Adjustment processes, cascading and protection against negative water species

1 Goals

This part explains the structure of APLPAR for both the L3MT and the LSTRAPRO cases.
This structure includes

- preventing the occurrence and use of negative values for the water phases specific contents;
- implementing the cascade which handles the fair combination of the parametrizations and allows running in the grey zone.

2 General layout

This is given at Figure 1.
The principle of the cascade is to use internal values for the water contents and the temperature, which are updated by the cascaded schemes. These local variables are water vapour $q_v^* \equiv$ ZQV, cloud ice $q_i^* \equiv$ ZQI, cloud droplets $q_\ell^* \equiv$ ZQL, snow $q_s^* \equiv$ ZQS, rain $q_r^* \equiv$ ZQR, temperature $T^* \equiv$ ZT.

3 Implementation details

Notations: below, we note $\delta\psi$ an increment in time and $\triangle\psi^l = \psi^{\bar{l}} - \psi^{\overline{l-1}}$ a downward vertical increment. Fluxes are counted positive downwards. The level indices range from 1 at the top of the atmosphere to L at the bottom. $\bar{l}$ is the lower interface of level l.

3.1 Corrections of negative specific contents

Any time a negative specific content can occur, it must be fixed before further using the variable. This correction made on the internal state must also impact on the evolution of the mean grid box variable.
For each of the 5 phases a correction flux accumulates the corrections which must contribute to the corresponding tendency calculation in CPTEND: arrays PFCQNG, PFCQING, PFCQLNG, PFCQRNG, PFCQSNG.
Negative condensed phases are brought back to zero by ”condensing” some water vapour. Water vapour can also become negative, either after some transport processes or when subtracting the amount necessary to fix the condensed phases.
In this case, the missing vapour is taken in the layers below, and eventually to the surface if a positive value cannot be restored before.
The evolution is based on a diffusion equation:

$$\left(\frac{d\psi}{dt}\right)_{\mathrm{cor}} = -g\frac{\partial J_\psi^{\mathrm{cor}}}{\partial p} \qquad \Longrightarrow \triangle J_\psi^{i\mathrm{cor}} = -\frac{\triangle p^l}{g\delta t}\delta\psi^l$$

In the code, $\frac{\triangle p^l}{g\delta t} \equiv$ ZPOID.

The correction of condensed phases (specific content q_n^*) proceeds as

$$\begin{array}{lr}
\text{Locally updated value after some parametrization :} & q_{n1}^* \\
\text{The fixed final value is} & q_n^* = \max(q_{n1}^*, 0) \\
\Rightarrow \text{The correction} & \mathsf{ZDQn} \equiv \delta q_n' = q_n^* - q_{n1}^* \geq 0 \\
\text{The corrective flux is} & J_n^{\mathrm{cor}\bar{l}} = J_n^{\mathrm{cor}\overline{l-1}} - \dfrac{\triangle p^l}{g\delta t}\delta q_n' \leq 0
\end{array} \qquad (1)$$

where index $_n$ stands for $_i$, $_\ell$, $_r$, or $_s$. The missing water has to be taken from the locally updated water vapour ZQX1 $\equiv q_{v1}$, in addition to the residual corrective flux for the layers above. If the vapour in current layer is not enough for this, a residual corrective flux has to be input at the lower interface of the layer. The goal is to take the missing water as much as possible in the layers directly below the level where it occurs, rather than bringing all the water from the surface.

(Fix negative contents from advection) $\left.\right\} \longmapsto J_\ell^{cor},\ J_i^{cor},\ J_r^{cor},\ J_s^{cor},\ J_v^{cor}$

$[q_s^*, q_r^*, q_i^*, q_\ell^*, q_v^*]^\swarrow$

$f^{st} \longleftarrow$ $\boxed{\text{ACNEBCOND — Resolved cloud fraction}}$

$f^* \leftarrow$ (ACNEBN)

$\downarrow$ (ACNPART) $\rightarrow$ low, med, high, conv clouds

(ACRANEB Radiation) $\rightarrow T_{\text{surf}}$

$\searrow$(ACDIFUS Turb diffusion) $\longrightarrow J_\ell^{td},\ J_i^{td},\ J_v^{td},\ J_S^{td},\ J_V^{td}$

$J_v^{cor\prime} + J_v^{td} \longleftarrow [q_i^*, q_\ell^*, T^*]^\swarrow \longmapsto J_\ell^{cor},\ J_i^{cor},\ J_v^{cor}$

$\downarrow$ $\boxed{\text{ACCDEV — Resolved condensation}}$ $\longrightarrow F_{vi}^{st},\ F_{v\ell}^{st}$

$[q_v^*, q_i^*, q_\ell^*, T^*]^\swarrow$

moisture convergence $\rightarrow$ $\boxed{\text{ACCVUD — Deep convective updraught}}$ $\longrightarrow F_{vi}^{cu},\ F_{v\ell}^{cu},\ J_v^{cu},\ J_i^{cu},\ J_\ell^{cu},\ J_S^{cu},\ J_V^{cu}$

$[q_v^*, q_i^*, q_\ell^*, T^*]^\swarrow \longmapsto J_\ell^{cor},\ J_i^{cor},\ J_v^{cor}$

ACUPU $\rightarrow f^{cu}, \alpha_{\text{cu}} = \dfrac{F_{vc}^{cu}}{F_{vc}^{cu} + F_{vc}^{st}}, f^{eq}$

$\boxed{\text{APLMPHYS — Microphysics}}$ $\longrightarrow F_{is},\ F_{\ell r},\ [F_{\ell i}],\ \mathcal{P}_r,\ \mathcal{P}_s,\ F_{rv},\ F_{sv}$

$F_{hP} \swarrow$ $[q_s^*, q_r^*, q_i^*, q_\ell^*, q_v^*, T^*]^\swarrow \longmapsto J_r^{cor},\ J_s^{cor},\ J_v^{cor}$

ACUPM

$\searrow$ $\boxed{\text{ACMODO — Moist downdraught}}$ $\longrightarrow F_{rv}^{dd},\ F_{sv}^{dd},\ J_v^{cu},\ J_i^{cu},\ J_\ell^{cu},\ J_S^{cu},\ J_V^{cu}$

$\mathcal{P}_r,\ \mathcal{P}_s \swarrow$ $[q_v^*, q_i^*, q_\ell^*]^\swarrow \longmapsto J_i^{cor},\ J_\ell^{cor},\ J_v^{cor}$

ACUPD

$\alpha_{\text{cu}} \longrightarrow$ (Convective/ Resolved) $\longrightarrow \mathcal{P}_r^{cu},\ \mathcal{P}_s^{cu},\ \mathcal{P}_r^{st},\ \mathcal{P}_s^{st}$

Figure 1: General layout

- ZQV0 $\equiv q_{v0}$ = water vapour after removal of the residual corrective fluxes of the layers above:

$$\begin{aligned} q_{v0}^* &= q_{v1}^* - \frac{g\delta t}{\triangle p}\{-J_v^{\mathrm{cor}\prime\overline{l-1}} - J_i^{\mathrm{cor}\prime\overline{l-1}} - J_\ell^{\mathrm{cor}\prime\overline{l-1}} - J_s^{\mathrm{cor}\prime\overline{l-1}} - J_r^{\mathrm{cor}\prime\overline{l-1}}\} \\ &= q_{v1}^* - \frac{g\delta t}{\triangle t}\{-J_v^{\mathrm{cor}\prime\overline{l-1}} - J_c^{\mathrm{cor}\prime\overline{l-1}}\} \end{aligned} \tag{2}$$

- Remove from it the correction for negative precipitation in current layer; if the result is positive, current layer provides all missing water and the final vapour content is the remainder; else the final vapour content is zero:

$$q_v^* = \max(0, q_{v0}^* - \delta q_c') \qquad \text{with} \qquad \delta q_c' = \delta q_i' + \delta q_\ell' + \delta q_s' + \delta q_r' \tag{3}$$

- The residual correction flux at the lower interface is obtained from

$$J_v^{\mathrm{cor}\prime\overline{l}} = J_v^{\mathrm{cor}\prime\overline{l-1}} - \frac{\triangle p}{g\delta t}\delta q_v' \tag{4}$$

  where $\delta q_v' = q_v^* - q_{v1}^*$ is the decrement brought by the fixing procedure to the local water vapour. For instance

  - If $q_v^* = 0$ it means that

$$\delta q_v' = -q_{v1}^*$$

  - If $q_v^* > 0$, no residual correction flux needs to be input at the lower interface: i.e.

$$J_v^{\mathrm{cor}\prime\overline{l}} + J_c^{\mathrm{cor}\prime\overline{l}} = 0 \Rightarrow J_v^{\mathrm{cor}\prime\overline{l}} = -J_c^{\mathrm{cor}\prime\overline{l}}$$

$$\Rightarrow \delta q_v' = -\frac{g\delta t}{\triangle p}\left(-J_c^{\mathrm{cor}\prime\overline{l}} - J_v^{\mathrm{cor}\prime\overline{l-1}}\right) = -\frac{g\delta t}{\triangle p}\left(-J_c^{\mathrm{cor}\prime\overline{l-1}} - J_v^{\mathrm{cor}\prime\overline{l-1}}\right) - \delta q_c' = q_{v0}^* - \delta q_c' - q_{v1}^*$$

  And in both cases we verify

$$\delta q_v' = \max(0, q_{v0}^* - \delta q_c') - q_{v1}^* \tag{5}$$

Remark that this construction makes that the tendency of each species (and also of water vapour) has only to include the corrective flux associated to this same species.

Remark: In the case J_v^{cor} is not zero at the lowest model level, the missing vapour should be brought by surface evaporation: for this we should add this flux to the surface evaporation fluxes used to compute the tendency of surface water.
Actually this is avoided because it can lead to unstable behaviour.

3.1.1 Correction of negative condensates after horizontal advection

Advection does not warrant that the specific contents keep values in their physical range, i.e. between 0 and 1. Since these contents are always much smaller then 1, there is no risk that they become bigger than 1, but well smaller than 0. The correction is adiabatic, since it fixes a mathematical problem and does not correspond to any physical phase change in the reality.
The cloud condensates and precipitation contents are fixed following 1. Hence it is assumed that all missing condensates are take directly from the vapour (rather than taking the missing precipitation from the cloud condensates).
The vapour correction follows Eq. 2, 3, 5, 4.
These first corrections yield the initial values of the water internal variables q_v^*, q_i^*, q_ℓ^*, q_s^*, q_r^*.
The mean grid box specific heat at each level is still in PCP, while ACTQSAT computes mean grid box values of the blue point (PTW, PQW)$\equiv (T_w, q_w)$, the saturation moisture PQSAT$\equiv q_{\mathrm{sat}}$, and relative humidity PRH.
The difference between the total moisture $q_v^* + q_i^* + q_\ell^*$ and the mean grid-box wet bulb moisture PQW is stored in ZQD, for later use by APLMPHYS.

3.2 Cloud fractions

ACNEBCOND computes a stratiform ("resolved") cloud fraction, in one of two ways:

- LXRCDEV: the Xu-Randall (-Chen ?)
- LSMGCDEV using Smith's approach (triangular pdf for total water in the grid box) as implemented by Ph. Lopez with some refinements of L. Gerard.

Initially, it was thought to combine this resolved cloud fraction with the pseudo-historic convective cloud fraction to yield a total cloud fraction to pass to the radiation and the turbulent diffusion schemes.
Presently, ACNEBN computes radiative cloud fractions f^* (total) and f^{cu*} (convective). These are based on condensates, the stratiform ones (advected from previous time step) and the convective ones based on the pseudo-historic convective cloud fraction f^{cu-}.
This radiative cloud fractions are also used by ACNPART to produce the classified cloudiness fields: low, medium, high and convective clouds.
f^* is then passed to the radiation scheme (ACRANEB or other)
The vertical turbulent diffusion ACDIFUS uses, for estimating the diffusion of condensates (passing through "conservative variables") a cloud fraction f_{diff} which can be: $f_{\text{diff}} = 0$ (NDIFFNEB = 0), $f_{\text{diff}} = f^{st}$ (NDIFFNEB = 1) or $f_{\text{diff}} = f^*$ (NDIFFNEB = 2).

3.3 Update of the internal state after vertical turbulent diffusion

Turbulent diffusion may affect the vertical profiles of temperature, water vapour, cloud condensates and momentum. Precipitation contents are not affected.
One updates the internal state using the turbulent diffusion fluxes $J_\psi^{td} \equiv \mathsf{PDIFT}\psi$.

$$\delta T^* = -\frac{g\delta t}{c_p \triangle p}\triangle J_S^{td}, \qquad \delta q_{i1}^* = -\frac{g\delta t}{\triangle p}\triangle J_i^{td}, \qquad \delta q_{\ell 1}^* = -\frac{g\delta t}{\triangle p}\triangle J_\ell^{td}, \qquad \delta q_{v1}^* = -\frac{g\delta t}{\triangle p}\triangle J_v^{td}$$

Horizontal momentum is not updated, since it would have a negligible impact.
The protection against negative cloud condensates is applied following equations 1. The corrections are cumulated in local arrays, which are then added to the output corrective fluxes.
The missing cloud condensates have to been taken from vapour, and the corrective flux of vapour is obtained (in a local array ZFQVNG) with Eq. 4. However the water vapour q_v^* itself is not updated, because we use J_v^{td} (and the present correction, i.e. $J_v^{td} + J_v^{cor\prime}$) in the closure of the updraught.

Remarks:

- *As mentioned in section 3.1, the* PFQ*NG *only affect the tendencies of water variables, not the heat.*
  Assuming here an adiabatic correction is justified if the negative values are associated to algorithmic formulation, less if they are associated to physics. But physically, a diffusion scheme based on local gradients is unlikely to induce negative values.
- *Using 2-D arrays for the increments of the 4 condensates is a waste of memory.*

Remark: in the case LSTRAPRO (i.e. not L3MT), correction fluxes computed as well, but there is no update of the internal state, which stays all along equal to the initial state.

3.4 Update after condensation/evaporation

The routine ACCDEV computes the resolved condensation/evaporation fluxes F_{vi}^{st}, $F_{v\ell}^{st}$. These are obtained with the "ACPLUIE_PROG" scheme (LXRCDEV) or with the Smith-Gerard scheme (LSMGCDEV.
In the case LSTRAPRO, updated internal values of the cloud condensates (q_i', q_ℓ') and the water vapourq_v') are passed to the microphysics routine APLMPHYS, together with the unmodified mean grid box temperature $\overline{T}$.
In this case, ACCDEV also outputs values of precipitation, precipitation generation and evaporation fluxes.

In the case L3MT, the precipitation (and precipitation-associated) fluxes are not affected (left to zero) at this stage. In APLPAR, the condensation fluxes are used to update the condensates, water vapour as well as the temperature:

$$\delta q_i^* = \frac{g\delta t}{\triangle p}\triangle F_{vi}^{st}, \qquad \delta q_\ell^* = \frac{g\delta t}{\triangle p}\triangle F_{v\ell}^{st}$$

$$\delta q_v^* = -\delta q_i^* - \delta q_\ell^*, \qquad \delta T^* = \big(L_{vi}\delta q_i^* + L_{v\ell}\delta q_\ell^*\big)/c_p$$

Presently, the latent heats L_{vi}, $L_{v\ell}$ are those calculated with the initial temperature $\overline{T}$, and c_p with the initial moisture $\overline{q}$.
The implementation of the condensation scheme prevents the occurrence of negative (water vapour) specific content at this stage.

3.5 Input for accvud

The updraught closure uses the moisture convergence, calculated as:

$$\mathsf{CVGQ} = -\overline{\mathbf{V}} \cdot \nabla \overline{q_v} - \overline{\omega}\frac{\partial \overline{q_v}}{\partial p} - \frac{g}{\triangle p}\big(\triangle J_v^{td} + \triangle \mathsf{ZFQVNG}\big)$$

(The three-dimensional divergence has actually been computed by CPPHINP call by MF_PHYS before APLPAR). Beware that under L3MT one *must* set GCOMOD=0.

A call to ACTQSAT yields updated values of $T_w^* = \mathsf{ZTW}$, $q_w^* = \mathsf{ZQW}$, $q_{\text{sat}}^* = \mathsf{ZQSAT}$, $\phi_{slc}^* = \mathsf{ZGEOSLC}$ (the geopotential for slanted convection).
These values are passed to the updraught routine ACCVUD.

3.6 Updates after updraught

The internal state is updated following the updraught condensation and transport; additional corrections against negative contents are brought subsequently.

$$\delta q_i^* = -\frac{g\delta t}{\triangle p}\big\{\triangle F_{vi}^{cu} - \triangle J_i^{cu}\big\} \qquad \delta q_\ell^* = -\frac{g\delta t}{\triangle p}\big\{\triangle F_{v\ell}^{cu} - \triangle J_\ell^{cu}\big\}$$

$$\delta q_v^* = -\frac{g\delta t}{\triangle p}\big\{-\triangle F_{vi}^{cu} - \triangle F_{v\ell}^{cu} - \triangle J_v^{cu}\big\} \qquad \delta T^* = -\frac{g\delta t}{c_p\triangle p}\big\{L_{vi}\triangle F_{vi}^{cu} + L_{v\ell}\triangle F_{v\ell}^{cu} - \triangle J_S^{cu}\big\}$$

Presently the updraught does not transport precipitation. Remark that the condensation fluxes F_{vi}, $F_{v\ell}$ are positive and increase downwards, while the transport fluxes are oriented upwards, i.e. negative.
The latent heats $L_{v\ell}$, L_{vi} are taken at the mean grid box temperature $\overline{T}$, and the specific heat c_p at the mean grid box moisture $\overline{q_v}$.
The corrections against negative specific moisture are applied as described in section 3.1 (but with $\delta q_r' = 0 = \delta q_s'$).
Local corrections are cumulated in local arrays $J_n^{\text{cor}\prime}$, which are added to the output corrective fluxes J_n^{cor}.

Actually taking the missing condensates from the water vapour represents an additional condensation: a more consistent approach would be to simply add $J_i^{\text{cor}\prime} + J_\ell^{\text{cor}\prime}$ to the convective condensation fluxes, so that the correction would also impact on the temperature.

3.7 Input to microphysics and subsequent routines

After the update of the internal state, subroutine ACUPU treats the updraught environment properties to pass to the subsequent routines.

- The environment vertical velocity is different from the mean grid box velocity:

$$\mathsf{ZOME} \equiv \omega_e\delta t = \overline{\omega}\delta t - \sigma_u\omega_u^*\delta t$$

  where $\mathsf{PUDOM} = \omega_u^*\delta t$ is the updraught relative vertical velocity.

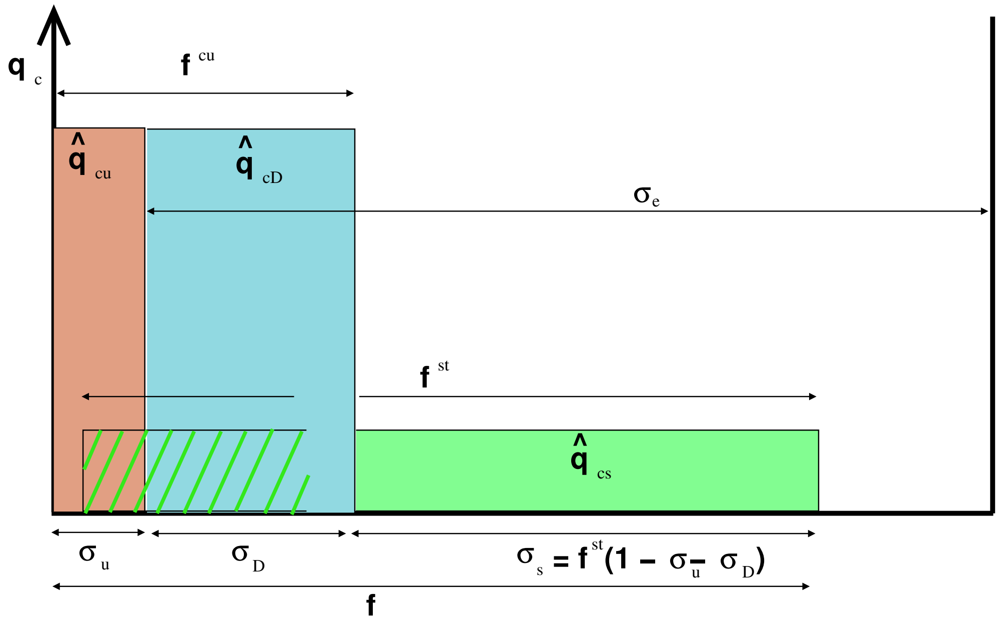

Figure 2: Grid box subareas.

- The fraction of the grid box area occupied by detrained material is limited to the available space out of the updraught:

$$\text{ZSIGDE} \equiv \sigma_D = \min(\sigma'_D, 1 - \sigma_u)$$

  where σ'_D is the detrainment fraction returned by ACCVUD, assuming the same condensate contents in the detrainment area and in the updraught: $q'_{cD} = q_{cu}$.

- An equivalent mesh fraction f^{eq} is computed, to be used in the microphysics for estimating the intensive condensate specific contents.

- Under LUDEN=T, the internal state is assumed to represent the updraught environment instead of the mean grid box. Then the microphysics and the downdraught are compute is this fraction $(1 - \sigma_u)$ of the grid box, and all the fluxes will have to be brought back to the mean grid box afterwards, in subroutine ACUPD.

The convective fraction of the condensates is estimated as

$$\text{ZFRCO} \equiv \alpha_c = \frac{F^{cu}_{vc}}{F^{cu}_{vc} + F^{st}_{vc}}$$

where F_{vc} are the condensation fluxes, The value of this ratio at the lowest model level is output in ZSIGPC, used in APLPAR to partition the total precipitation between convective and stratiform contributions.
Noting $q_c = q_i + q_\ell$ the total cloud condensate, $\overline{\psi}$ the mean grid box value of ψ, $\widehat{\psi}$ its intensive in-cloud value, the mean grid box values of condensate (total, convective and stratiform) are

$$\overline{q_c} = \overline{q_{cc}} + \overline{q_{cs}} \qquad \overline{q_{cc}} = \alpha_c \overline{q_c} \qquad \overline{q_{cs}} = (1 - \alpha_c)\overline{q_c}$$

while intensive values are

$$\widehat{q_{cs}} = \frac{\overline{q_{cs}}}{f^{st}} \qquad \widehat{q_{cc}} = \frac{\overline{q_{cc}}}{f^{cu}}$$

The convective f^{cu} and stratiform f^{st} cloud fraction can overlap, the total cloud being given by $f = f^{cu} + f^{st} - f^{cu} f^{st}$. The "resolved" condensate is assumed to be homogeneously distributed over a fraction f^{st} of the grid box, so that a fraction $f^{st} f^{cu} \widehat{q_{cs}} = f^{cu} \overline{q_{cs}}$ contributes to the intensive condensates in the updraught and detrainment area. The purely convective condensate is then

$$\overline{q_{cc}} = \sigma_u(\widehat{q_{cu}} - \overline{q_{cs}}) + \sigma_D(\widehat{q_{cD}} - \overline{q_{cs}})$$

We can consider the following separated fractions of the grid box area each with different intensive contents:

$$\widehat{q_{cu}} = \frac{\alpha_c + (\sigma_u + \sigma_D)(1-\alpha_c)}{\sigma_u + \sigma'_D}\overline{q_c} \qquad \text{over updraught area } \sigma_u$$

$$\widehat{q_{cD}} = \frac{\sigma'_d}{\sigma_D}\widehat{q_{cu}} \qquad \text{over detrainment area } \sigma_D$$

$$\widehat{q_{cs}} = \frac{(1-\alpha_c)}{f^{st}}\overline{q_c} \qquad \text{over purely stratiform area } \sigma_{st} = f^{st}(1-\sigma_u-\sigma_D)$$

The microphysics calculation works with on single value of intensive condensates. If we assume that the mean grid box condensate $\overline{q_c}$ is homogeneously distributed over the total cloud fraction f, we introduce a bias, because the peak values in the convective cloud will be smoothed, and the precipitation will be underestimated. On the other hand, we cannot assume that all the condensate is only over f^{cu}. For this reason, we are seeking an intermediate "equivalent situation" where the grid box condensate would be distributed on an area smaller than f, but bigger than f^{cu}.

3.7.1 Case LUDEN=F

This cases assumes that microphysics and downdraughts can occur over the whole grid box, hence with a small contradiction to the hypothesis that there is no precipitation inside the updraught.
First we define separate fractions f'_{cu} and f'_{st} such that $f = f'_{st} + f'_{cu}$:

$$f'_{cu} = f^{cu}\frac{f}{f^{st}+f^{cu}} \qquad f'_{st} = f^{st}\frac{f}{f^{st}+f^{cu}}$$

Corresponding intensive values are

$$\widehat{q'_{cc}} = \frac{\overline{q_{cc}}}{f'_{cu}} = \frac{\overline{q_c}\alpha_c}{f'_{cu}} \qquad \widehat{q'_{cs}} = \frac{\overline{q_{cs}}}{f'_{st}} = \frac{\overline{q_c}(1-\alpha_c)}{f'_{st}}$$

One writes that "equivalent intensive condensate" is a weighted sum of the intensive convective and stratiform values, the weights being their respective proportions:

$$\widehat{q_{ce}} = \frac{\overline{q_c}}{f^{eq}} = \widehat{q'_{cs}}(1-\alpha_c) + \widehat{q'_{cc}}\alpha_c = \{\frac{(1-\alpha_c)^2}{f'_{st}} + \frac{\alpha_c^2}{f'_{cu}}\}\overline{q_c}$$

which yields the "equivalent cloud fraction" f^{eq}.

$$\frac{1}{f^{eq}} = \{\frac{(1-\alpha_c)^2}{f^{st}} + \frac{\alpha_c^2}{f^{cu}}\}\frac{f^{st}+f^{cu}}{f} \tag{6a}$$

An alternative would be to interpolate the cloud fractions instead of the condensates, for instance

$$f^{eq} = f^{cu} + (1-\alpha_c)\sigma_{st} = f^{cu} + (1-\alpha_c)(1-\sigma_u-\sigma_D)f^{st} \tag{6b}$$

3.7.2 Case LUDEN=T

The goal is to handle correctly the case where the updraught (assumed with no precipitation) covers a significant fraction of the grid box. The microphysics and the downdraught are then computed on the remaining part, $(1-\sigma_u)$. Noting $\widetilde{\psi}$ the mean value of ψ over $(1-\sigma_u)$:

$$\widetilde{\psi} = \frac{\overline{\psi} - \sigma_u\psi_u}{1-\sigma_u}$$

where $_u$ stands for values in the updraught, $_e$ in the environment.
The stratiform condensates are

$$\overline{q_{\ell s}} = (1-\alpha_c)\overline{q_\ell} \qquad \overline{q_{is}} = (1-\alpha_c)\overline{q_i} \qquad \widehat{q_{\ell s}} = \frac{\overline{q_{\ell s}}}{f^{st}} \qquad \widehat{q_{is}} = \frac{\overline{q_{is}}}{f^{st}}$$

and the intensive values in the updraught and the detrainment areas are

$$\widehat{q_{iu}} = \frac{\alpha_c\overline{q_i}}{\sigma_u+\sigma'_D} + \overline{q_{is}} \qquad \widehat{q_{\ell u}} = \frac{\alpha_c\overline{q_\ell}}{\sigma_u+\sigma'_D} + \overline{q_{\ell s}} \qquad \widehat{q_{iD}} = \frac{\sigma'_D}{\sigma_D}\widehat{q_{iu}} \qquad \widehat{q_{\ell D}} = \frac{\sigma'_D}{\sigma_D}\widehat{q_{\ell u}}$$

The convective fraction in the updraught environment is then

$$\widetilde{\alpha_c} = \frac{\sigma_D(\widehat{q_{cD}} - \widehat{q_{cs}} f^{st})}{\sigma_D \widehat{q_{cD}} + (1-\sigma_u-\sigma_D)\widehat{q_{cs}} f^{st}} = \frac{\sigma_D(\widehat{q_{cD}} - \overline{q_{cs}})}{\sigma_D(\widehat{q_{cD}} - \overline{q_{cs}}) + (1-\sigma_u)\overline{q_{cs}}}$$

Replacing to eliminate the condensates yields

$$\widetilde{\alpha_c} = \frac{\alpha_c \frac{\sigma'_D}{\sigma'_D+\sigma_u} + (1-\alpha_c)(\sigma'_D - \sigma_D)}{\alpha_c \frac{\sigma'_D}{\sigma'_D+\sigma_u} + (1-\alpha_c)(1-\sigma_u+\sigma'_D-\sigma_D)} \tag{7}$$

The equivalent mesh fraction can be estimated

- by interpolating the condensates in the updraught environment:

$$\widehat{\widehat{q_{ce}}} = \frac{\widetilde{q_c}}{f^{eq}} = \frac{\overline{q_c}(1-\sigma_u)}{f^{eq}} = \widehat{\widehat{q_{cs}}}(1-\widetilde{\alpha_c}) + \widehat{\widehat{q_{cc}}}\widetilde{\alpha_c} = \widehat{q_{cs}}(1-\widetilde{\alpha_c}) + \widehat{q_{cD}}\widetilde{\alpha_c} \quad \Rightarrow f^{eq} = \frac{\overline{q_c}(1-\sigma_u)}{\widehat{q_{cs}}(1-\widetilde{\alpha_c}) + \widehat{q_{cD}}\widetilde{\alpha_c}}$$

  Eliminating the condensate yields:

$$f^{eq} = \frac{(1-\sigma_u)}{\frac{(1-\widetilde{\alpha_c})(1-\alpha_c)}{f^{st}} + \widetilde{\alpha_c}\frac{\sigma'_D}{\sigma_D}\left(1-\alpha_c+\frac{\alpha_c}{\sigma_u+\sigma'_D}\right)} \tag{8a}$$

- by interpolating the cloud fractions:

$$f^{eq} = \sigma_D + (1-\widetilde{\alpha_c})\sigma_{st} \tag{8b}$$

In addition, with LUDEN=T we make all the subsequent calculations over the fraction $\sigma_e = 1-\sigma_u$ of the grid box. For this we put in the internal variables the properties in this area:

$$\widetilde{q_i^*} = \frac{\overline{q_i^*} - \sigma_u q_{iu}}{1-\sigma_u} \qquad \widetilde{q_\ell^*} = \frac{\overline{q_\ell^*} - \sigma_u q_{\ell u}}{1-\sigma_u} \qquad \widetilde{q_v^*} = \frac{\overline{q_v^*} - \sigma_u q_{vu}}{1-\sigma_u}$$

$$\widetilde{T^*} = \frac{\overline{T^*} - \sigma_u T_u}{1-\sigma_u} \qquad \widetilde{\mathbf{V}^*} = \frac{\overline{\mathbf{V}^*} - \sigma_u \mathbf{V}_u}{1-\sigma_u}$$

The fluxes which will be modified by the microphysics and the downdraught must also be referred to this area:

$$\widetilde{J_S^{cu}} = \frac{\overline{J_S^{cu}}}{1-\sigma_u} \qquad \widetilde{J_v^{cu}} = \frac{\overline{J_v^{cu}}}{1-\sigma_u}$$

$$\widetilde{J_i^{cu}} = \frac{\overline{J_i^{cu}}}{1-\sigma_u} \qquad \widetilde{J_\ell^{cu}} = \frac{\overline{J_\ell^{cu}}}{1-\sigma_u} \qquad \widetilde{J_V^{cu}} = \frac{\overline{J_V^{cu}}}{1-\sigma_u}$$

$$\widetilde{F_{v\ell}^{cu}} = \frac{\overline{F_{v\ell}^{cu}}}{1-\sigma_u} \qquad \widetilde{F_{vi}^{cu}} = \frac{\overline{F_{vi}^{cu}}}{1-\sigma_u}$$

$$\widetilde{F_{v\ell}^{st}} = \frac{\overline{F_{v\ell}^{st}}}{1-\sigma_u} \qquad \widetilde{F_{vi}^{st}} = \frac{\overline{F_{vi}^{st}}}{1-\sigma_u}$$

$$\widetilde{J_v^{\rm cor}} = \frac{\overline{J_v^{\rm cor}}}{1-\sigma_u} \qquad \widetilde{J_i^{\rm cor}} = \frac{\overline{J_i^{\rm cor}}}{1-\sigma_u} \qquad \widetilde{J_\ell^{\rm cor}} = \frac{\overline{J_\ell^{\rm cor}}}{1-\sigma_u}$$

$$\widetilde{J_r^{\rm cor}} = \frac{\overline{J_r^{\rm cor}}}{1-\sigma_u} \qquad \widetilde{J_s^{\rm cor}} = \frac{\overline{J_s^{\rm cor}}}{1-\sigma_u}$$

This is nothing more than a mathematical trick: for instance it does not mean that the convective transport flux would be actually concentrated over σ_e. In the downdraught we will add to it a contribution occurring completely in σ_e and referred to this area: so if we concentrate now the flux on σ_e, the combination is much simpler, we can add contributions referring to σ_e and re-dilute the sum over the mean grid box (see ACUPD).

3.8 Updates after microphysics

The microphysics produces fluxes of precipitation, precipitation generation (including auto-conversion and collection processes) and precipitation evaporation. In addition, it has modified total condensation fluxes following transfers between solid and liquid phases. The respective increments are found back by difference between the condensation fluxes after F^+_{vi}, $F^+_{v\ell}$ and before microphysics F^-_{vi}, $F^-_{v\ell}$.

$$\triangle F^{\mathrm{mic}}_{vi} = \triangle F^+_{vi} - \triangle F^-_{vi} \qquad \triangle F^{\mathrm{mic}}_{v\ell} = \triangle F^+_{v\ell} - \triangle F^-_{v\ell}$$

These microphysics increments are cumulated into arrays ZFCQLDM, ZFCQIDM.
For water variables, the local updates are written:

$$\delta q^*_r = -\frac{g\delta t}{\triangle p}\{-\triangle F_{Pr} + \triangle \mathcal{P}_r + \triangle F_{rv}\} \qquad \delta q^*_s = -\frac{g\delta t}{\triangle p}\{-\triangle F_{Ps} + \triangle \mathcal{P}_s + \triangle F_{sv}\}$$

$$\delta q^*_\ell = -\frac{g\delta t}{\triangle p}\{\triangle F_{Pr} - \triangle F^{\mathrm{mic}}_{v\ell}\} \qquad \delta q^*_i = -\frac{g\delta t}{\triangle p}\{\triangle F_{Ps} - \triangle F^{\mathrm{mic}}_{vi}\}$$

$$\delta q^*_v = q^*_{v1} - q^*_v = -\frac{g\delta t}{\triangle t}\{-\triangle F_{rv} - \triangle F_{sv} + \triangle F^{\mathrm{mic}}_{vi} + \triangle F^{\mathrm{mic}}_{v\ell}\}$$

It is assumed here that (by construction) no negative cloud condensate results from the microphysics.
Negative precipitation can occur; the missing water has to be taken from *another phase* – which logically should be the cloud condensates, but presently is directly the water vapour. The corrections follow equations 1 (for rain and snow) and 2, 3, 5, 4 for water vapour.
Local corrective increments are accumulated in local arrays $J^{\mathrm{cond}\prime}_n$ which are added to the total corrective fluxes J^{cond}_n.

Again, the corrections are taken adiabatic. A more consistent approach would be to take the missing precipitation form the local cloud condensates (i.e. at the same level), then the missing cloud condensates form the vapour while adding $J^{\mathrm{cor}\prime}_i$ + $J^{\mathrm{cor}\prime}_\ell$ to the resolved condensation fluxes instead of the total corrective fluxes. The calculation of the vapour corrective fluxes would not change.

3.9 Input for ACMODO

These are prepared by subroutine ACUPM.
The closure of the downdraught is based on the cooling associated to the precipitation flux. This is represented by the precipitation enthalpy flux F_{hP}, normally computed by CPFHPFS. This includes three phenomena:

- precipitation evaporation;
- precipitation melting;
- precipitation taking the temperature of the layers it crosses.

Updated values, of $c_p(q^*_v)$ for the latter effect and of the latent heats $L_{vi}(T)$, $L_{v\ell}(T)$ for the two others, would be preferable. This has not yet been implemented, and *it has been decided* to neglect here the cooling by bringing the precipitation to the local temperature. A precipitation heat flux is thus estimated here as

$$\triangle F_{hP} = L_{vi}(\overline{T})\triangle F_{sv} + L_{v\ell}(\overline{T})\triangle F_{rv}$$

which includes the evaporation and melting effects.
Since this cooling is used in the closure of the downdraught, we do not make it contribute to the temperature profile input to it. But actually the downdraught only uses a fraction GDDEVF of $\triangle F_{hP}$ for its closure. If LCDDEVPRO=T the remaining par (1-GDDEVF) still contributes to the temperature profile input to the downdraught:

$$\delta T^* = -(1 - \mathsf{GDDEVF})\frac{g\delta t}{c_p\triangle p}\triangle F_{hP}$$

3.10 Updates after downdraught

ACMODO outputs separate transport fluxes (ZDIFCQ*D, ZDIFCSD, ZSTRC*D) which must be added to the already existing ones:

$$J^{cu}_v = J^{cu}_v + J^{dd}_v \qquad J^{cu}_i = J^{cu}_i + J^{dd}_i \qquad J^{cu}_\ell = J^{cu}_\ell + J^{dd}_\ell \qquad J^{cu}_{\mathbf{V}} = J^{cu}_{\mathbf{V}} + J^{dd}_{\mathbf{V}}$$

Presently we do not include a transport of the precipitation q_r^*, q_s^* by the downdraught.
ACMODO also outputs precipitation evaporation fluxes (PFPEVPCL $\equiv F_{rv}^{dd}$, PFPEVPCN $\equiv F_{sv}^{dd}$) which reduce the precipitation flux:

$$\mathcal{P}_r = \mathcal{P}_r - F_{rv}^{dd} \qquad\qquad \mathcal{P}_s = \mathcal{P}_s - F_{rv}^{dd}$$

This last update should actually be moved to ACUPD, to allow a delayed update in relation to sedimentation time (section 3.11.1).
The internal state is updated in order to check against negative contents.

$$\delta q_i^* = -\frac{g\delta t}{\triangle p}\triangle J_i^{dd} \qquad\qquad \delta q_\ell^* = -\frac{g\delta t}{\triangle p}\triangle J_\ell^{dd}$$

$$\delta q_v^* = -\frac{g\delta t}{\triangle p}\{\triangle J_v^{dd} - \triangle F_{sv}^{dd} - \triangle F_{rv}^{dd}\}$$

There is presently no update of the precipitation contents which paradoxically have not been changed by the downdraught (see 3.11.1). The corrections are applied as described in section 3.1.

3.11 Final calculations

3.11.1 ACUPD

Subroutine ACUPD can

- bring back all fluxes to the mean grid box in the case they were computed for the updraught environment (LUDEN=T);

$$\begin{array}{llll}
\overline{J_S^{cu}} = \widetilde{J_S^{cu}}(1-\sigma_u) & \overline{J_v^{cu}} = \widetilde{J_v^{cu}}(1-\sigma_u) & & \\
\overline{J_i^{cu}} = \widetilde{J_i^{cu}}(1-\sigma_u) & \overline{J_\ell^{cu}} = \widetilde{J_\ell^{cu}}(1-\sigma_u) & \overline{J_V^{cu}} = \widetilde{J_V^{cu}}(1-\sigma_u) & \\
\overline{F_{v\ell}^{cu}} = \widetilde{F_{v\ell}^{cu}}(1-\sigma_u) & \overline{F_{vi}^{cu}} = \widetilde{F_{vi}^{cu}}(1-\sigma_u) & & \\
\overline{F_{v\ell}^{st}} = \widetilde{F_{v\ell}^{st}}(1-\sigma_u) & \overline{F_{vi}^{st}} = \widetilde{F_{vi}^{st}}(1-\sigma_u) & & \\
\overline{J_v^{\mathrm{cor}}} = \widetilde{J_v^{\mathrm{cor}}}(1-\sigma_u) & \overline{J_i^{\mathrm{cor}}} = \widetilde{J_i^{\mathrm{cor}}}(1-\sigma_u) & \overline{J_\ell^{\mathrm{cor}}} = \widetilde{J_\ell^{\mathrm{cor}}}(1-\sigma_u) & \\
\overline{J_r^{\mathrm{cor}}} = \widetilde{J_r^{\mathrm{cor}}}(1-\sigma_u) & \overline{J_s^{\mathrm{cor}}} = \widetilde{J_s^{\mathrm{cor}}}(1-\sigma_u) & & \\
\overline{F_{Pr}} = \widetilde{F_{Pr}}(1-\sigma_u) & \overline{F_{Ps}} = \widetilde{F_{Ps}}(1-\sigma_u) & \overline{\mathcal{P}_r} = \widetilde{\mathcal{P}_r}(1-\sigma_u) & \overline{\mathcal{P}_s} = \widetilde{\mathcal{P}_s}(1-\sigma_u) \\
\overline{F_{rv}} = \widetilde{F_{rv}}(1-\sigma_u) & \overline{F_{sv}} = \widetilde{F_{sv}}(1-\sigma_u) & \overline{F_{rv}^{dd}} = \widetilde{F_{rv}^{dd}}(1-\sigma_u) & \overline{F_{sv}^{dd}} = \widetilde{F_{sv}^{dd}}(1-\sigma_u)
\end{array}$$

- Handle the combination of downdraught evaporation and precipitation fluxes and contents If the downdraught evaporation fluxes are simply added to the precipitation fluxes, we forget completely the sedimentation problem which would normally induce a delay or vertical offset between the occurrence of evaporation and the decrease of precipitation. Moreover, the decrease of precipitation by downdraught evaporation should imply modification of the collection processes and the entire sedimentation calculation of the microphysics should be redone, which on its turn would affect the downdraught input and so on.

  We have different possibilities:

  - We simply add the evaporation to precipitation:

    $$\mathcal{P}_r \to \mathcal{P}_r - F_{rv}^{dd} \qquad F_{rv} \to F_{rv} + F_{rv}^{dd} \qquad \frac{\delta q_r}{\delta t} = -g\big(\frac{\triangle F_{rv}}{\triangle p} + \frac{\triangle \mathcal{P}_r}{\triangle p}\big)$$

    which implies that q_r would not be modified by downdraught evaporation.
    However we can make the downdraught activity interfere in the microphysics calculation at the next time step, through a modification sedimentation velocity and/or an increase of evaporation.
  - To take the delay / vertical offset into account, we could use the P_3 pdf of the microphysics calculation, which accounts for the impact of production/destruction in a layer on the evolution of the precipitation at its lower interface:

    $$F_{rv} \to F_{rv} + F_{rv}^{dd} \qquad \mathcal{P}_r^{\bar{l}} \to \mathcal{P}_r^{\bar{l}} - \{P_3 F_{rv}^{dd^{\bar{l}}} + (1-P_3)F_{rv}^{dd^{\overline{l-1}}}\}$$

  This way the precipitation content q_r and q_s would also be modified by downdraught evaporation

3.11.2 Completing other outputs

Finally, the precipitation is partitioned between a convective and a "stratiform" (resolved) part, using the convective fraction at lowest model level ZSIGPC $= \alpha_c$ computed in ACUPU.

$$\mathcal{P}_r^{cu} = \alpha_c \mathcal{P}_r \qquad \mathcal{P}_s^{cu} = \alpha_c \mathcal{P}_s$$
$$\mathcal{P}_r^{st} = (1-\alpha_c)\mathcal{P}_r \qquad \mathcal{P}_s^{st} = (1-\alpha_c)\mathcal{P}_s$$

Immediately after ACUPU, the pseudo-historic convective cloud fraction has been stored in PUNEBH:

$$\mathsf{PUNEBH} = \sigma_D + \sigma_u$$

Under LPHSPSH=T (normal case), the pseudo-historic precipitation sensible heat flux is stored in PFHPS.

$$F_{sPs} = (\overline{T}^L - \overline{T_s})\big\{(c_w - c_{pa})\mathcal{P}_r^{\overline{L}} + (c_i - c_{pa})\mathcal{P}_s^{\overline{L}}\big\}$$