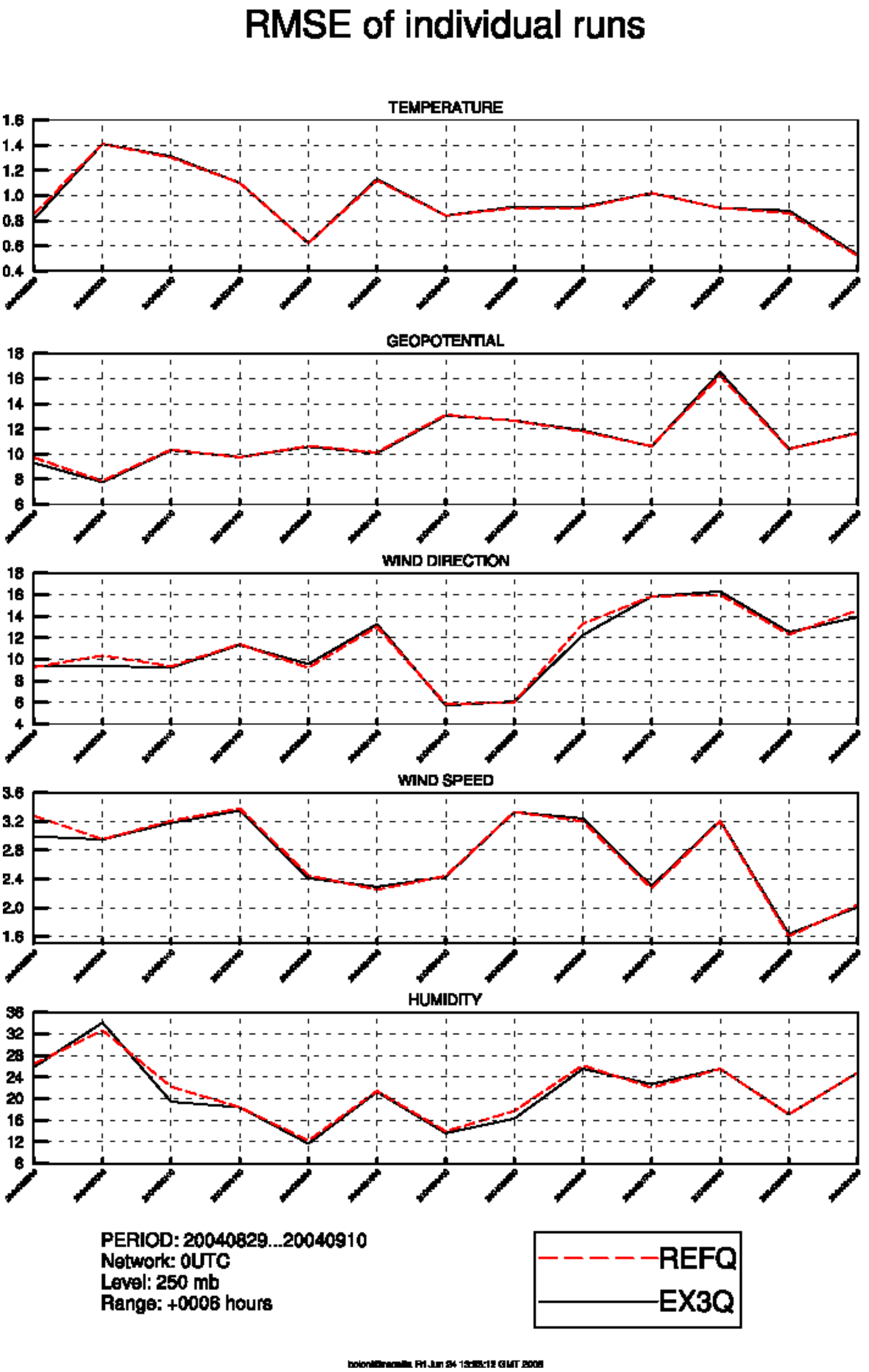

Fig 7.b: RMSE of the individual runs at 250 hPa. REFQ is the reference experiment (ALD/HU operational setup) and EXP3 experiment with no *sb(q)* tuning in the stratosphere.

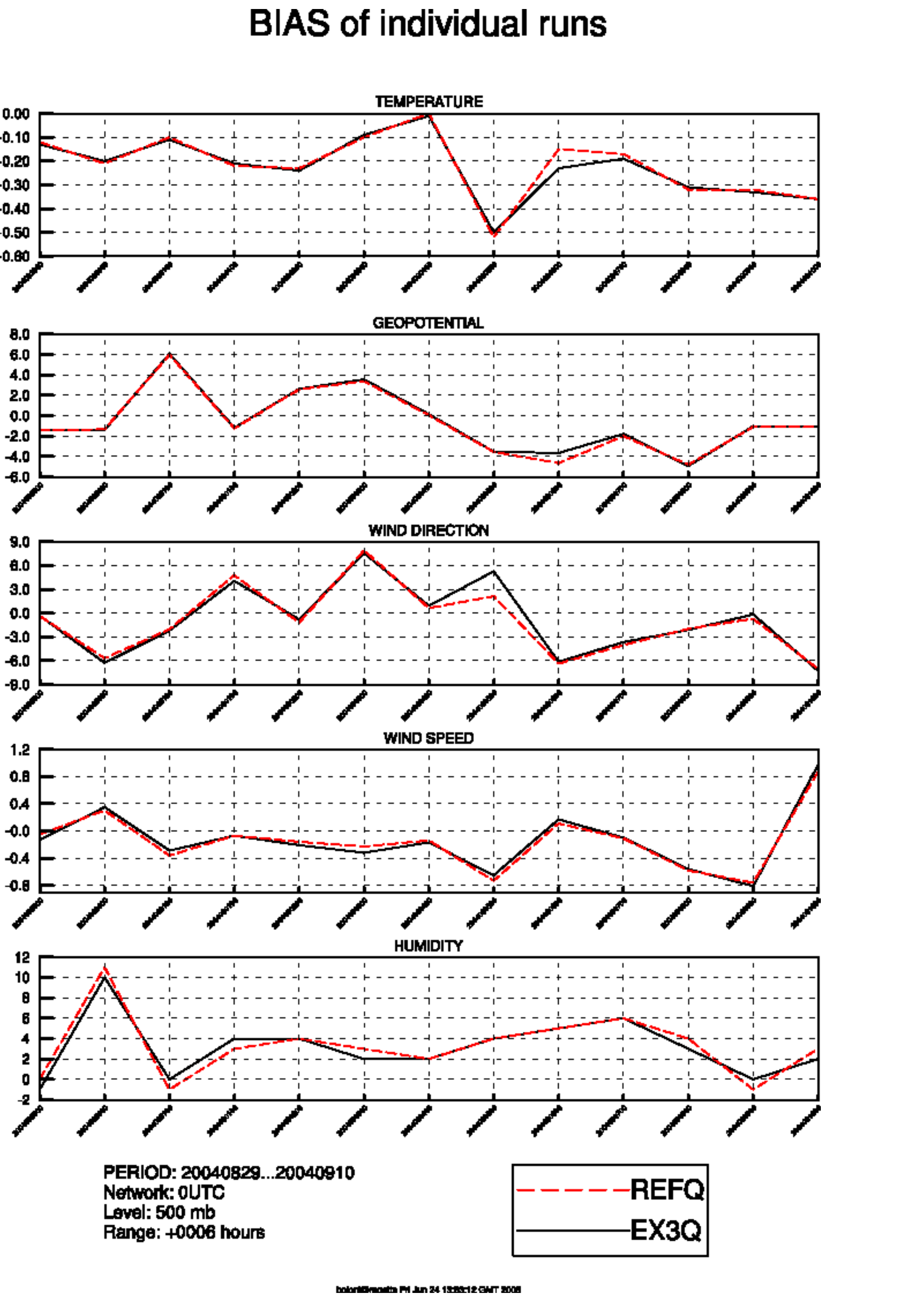

Fig 7.c: BIAS of the individual runs at 500 hPa. REFQ is the reference experiment (ALD/HU operational setup) and EXP3 experiment with no *sb(q)* tuning in the stratosphere.

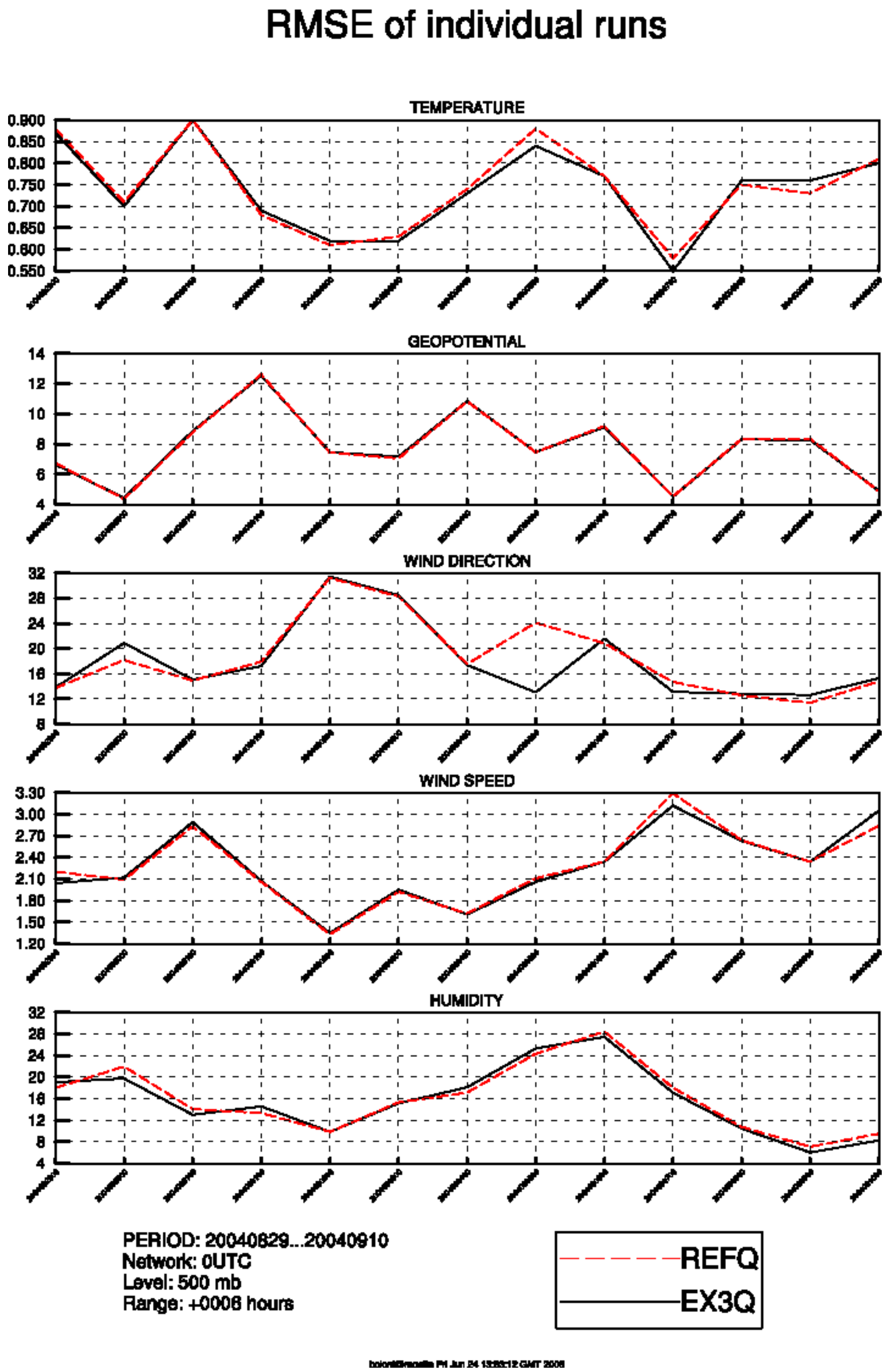

Fig 7.d: RMSE of the individual runs at 500 hPa. REFQ is the reference experiment (ALD/HU operational setup) and EXP3 experiment with no *sb(q)* tuning in the stratosphere.

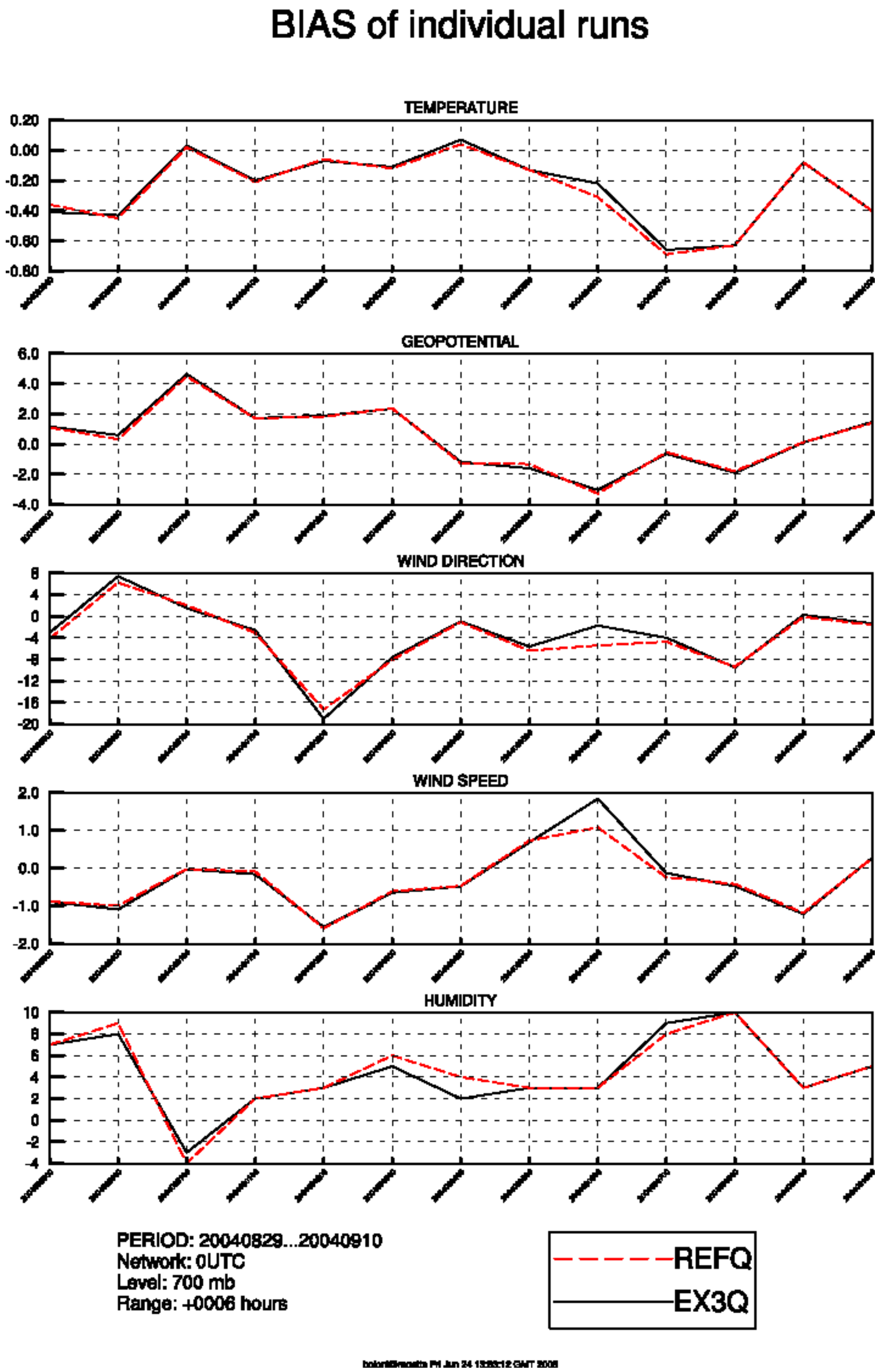

Fig 7.e: BIAS of the individual runs at 700 hPa. REFQ is the reference experiment (ALD/HU operational setup) and EXP3 experiment with no *sb(q)* tuning in the stratosphere.

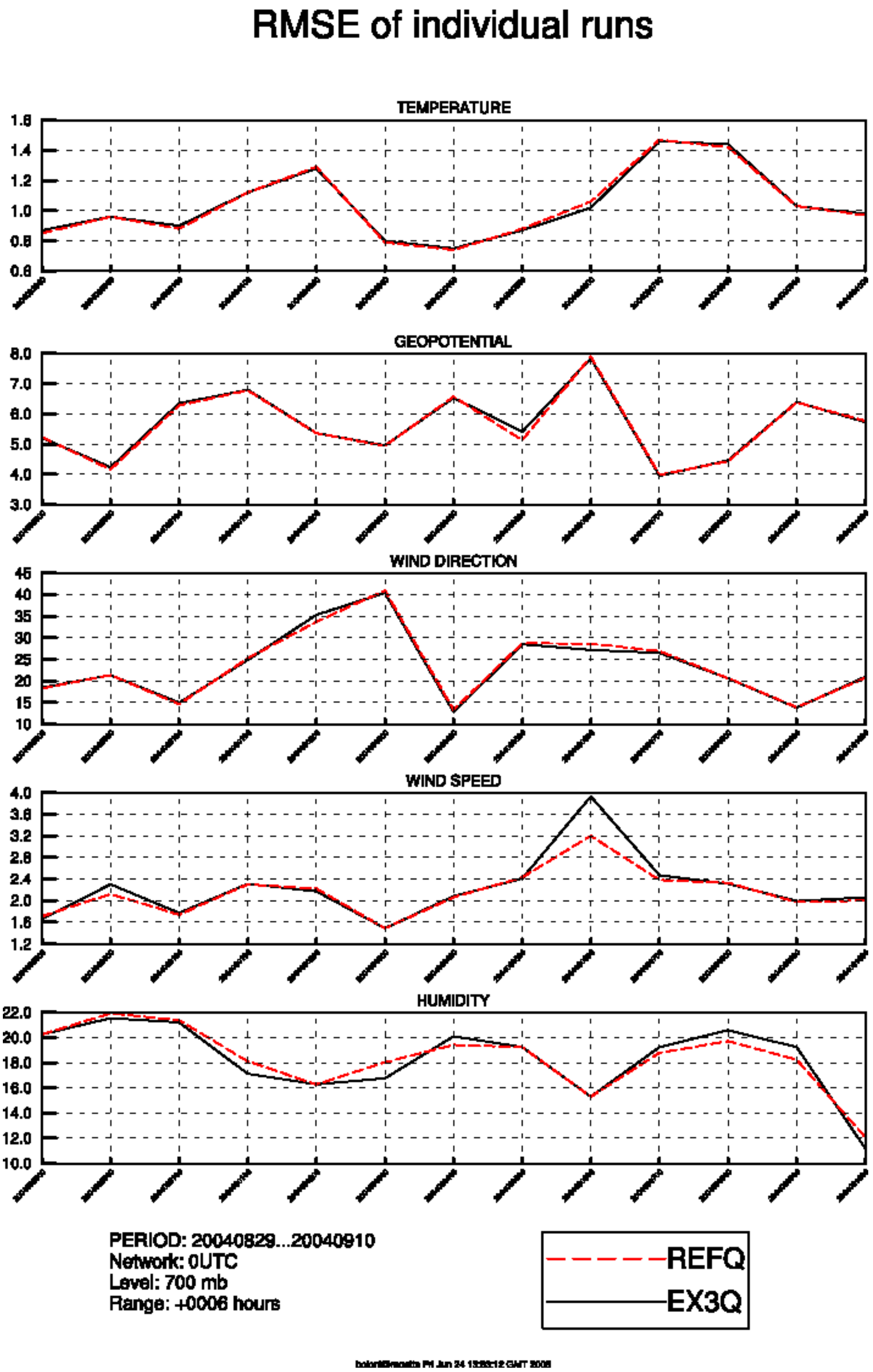

Fig 7.f: RMSE of the individual runs at 500 hPa. REFQ is the reference experiment (ALD/HU operational setup) and EXP3 experiment with no *sb(q)* tuning in the stratosphere.