

Study of the vertical structure of 3D-Var analysis increments

RC LACE internal report

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Introduction

3D-Var belongs to the group of intermittent data assimilation strategies. It is an approximate, iterative minimisation procedure of finding the best possible state of atmosphere. In order to do so, it uses background field, observations and known physical properties of the atmosphere, and combines all three of them. For each iteration an approximate analysis increment is calculated from an initial approximation (usually the first initial approximation is background). The solution is sought by performing several evaluations of the cost function J and of its gradient ∇J , in order to approach the minimum:

$$J(\delta\mathbf{x}) = \frac{1}{2} \delta\mathbf{x}^T \mathbf{B}^{-1} \delta\mathbf{x} + \frac{1}{2} (\mathbf{H}\delta\mathbf{x} - \mathbf{d})^T \mathbf{R}^{-1} (\mathbf{H}\delta\mathbf{x} - \mathbf{d})$$

$$\nabla J = \mathbf{B}^{-1} \delta\mathbf{x} + \mathbf{H}^T \mathbf{R}^{-1} (\mathbf{H}\delta\mathbf{x} - \mathbf{d})$$

where $\delta\mathbf{x}$ is analysis increment to the background \mathbf{x}_b , \mathbf{B} background error covariance matrix, \mathbf{R} covariance matrix of observation error and \mathbf{d} innovation vector

$$\mathbf{d} = \mathbf{y} - \mathbf{H}\mathbf{x}_b$$

where \mathbf{y} is the observation vector and \mathbf{H} is observation operator that transforms background field from model state to observation state. In practice, it is a set of interpolation operators that transform the model grid point values to observation point values, as well as perform some conversions from model variables to observed parameters. Note that in cost function we have rather \mathbf{H} , linearised observation operator, because it is reasonable to linearize \mathbf{H} in the vicinity of background state (tangent linear hypothesis, see Bouttier F. and Courtier P., 1999 for more information).

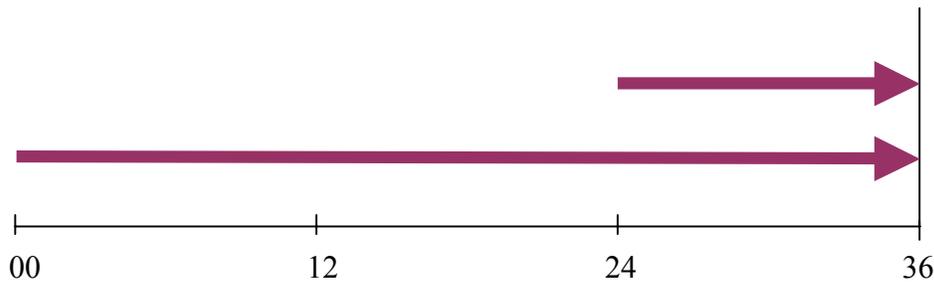
It is worth mentioning that background error covariance matrix \mathbf{B} determines the final shape of analysis increments. In particular, background error variances are responsible for magnitude of an increment, while background error correlations determine information spreading, information smoothing as well as some balance properties (Bouttier F., Courtier P.).

Finally, analysis \mathbf{x}_a is given simply by

$$\mathbf{x}_a = \mathbf{x}_b + \delta \mathbf{x}$$

Although this procedure is iterative, however, there is no guarantee that the iterations will converge.

In general, there are several kinds of statistics that estimate background error covariance matrix \mathbf{B} . Most common one is so-called standard NMC statistics in which background error covariances are derived from a set of differences between forecasts that are valid at the same time, but at different ranges (usually 12 and 36 hour forecasts). Although this method is still widely used for global models, for LAM models it showed not to be so appropriate. Namely, this method results in such a structure of \mathbf{B} that is more concentrated on ‘large scales’ (macroscale), compared to ‘small scales’ (mesoscale), in a sense that it acts rather on macroscale ranges of the analysis fields. This effect is recognised to exist due to impact of lateral boundary conditions (LBC) introduced in process of coupling. However, because in LAM we rather want emphasis on smaller scales, lagged NMC statistics was derived to account for this disadvantage. It follows the same procedure as before, but uses the same LBC for the two forecasts defining forecast errors what is supposed to give more impact to small scales in the analysis (pic 1).



Picture 1. Both statistics are derived from a set of differences between forecasts that are valid at the same time, but at different ranges (usually 12 and 36 hour forecasts). In contrast to standard, lagged statistics uses the same lateral boundary conditions (LBC) for the two forecasts defining forecast errors.

Experiments

Structure functions were investigated by plotting vertical and horizontal cross-sections of single observation experiments. Observation increments (observation – background) used in the experiments were:

AIREP observations at 500 and 850 hPa levels:

- 1) temperature – 1 degree increment
- 2) wind component in y-direction (V) – 1ms^{-1} increment

3) specific humidity – 5% of the background field value

TEMP observations at 500 and 850 hPa levels:

1) relative humidity – 5% increment

SYNOP observations:

1) temperature – 1 degree increment

2) relative humidity – 5% increment

3) wind component in y-direction (V) – 1ms^{-1} increment

All experiments were done using both standard and lagged statistics. Extensive number of plots has been made, exploring analysis increments for temperature, relative humidity, specific humidity, U wind component, V wind component, vorticity and divergence. While vertical analysis increments structures were plotted for all experiments and all variables, horizontal structures were plotted at model level 18 (≈ 510 hPa).

As a background field, initial field for model run on Apr 25th, 2003, 12 UTC was used. Although the date was selected randomly (by Gergo Bölöni), it proved to have some very special features (author's birthday!).

Observations were introduced to the location of meteorology station Kecskemet in the middle of Hungary (47N, 20E). The main domain of vertical cross sections (passing through observation point) was latitudinal, 24.35 degrees wide. Secondary domains used were wide latitudinal domain 34.9 degrees wide and 16 degrees wide longitudinal domain.

For temperature, relative humidity and wind on 500 and 850 hPa, we used Mandalay, while for specific humidity Bator had to be introduced. The inclusion of 10 m wind observation into an analysis demanded changes in choice of the screening blacklisting decisions.

Apriori expectations

In general, from properties of lagged statistics, following expectations were made:

- a) reduction in variances (smaller forecast differences, i.e. giving more weight to background field)
- b) change in horizontal and vertical structure functions, in physical space in sense of for ex. Gaussian curve with smaller halfwidth (in spectral space this can be illustrated by the shift of the variance maxima towards higher wavenumbers).

The presumption a) for single observation experiments was already demonstrated, but only for SYNOP relative humidity observation increment and its influence in vertical (Jurasek M., 2001).

Results

Comparison in variance

Because of a vast ‘zoo’ of vertical and horizontal plots for all pairs of variables, only a minor part of figures will be shown in this report. However, moderate-size database is going to be created in a near future (Nov-Dec 2003). All cross-sections are available on request from the author.

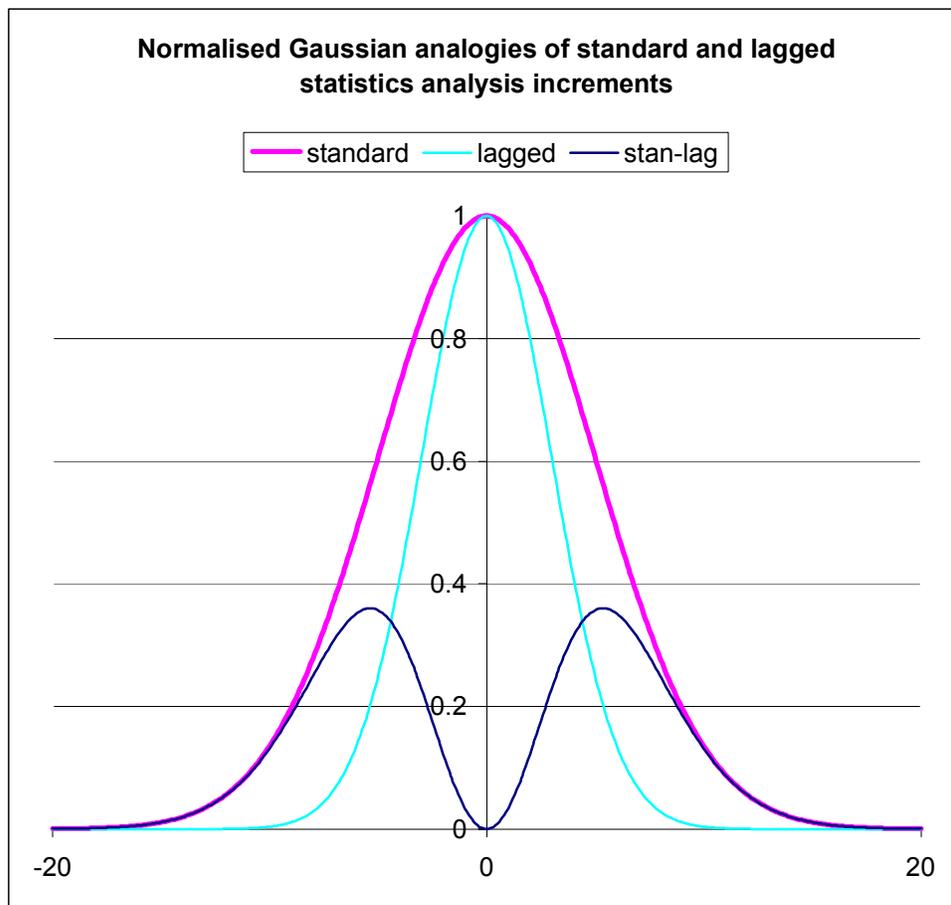
Comparison between standard and lagged statistics for temperature observation increment at different levels and its influence on temperature analysis increments can be investigated with vertical cross-sections (fig 1-6). The first thing to notice is a quite big reduction in variance (for some other increments reaching up to 10 times), clearly seen on cross, as well as on horizontal sections (fig 7-8). However, it should be noticed that this reduction is smaller at lower level of the atmosphere. The same behaviour is noticed for analysis increments of other variables and for other observation increments as well. The reason for that is that processes of macroscales are more dominant at 500 hPa level than at surface level, where smaller scales gain more dominance. Therefore, reduction of big scales performed in lagged statistics has greater influence at higher levels of atmosphere. This conclusion is fortified by insight into a normalised variance versus wave number dependencies (fig 9), that show the weight of different scales in a statistics of model error for a given variable at one level (Bölöni Gergely, personal communication). The reduction performed by lagged statistics is not the same for analysis increments of different variables as can be seen for relative humidity (fig 10-11). A greater or smaller asymmetry in relative humidity increments is observed for other observation increments, and it is interesting to notice that it is always at the same height, no matter at what level an observation is introduced. On horizontal cross section it can be seen that isolines are noisy above mountains (fig 12). This behaviour is noticed only for relative humidity increments, probably due to the design of relative humidity ‘operator’ because relative humidity is not a control variable, i.e. in 3Dvar it has to be calculated after minimisation (we call these calculations ‘operator’, but it is not an operator in mathematical point of view, rather a set of routines to compute relative humidity field out of fields of other variables).

Moreover, although in general property of lagged statistics is reduction, in some cases that is not the case, as can be seen for an influence of specific humidity observation increment on divergence analysis increments (fig 13-14). The same can be noticed at all three levels, as well as for wind and relative humidity observation increments. On the contrary, for temperature observation increment, regular reduction is present. Probably, the reason for such a behaviour is that for divergence different scales have very different impacts on analysis increments, and by removing one part of the scale spectrum, an effect of the rest of spectrum is even fortified.

Verifying change in structure

As can be seen from figures discussed above, comparison between structure of the analysis increments made with two statistics is a hard nut. Because of reduction in variance, the only way how it can be done is to look not at isolines, but at their density. If more weight is given to small scales in lagged statistics, we will expect to find isolines to be denser in the vicinity of observation and rarer far from it, compared to the standard statistics. With some effort, this can be observed for temperature analysis increment at 500 hPa (fig 1-2), but for virtually all other structure functions it is not sure that this effort will give a positive result.

Therefore, a new idea of confirming the change in structure functions has been tested. First, analysis increments should be normalised with their maximum grid point value in the vicinity of observation for both statistics. Secondly, the two normalised fields are



Picture 2. Visualisation of Gaussian analogies of normalised influence of standard and lagged statistics and their difference. Change in structure functions is verified if there is a well defined maxima of their difference.

then to be simply subtracted. It is expected that far from observations, difference will be zero, but will grow as we approach towards the observation point. However, at some distance, the difference will reach its maxima and start to decline to zero as we approach

more to the very point of observation (pic 2). This approach was used to analyse temperature analysis increments (fig 15-17) and expected behaviour is verified. For the time being, this idea is applicable only on circle like structure functions, what, without modifications, makes it unusable for wind analysis increments.

Why such a shape?

First thing to notice is that vertical cross-correlations and auto-correlations from B are responsible for the very shape analysis increments have. Therefore structure functions are basically an output of a model. If the model is reasonably good in prediction of atmospheric processes, we can try to understand structure functions in terms of atmospheric dynamics (physics). Such an approach will be presented here on a very few selected cases.

The shape of temperature, relative humidity and divergence analysis increments have one positive and one negative pole in vertical. However, one may notice that in general there exists a variety of increment shapes in horizontal and vertical which can show as circles, butterflies and double butterflies. One way of thinking why do analysis increments have the very shape on the figures is to learn about the situations when model makes errors, because these situations make the statistics that determine the increment outlook. Therefore, if we know that model is making a mistake in temperature field predominantly in advection of warm and cold fronts, we can analyse those situations specifically and conclude about the shape of the increment. In particular, if we know that warm fronts are warmer in lower atmosphere and cooler in high atmosphere (for warm fronts vice versa), we can understand why temperature increments have two poles.

Nevertheless, a somewhat different, "dynamical" way of analysing the shape of increment and its realism is proposed here. It is to ask what happens with the atmosphere if it is heated in one region. First, that heat will be spread in surroundings, but moreover that will induce more or less intensive dynamical effects in vertical (e.g. redistribution of air to satisfy vertical pressure gradient constraints). More precisely, heating causes rise of the tropopause therefore forcing the stable vertical layers in upper troposphere and in lower stratosphere to lift up and cool adiabatically. Mainly for this reason, the air at higher levels of the atmosphere will tend to be cooler than before, specially in stratosphere where stable stratification is present. Although in this case it is maybe more straightforward to think about the shape of temperature increment in terms of cold and warm fronts, explanation of relative humidity analysis increments as a consequence of errors in front prediction is not so simple (fig 10-11). However, dynamical approach very simply says that if we heat air, locally it will be able to accept more water vapour (Clausius-Calpeyrone's law), and relative humidity value will be lower. Of course, rise in specific humidity is present as well, but in middle and higher troposphere it does not seem to be a dominant factor. But adding heat in the middle troposphere means that a new vertical pressure structure in the atmosphere will take place, more specifically, air will be forced to lift. While lifting, air cools itself and relative humidity raises, what produces positive analysis increments further up from the observation. Lifting caused by heating can explain divergence analysis increments as well (fig 13-14). Namely, lifting forces convergence down from the observation and divergence in the upper part of atmosphere.

It should be noted that ‘dynamical’ approach does not demand the apriori knowledge about situations when a model makes mistakes. Moreover, it is believed that it might be a helpful additional tool for conclusion about the realism of a shape and a size of analysis increments.

Bugs in the attic!

For the sake of investigating analysis increments for which a symmetry line was parallel with our main (latitudinal) vertical cross section we introduced an orthogonal (longitudinal) cross section. Nevertheless, that procedure yielded some unexpected structure functions (fig 18). The same behaviour is spotted for orthogonal U and V wind cross sections (for other variables orthogonal cross-sections were not done). It is believed that the presented shape of wind analysis increments is due to a bug in the system.

Conclusions

Comparison between standard and lagged NMC statistics confirmed that influence of lagged statistics is, in general, reduction of analysis increments. However, this reduction varies among different levels and different observation and analysis increments. In particular, it is determined by the scale dominant for a given variable at a given level. If it is predominantly mesoscale, reduction of big scale influence performed in lagged statistics will not have an expressed impact. Rarely, because of the same reason and the fact that different scales can have very different impact in nature, lagged statistics can cause even greater analysis increments than standard one.

Change in structure functions showed to be very difficult to verify by looking at analysis increments themselves. Therefore, a new idea has been tested in which analysis increments of different statistics are normalised and subtracted. By looking at the difference, change in structure functions with this procedure can be confirmed. Nevertheless, for the time being, it is applicable for simple, ‘circle’ shaped structure functions.

Relative humidity increments showed to have two unwanted (maybe dependent) properties: noise above the mountains, and asymmetry in shape of increments. Both is believed to exist, at least partly, due to design of relative humidity operator.

Existence of a probable bug that heavily distorts wind analysis increments north from aprox. 48°N is found in the system.

It proved not to be so simple and straightforward to analyse structure functions at all. For conclusions about modifications in model error covariance matrix B, a knowledge about the model quality and influence of different scales is demanded. Furthermore, it is believed that dynamical approach might be a helpful tool for understanding structure functions and maybe deciding about realism of analysis increments as well.

References

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TEMPERATURE

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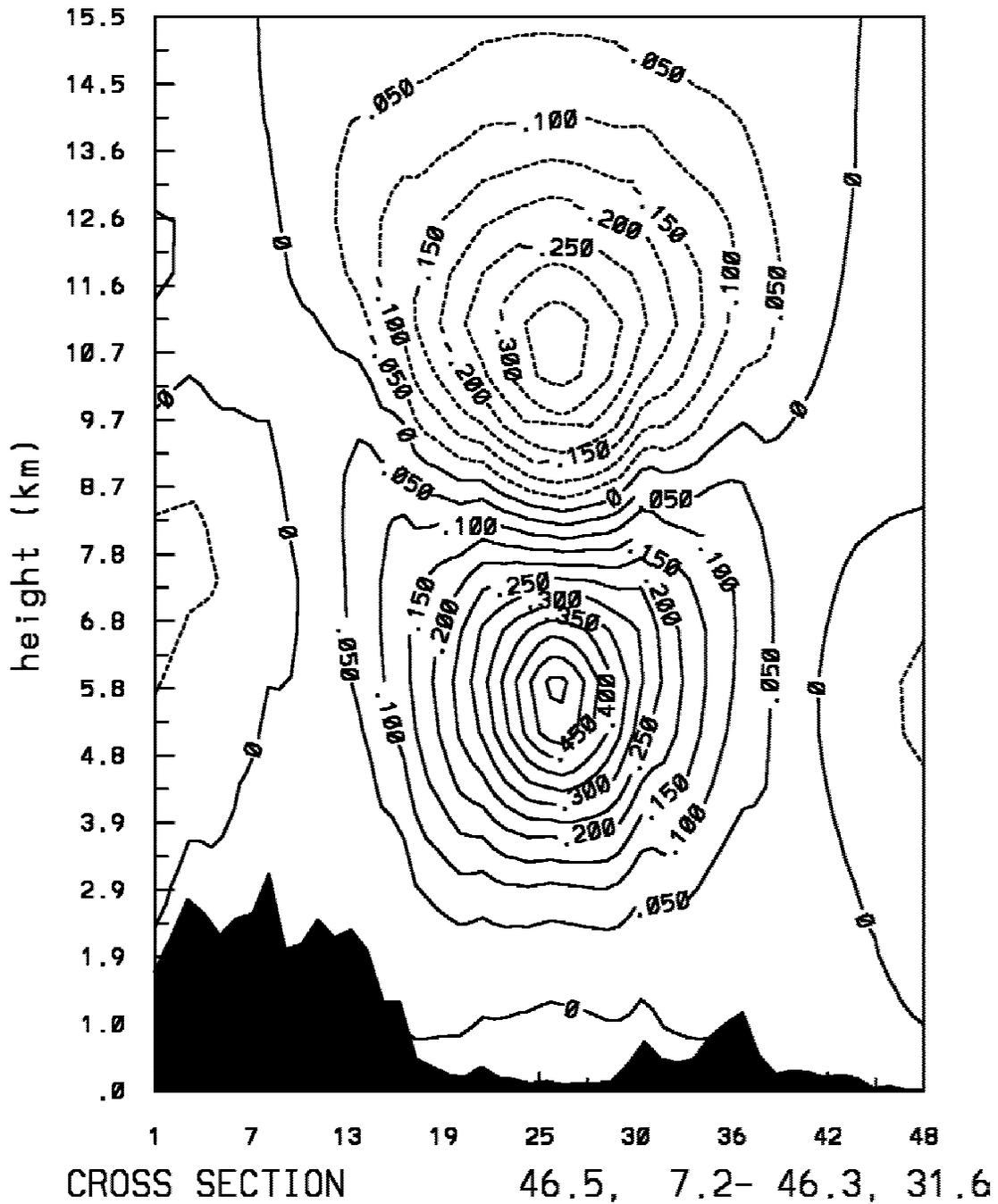


Figure 1. Temperature analysis increments due to temperature observation increment at 500 hPa derived using standard NMC statistics. Note the value of absolute maximum ($\approx 0.55^{\circ}\text{C}$) as well as great negative impact in upper troposphere and stratosphere.

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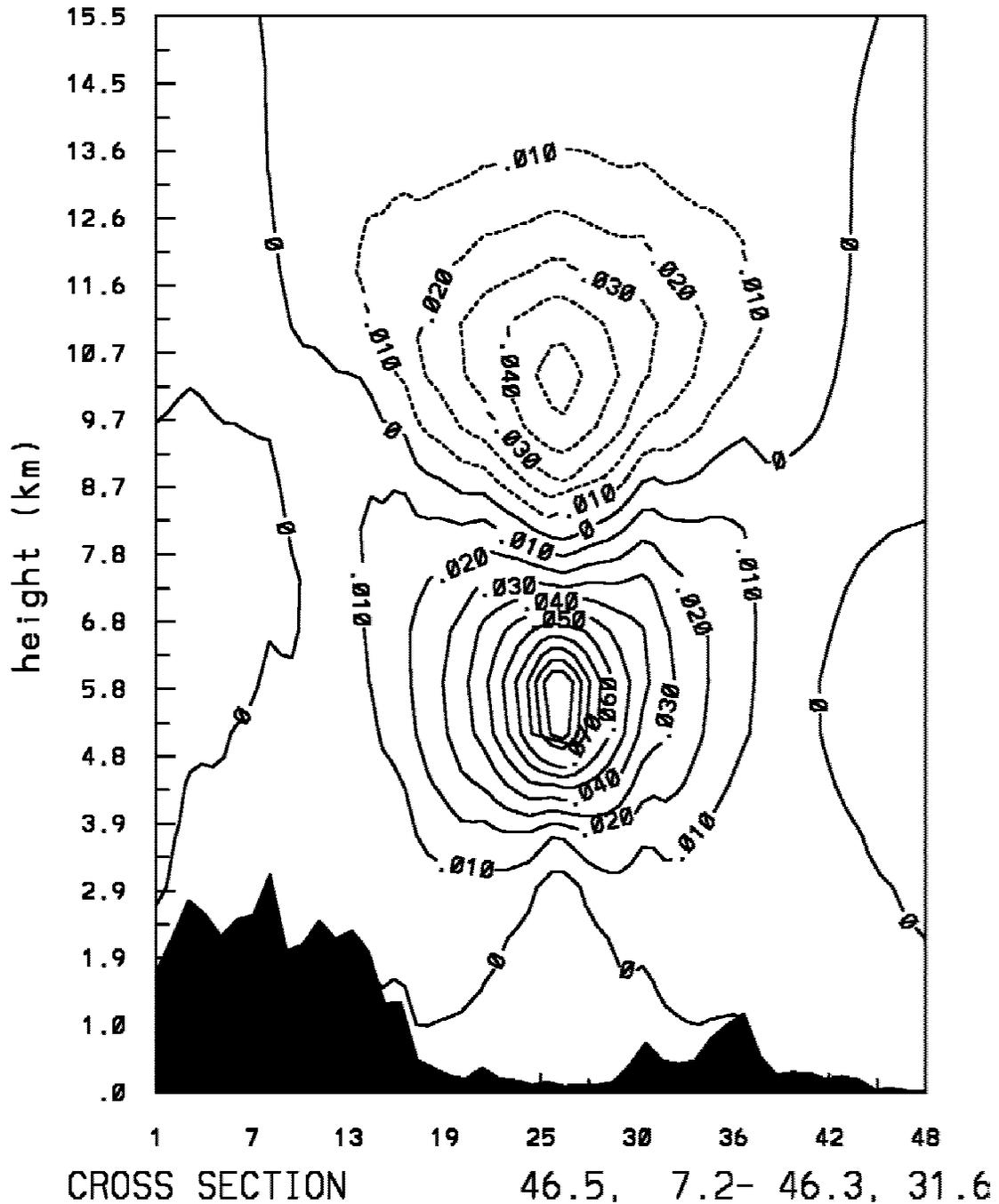


Figure 2. Temperature analysis increments due to temperature observation increment at 500 hPa derived using lagged NMC statistics. Note that the value of absolute maximum ($\approx 0.1^{\circ}\text{C}$) is reduced about 5 times.

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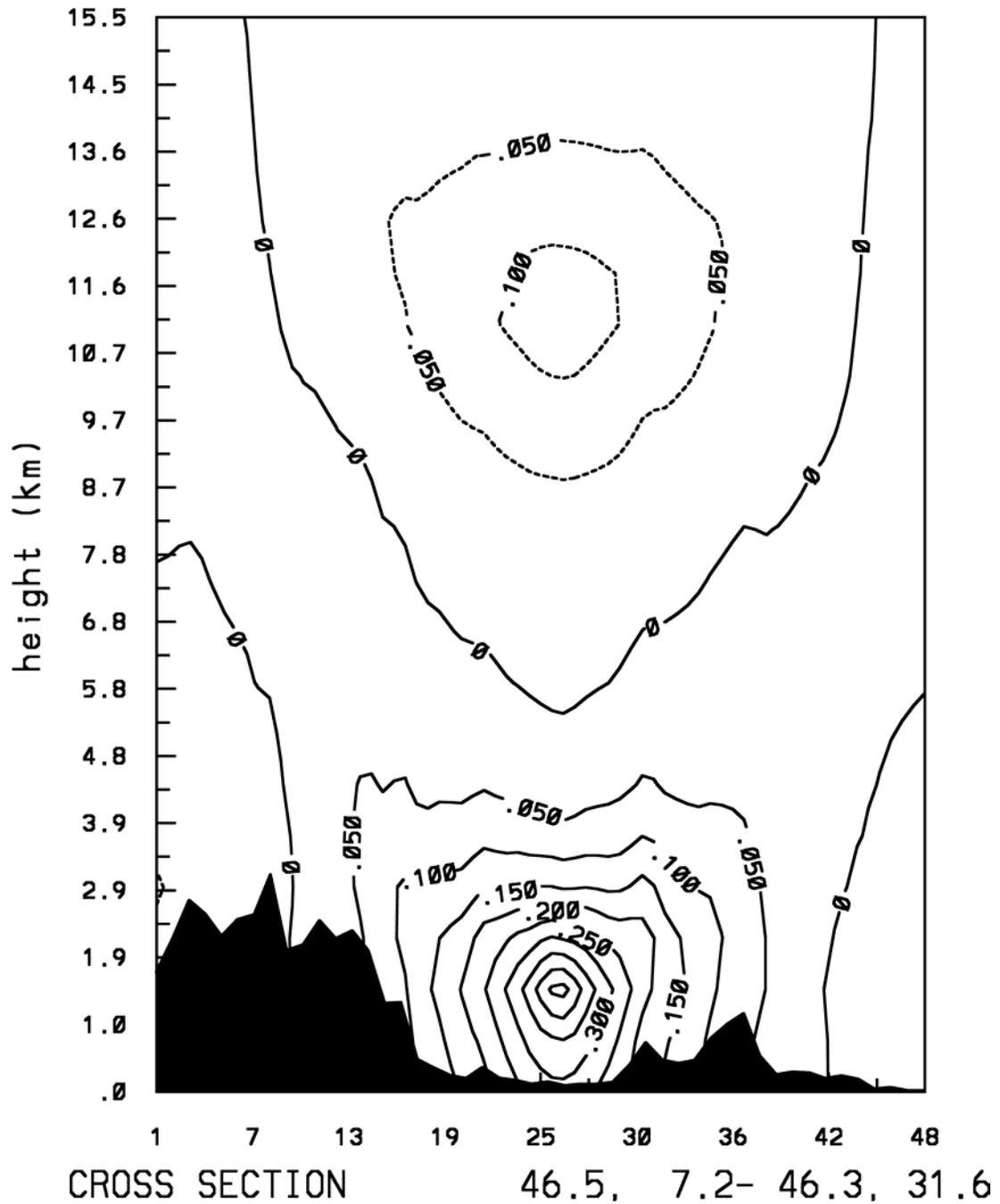


Figure 3. Temperature analysis increments due to temperature observation increment at 850 hPa derived using standard NMC statistics. Note the value of absolute maximum ($\approx 0.45^{\circ}\text{C}$) and relatively reduced negative increments stratosphere.

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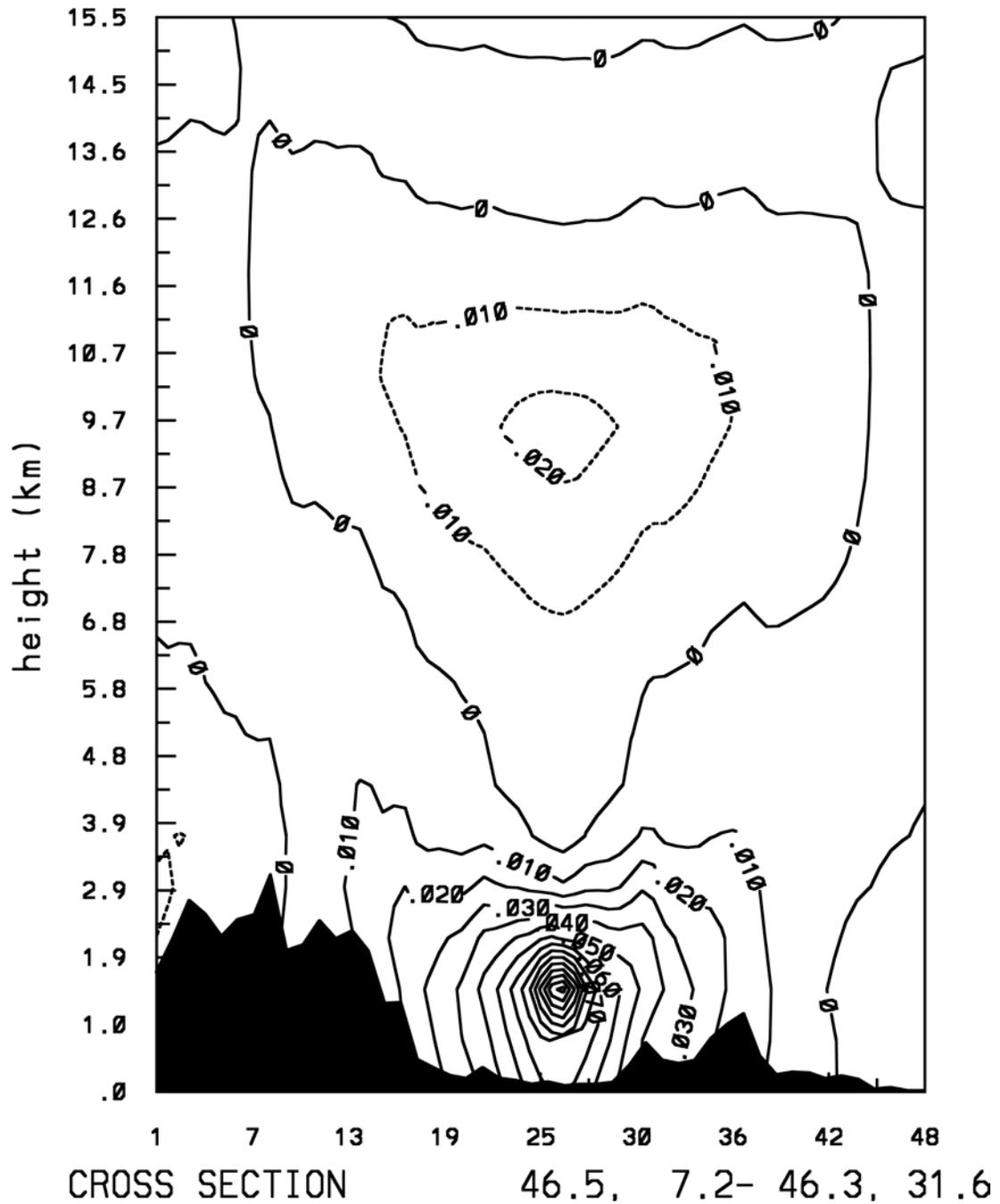


Figure 4. Temperature analysis increments due to temperature observation increment at 850 hPa derived using lagged NMC statistics. Note that the value of absolute maximum ($\approx 0.14^{\circ}\text{C}$) is reduced about 3 times.

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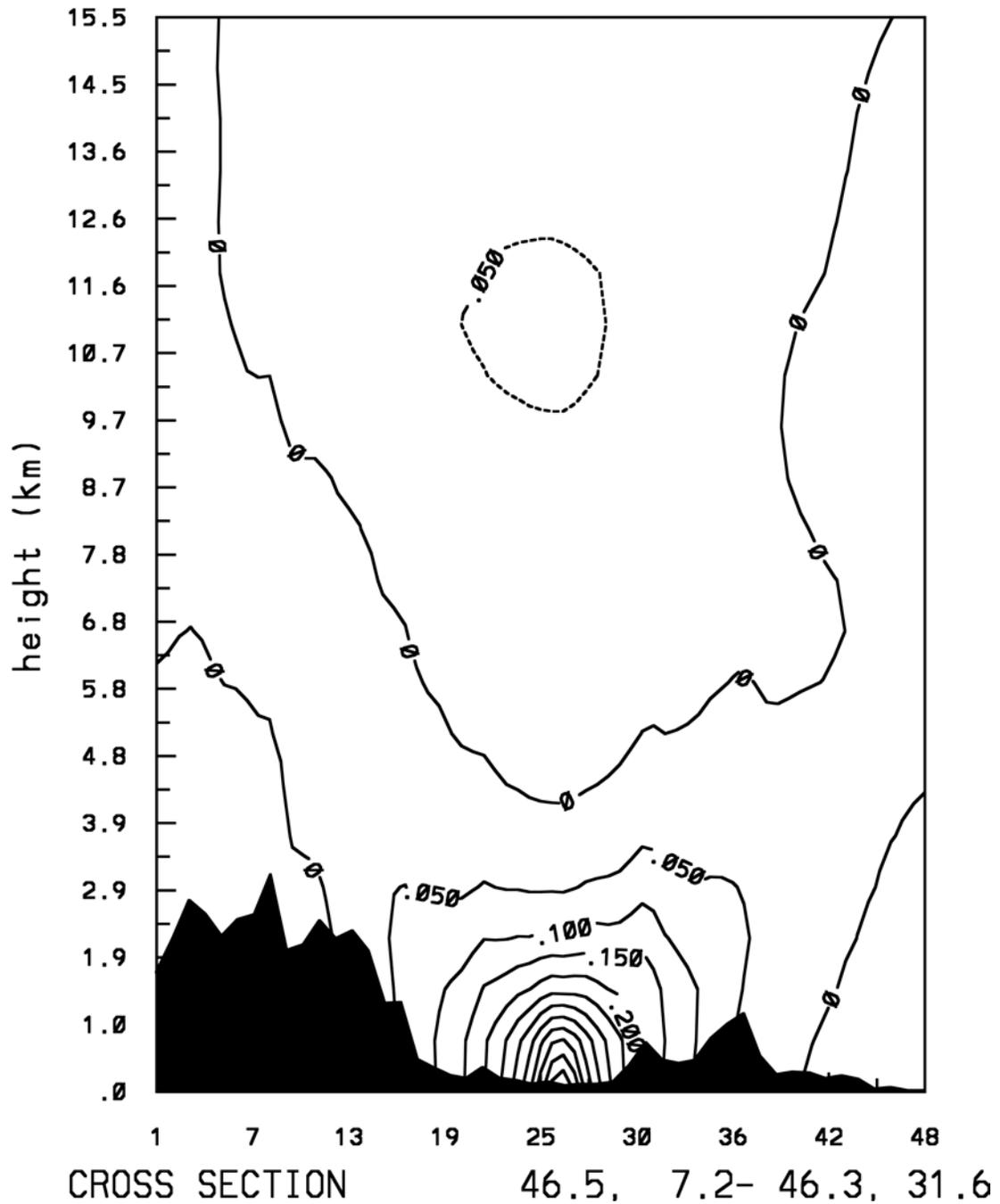


Figure 5. Temperature analysis increments due to temperature observation increment at 2m level derived using standard NMC statistics. Note the value of absolute maximum ($\approx 0.55^{\circ}\text{C}$) and almost no negative increments in stratosphere.

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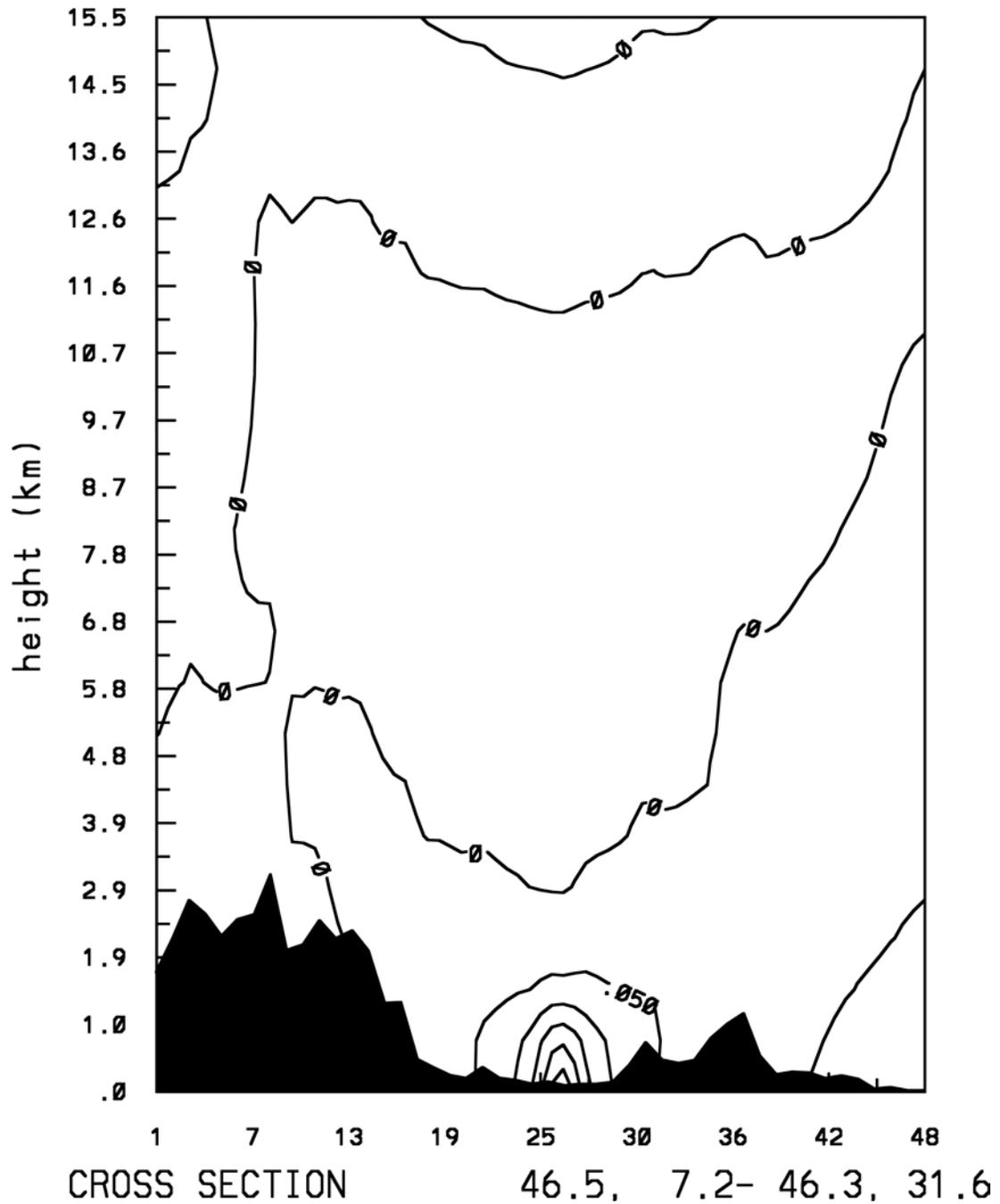


Figure 6. Temperature analysis increments due to temperature observation increment at 2m level derived using lagged NMC statistics. Note that the value of absolute maximum ($\approx 0.25^{\circ}\text{C}$) is reduced about 2 times.

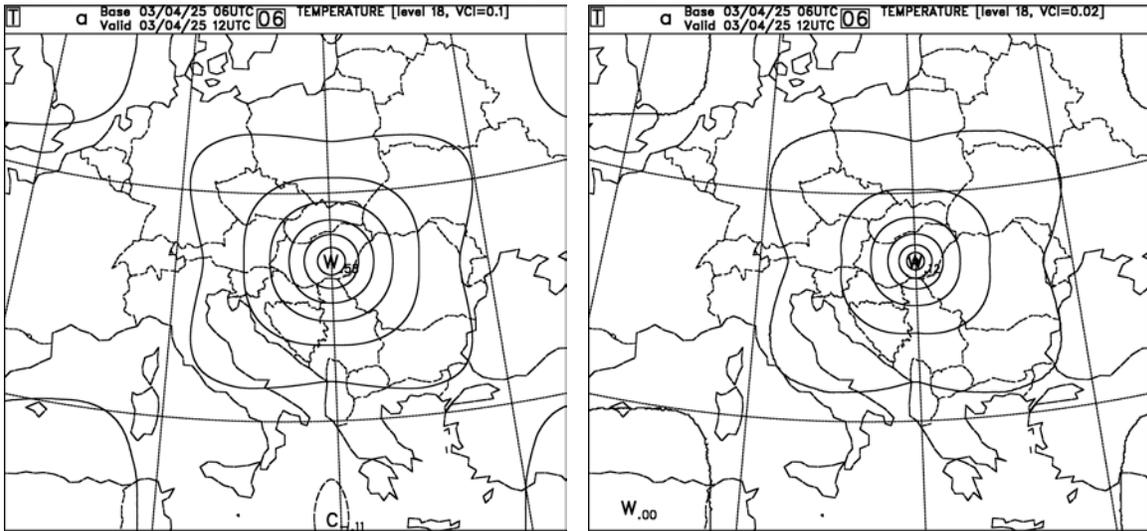


Figure 7 and 8. Horizontal temperature analysis increments at level 18 (≈ 510 hPa) due to temperature observation increment at 500 hPa level derived using standard (left) and lagged (right) NMC statistics. Note that isolines are more dense in the vicinity of observation increment for lagged than for standard statistics, therefore implying change in structure of the structure functions. However, for other structure functions this is very hard to notice, specially in vertical.

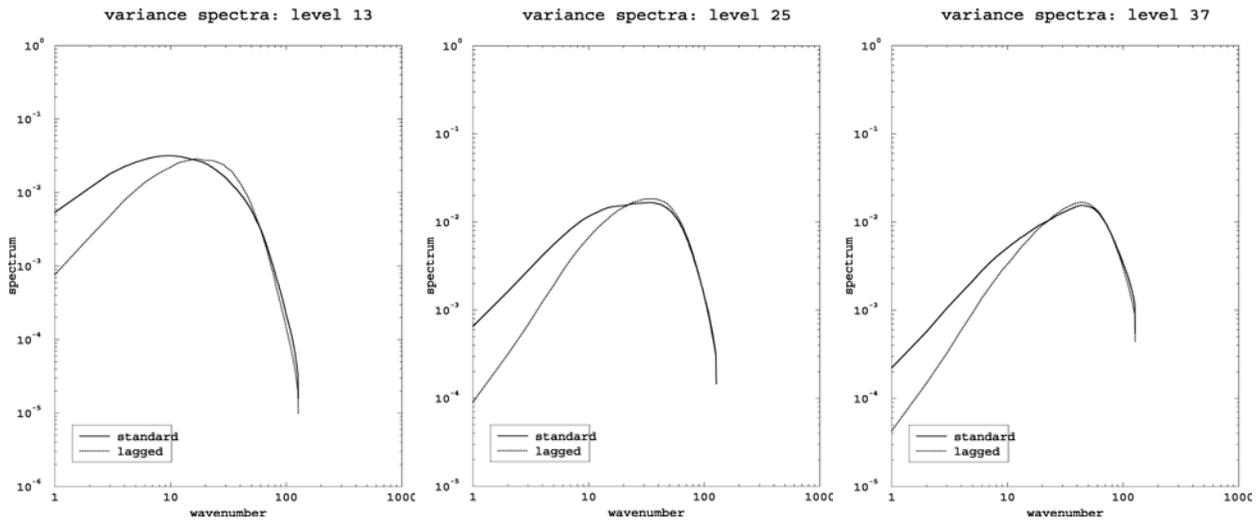


Fig 9. Variance spectra for divergence at 13, 25 and 37 (lowest) model levels. Because of smaller importance of macroscales in lower atmosphere, reduction of macroscale influence performed by lagged statistics makes smaller effect in lower than in higher atmosphere.

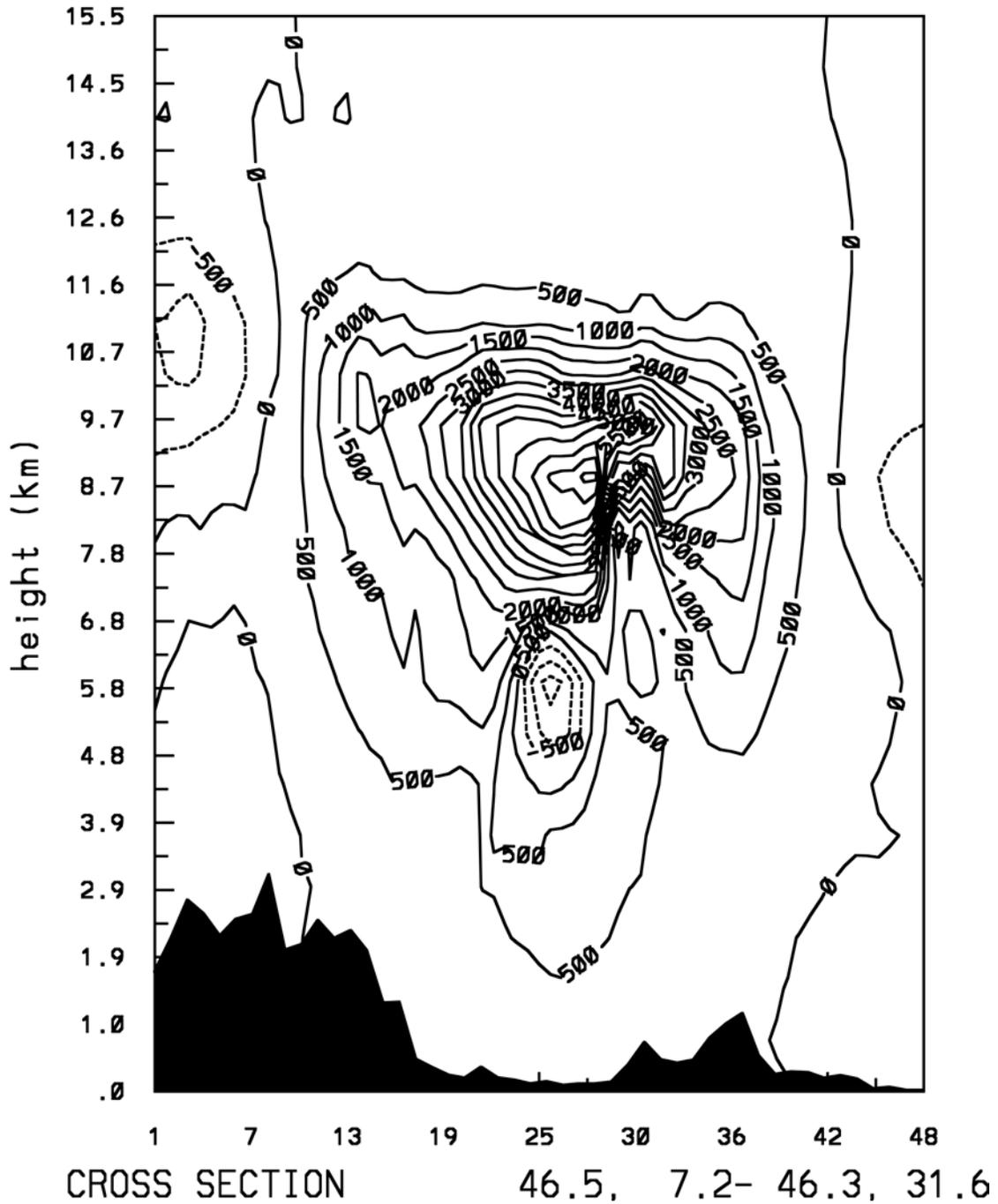


Figure 10. Relative humidity analysis increments due to temperature observation increment at 500 hPa level derived using standard NMC statistics. Note the local negative increments in the vicinity of observation increment and huge positive values further as well as asymmetry in the upper troposphere. Scaling factor is 1000 (1000 = 1% rh).

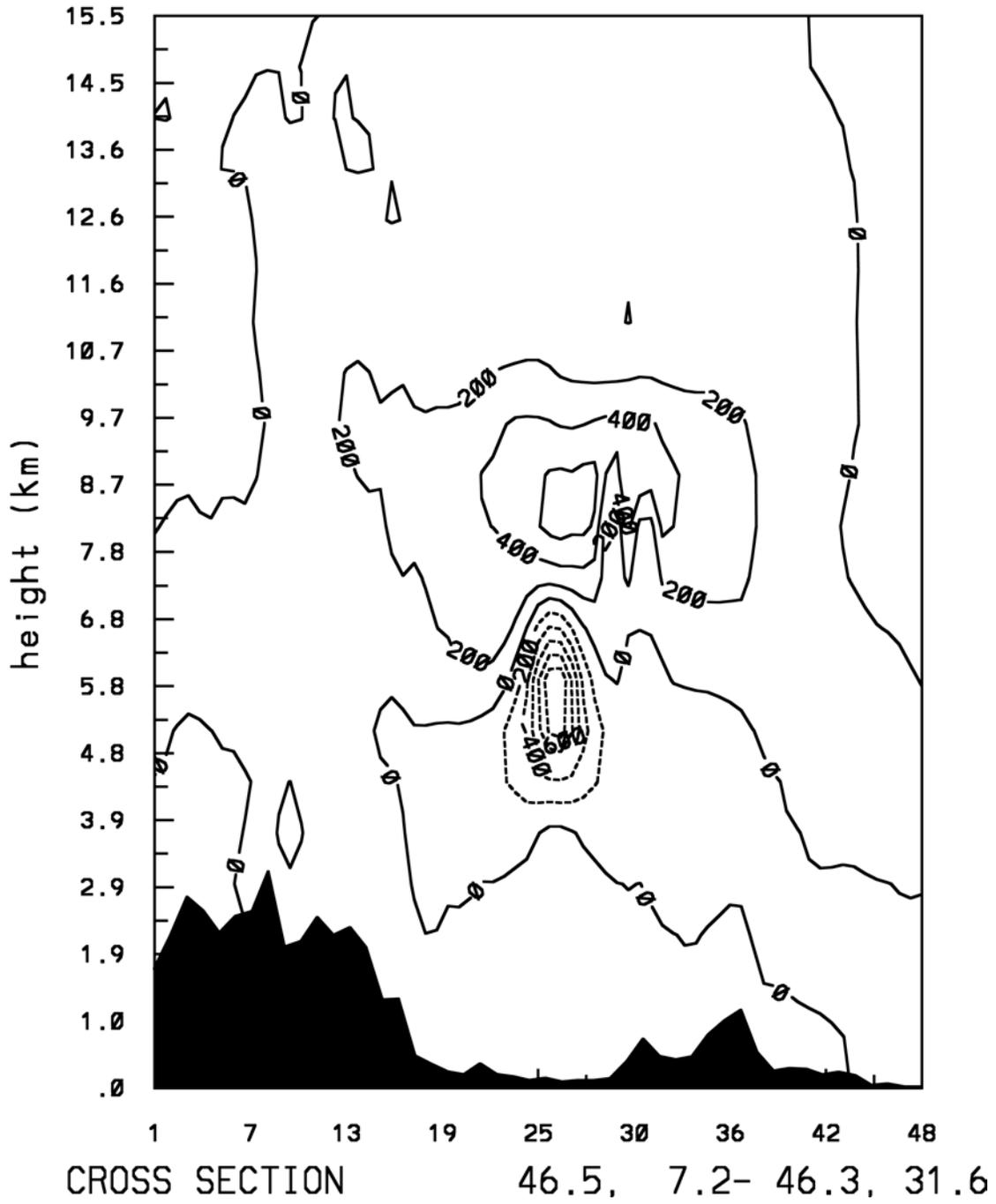


Figure 11. Temperature analysis increments due to temperature observation increment at 500 hPa level derived using lagged NMC statistics. The reduction of positive increments further from the observation is much stronger than reduction of negative ones around the observation point. Asymmetry is also present in lagged statistics and stays at the same level for observation increments at other levels as well.

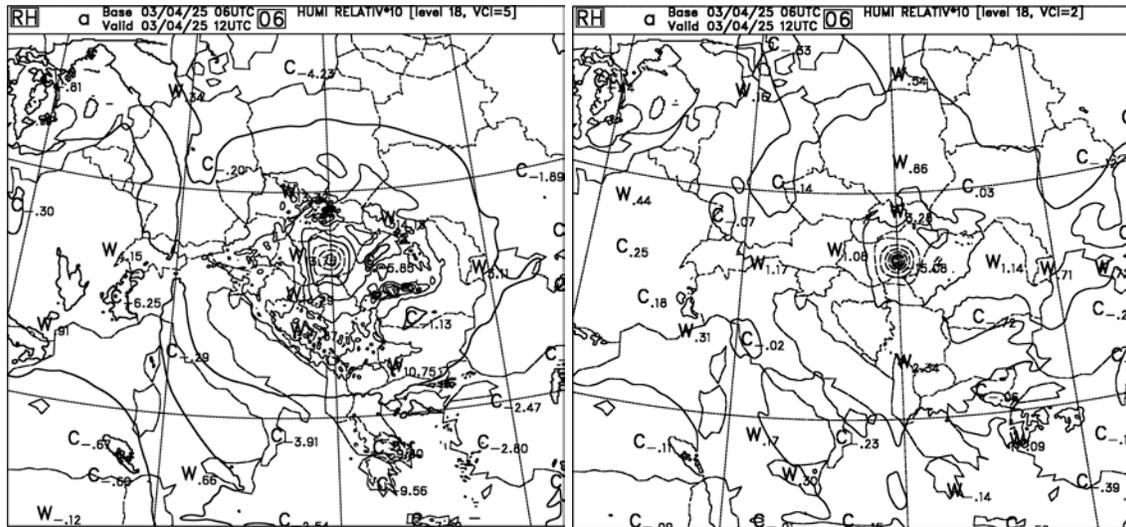


Figure 12. Horizontal relative humidity analysis increments at level 18 (≈ 510 hPa) due to temperature observation increment at 500 hPa level derived using standard (left) and lagged (right) NMC statistics. Besides asymmetry, second unwanted property of relative humidity analysis increments is noisy structure above mountains. This is due probably to the design of relative humidity observation operator. Scaling factor is 10.

DIVERGENCE

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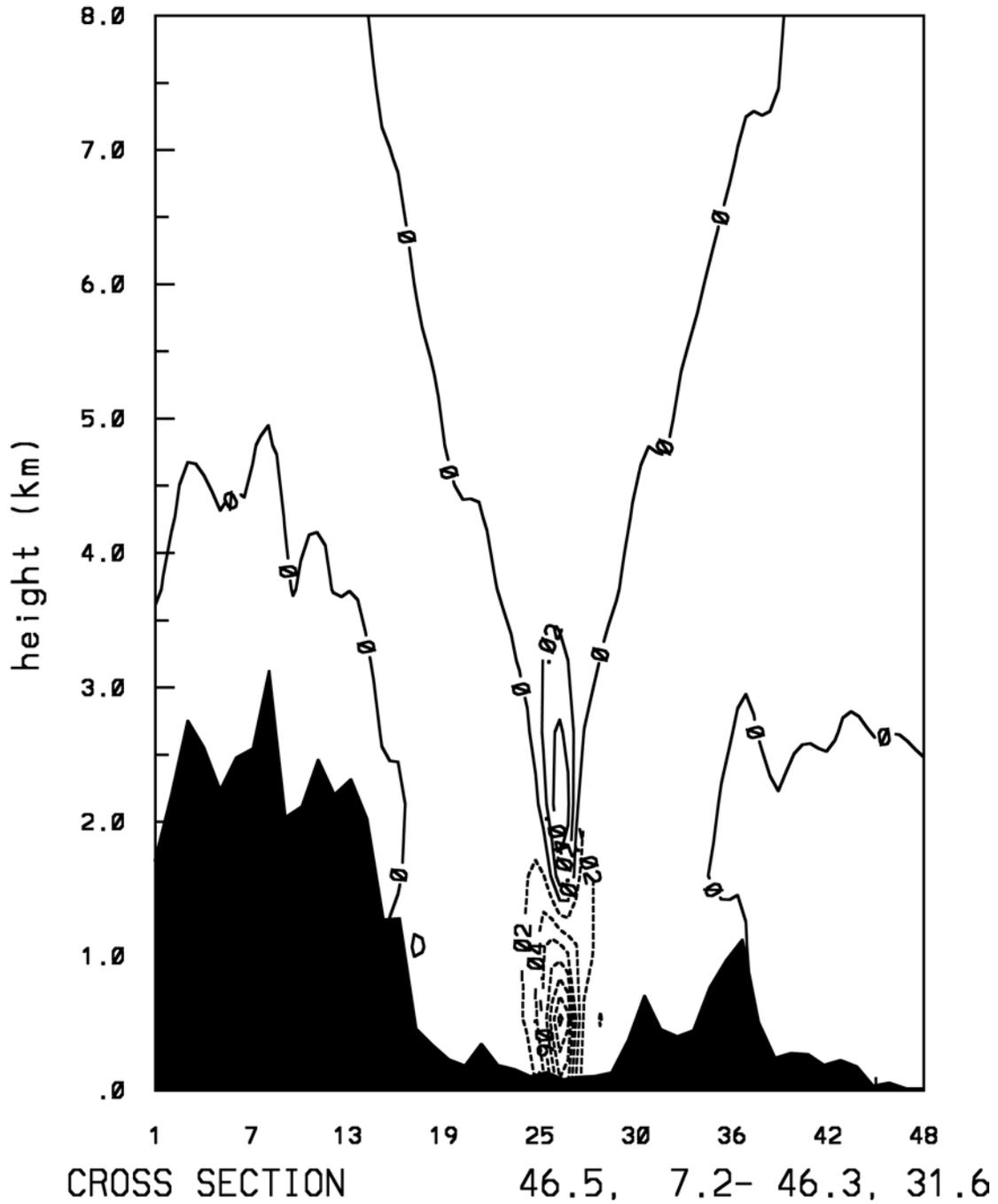


Figure 13. Divergence analysis increments due to specific humidity observation increment at 850 hPa level derived using standard NMC statistics. In the vicinity of observation increment, negative analysis increment reaches 0.14. Scaling factor is 100000.

DIVERGENCE

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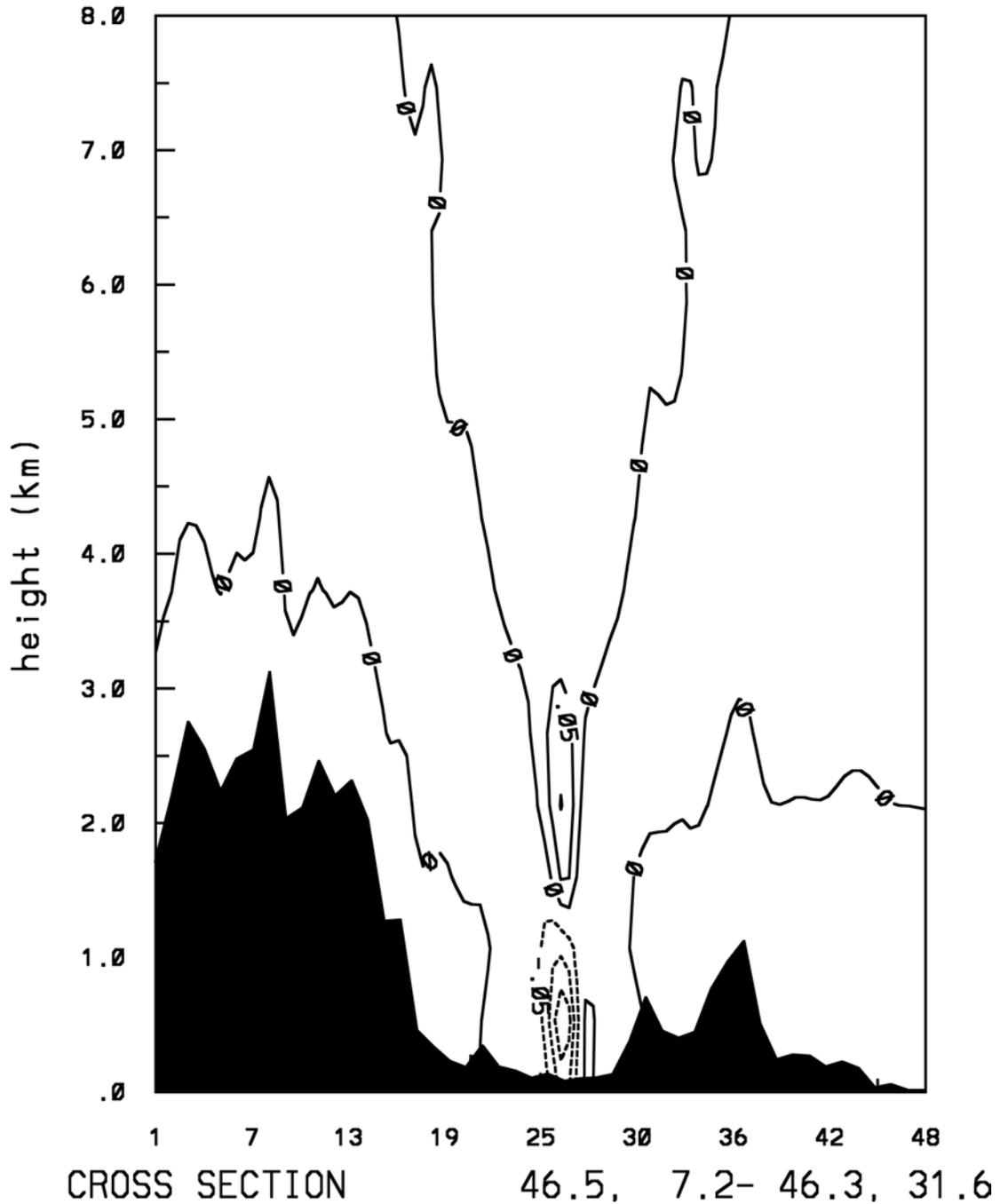
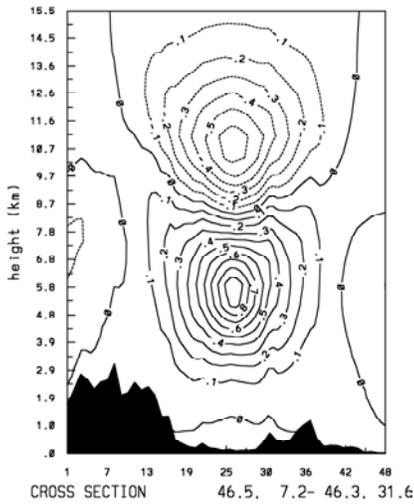
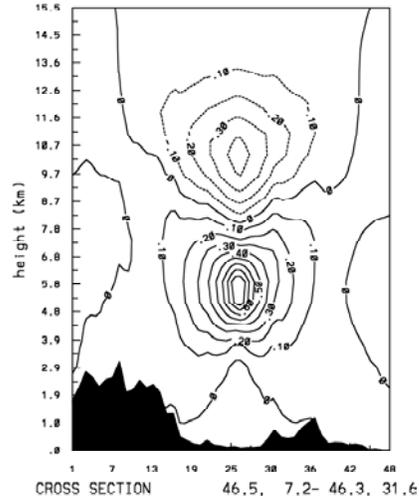


Figure 14. Divergence analysis increments due to specific humidity observation increment at 850 hPa level derived using lagged NMC statistics. In the vicinity of observation increment, negative analysis increments reaches around 0.18. Scaling factor is 100000. Although in general, influence of lagged statistics is reduction, but as presented, in some cases lagged statistics can be responsible for even greater variances.

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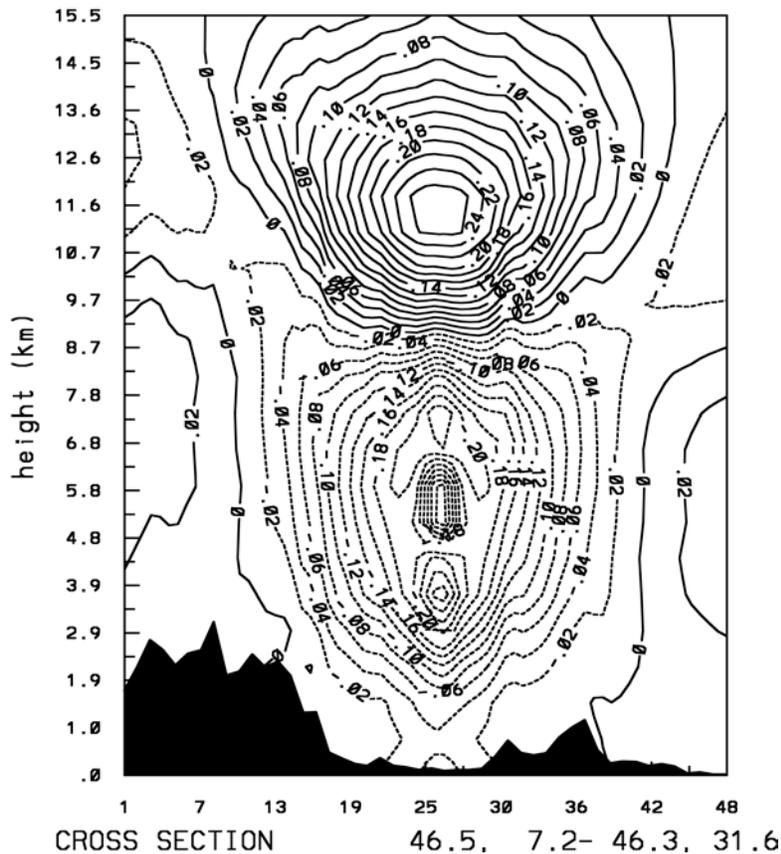


Figure 15-17. Normalised vertical structure for standard and lagged statistics (upper left and right) and their difference, in this case lagged – standard (down). As we approach to the observation point, absolute difference grows to its maximum value and then descents towards zero.

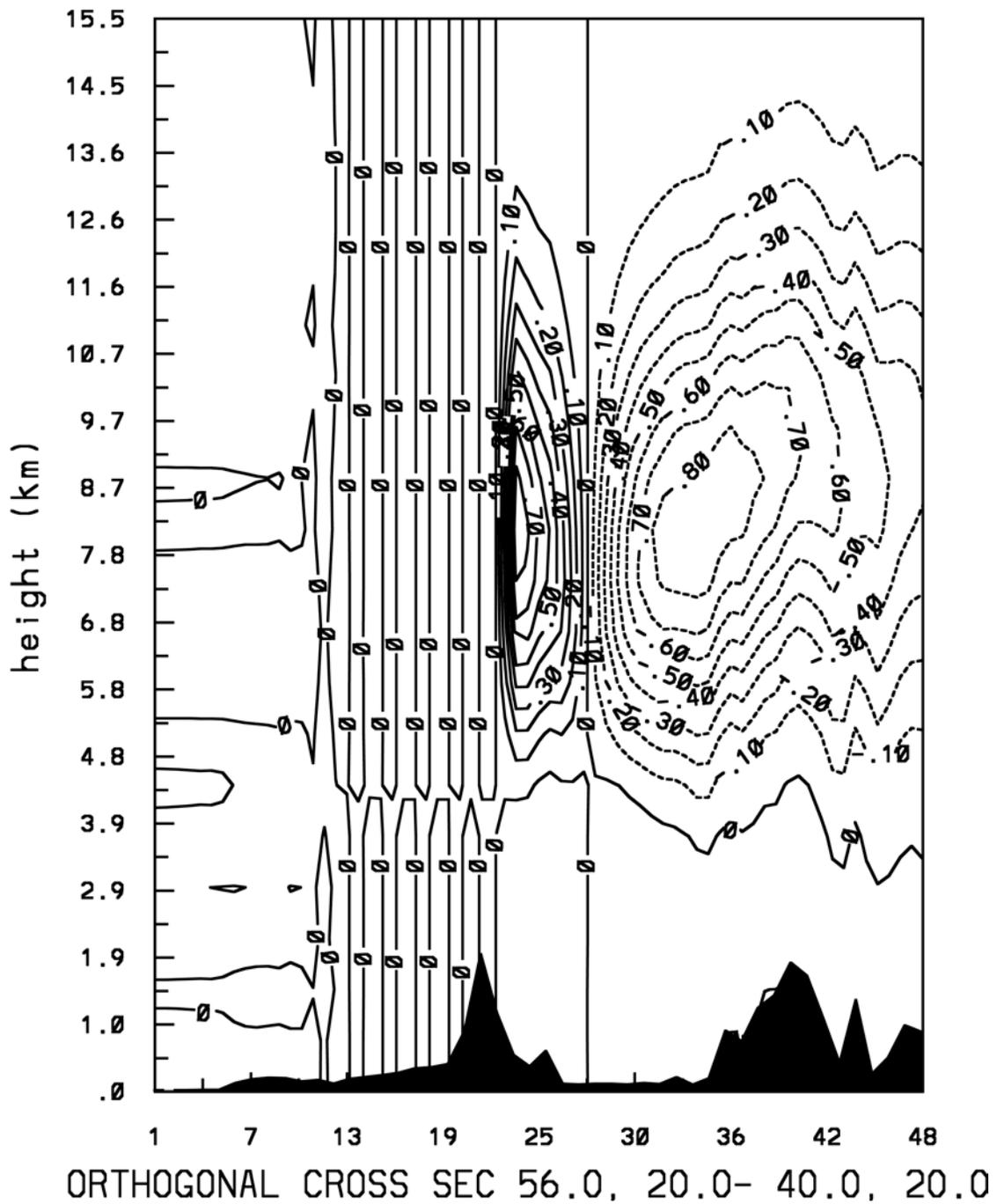


Figure 18. Distorsion of U wind component analysis increments north from $\approx 48^\circ\text{N}$ due to a bug in the system.