Prognostic Turbulence Kinetic Energy (TKE) scheme in IFS (CY49)

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Prognostic TKE scheme in IFS:

Current computation:

– Eddy Mass-Flux (EDMF):

$$\rho\overline{w'\psi'} = -\rho \mathbf{K}_\psi \frac{\partial \psi}{\partial z} + M_u\left(\psi_u - \overline{\psi}\right)$$

– Louis / M-O scheme:

$$\mathbf{K}_\psi = c_\psi \cdot l \cdot f_\psi\left(Ri, \zeta\right) \cdot \sqrt{\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2}$$

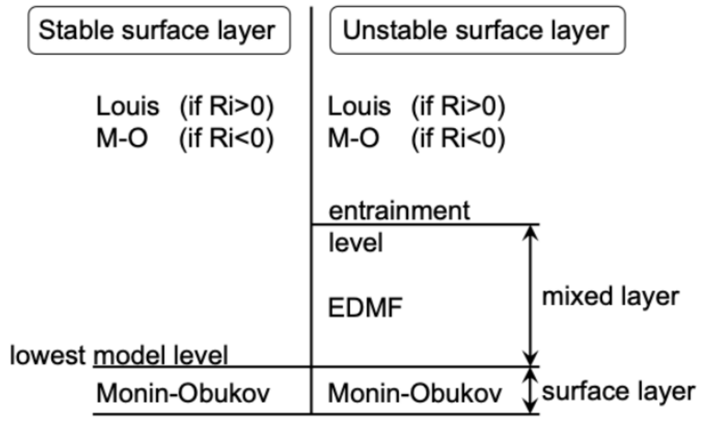

Figure 3.1 *Schematic diagram of the different boundary layer regimes.*

Turbulence Kinetic Energy (TKE=$\mathbf{e_k}$) computation:

$$\mathbf{K}_\psi = C_\psi \cdot L \cdot F_\psi\left(Ri\right) \cdot \sqrt{\mathbf{e_k}} \qquad \frac{\partial \mathbf{e_k}}{\partial t} = \mathrm{ADV} + \frac{\partial\left(K_{e_k}\frac{\partial \mathbf{e_k}}{\partial z}\right)}{\partial z} + \mathbf{K_M} S^2 - \mathbf{K_H} N^2 - \frac{C_\epsilon \mathbf{e_k}^{\frac{3}{2}}}{L}$$

$K_{M/H}$ – turbulence exchange coefficients for momentum and heat/moisture, L – turbulence length scale, F_ψ – stability dependency function, C_ψ, $C\varepsilon$ – closure constants, N^2 – Brunt–Väisälä frequency, S – wind shear, ADV – advection term, Ri – gradient Richardson number

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- Advantages of a prognostic TKE scheme:
  - Regime-adaptive representation of the turbulent flow (expected benefit at higher resolutions)
  - Advection of turbulence intensity (TKE)
  - Improved representation of stable boundary layer
  - Possibility of better interaction with other parameterization via TKE (length scale adjustment + additional source/sink terms)
  - More degrees of freedom in the turbulence scheme

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Thank you for your attention!

- The original (Meteo-France) TKE scheme has been updated and recalibrated (CY49):
  - The turbulence length scale reformulation
  - The stability dependency functions updated
  - Cloud-dependent stability parameter introduced
  - Partial equilibrium computation of the TKE source terms
  - Explicit inclusion of the advection term in the TKE solver
  - Utilization of the Estimated Inversion Strength (EIS) index, friction velocity, and heat and moisture surface fluxes to identify the stratocumulus (StCu) regime
  - Interaction with deep convection via length scale modification

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Prognostic TKE scheme in IFS:

Turbulent diffusion coefficients in TKE scheme

$$\mathbf{K_M} = C_K \chi_3(Ri_f^*)\sqrt{e_k}L, \quad \mathbf{K_H} = C_3 C_K \phi_3(Ri_f^*)\sqrt{e_k}L$$

- TKE - measure of turb. intensity
- length scale - scale of the problem
- stability functions - influence of stratification
- closure constants

C_K - closure constant, C_3 - inverse Prandtl number at neutrality, Ri_f^* - stability parameter in the form of flux Richardson number:
$Ri_f \equiv (\frac{g}{\theta_v}\overline{\theta'_v w'})/(\overline{u'w'}\frac{\partial u}{\partial z} + \overline{v'w'}\frac{\partial v}{\partial z})$

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Prognostic TKE scheme in IFS:

Prognostic TKE equation

$$\frac{d\mathbf{e_k}}{dt} = \frac{\partial}{\partial z}\left(K_{e_k}\frac{\partial \mathbf{e_k}}{\partial z}\right) + ST + BT - \epsilon_k,$$

$$\mathbf{e_k} \equiv \frac{\overline{u'u'} + \overline{v'v'} + \overline{w'w'}}{2} \quad \text{-Turbulence Kinetic Energy (TKE)},$$

$$ST \equiv -\overline{u'w'}\frac{\partial u}{\partial z} - \overline{v'w'}\frac{\partial v}{\partial z} \approx \mathbf{K_M S^2} \quad \text{-Shear term},$$

$$BT \equiv \frac{g}{\theta_v}\overline{\theta'_v w'} = E_{q_t}\overline{q'_t w'} + E_{\theta_l}\overline{\theta'_l w'} \approx -\mathbf{K_H N^2} \quad \text{-Buoyancy term}$$

$$\epsilon_k \equiv \frac{2\,\mathbf{e_k}}{\tau_k} = \mathbf{C}_\epsilon \frac{\mathbf{e_k}^{\frac{3}{2}}}{\mathbf{L}} \quad \text{-Dissipation term}$$

ST - shear term, BT - buoyancy term, K_{e_k} - turb. exchange coefficients for e_k; τ_k and τ_k - dissipation time scale; E_{q_t} and E_{θ_l} - cloud-dependent weights, C_ϵ - closure constant, S - wind shear, N - 'moist' Brunt-Väisälä Frequency Frequency (BVF).

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The stability dependence functions:

$$\chi_3(Ri) = \frac{1-\frac{Ri_f}{R}}{1-Ri_f} \quad , \quad \phi_3(Ri) = \frac{1-\frac{Ri_f}{P}}{1-Ri_f} \quad ,$$

$$1 \geq R > 0 \quad , \quad R \geq P > 0 \quad ,$$

- more flexible

P and R - determine the shape of the stability functions according to Bastak Duran et al. (2014)

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The stability dependence functions:

Cloud cover dependence:

- according to Marquet and Geleyn (2013):

$$
\begin{aligned}
Ri_f &= \frac{K_H\,(Ri_f)}{K_M\,(Ri_f)} Ri = \frac{C_3 R\,(P - Ri_f)}{R\,(P - Ri_f)} Ri, \\
Ri &= \frac{N_1^2\,(\mathbf{C})}{S^2} \\
N_1^2\,(\mathbf{C}) &= g\,M\,(\mathbf{C})\,\frac{c_{pd}}{c_p}\frac{\partial \ln(\theta_s)}{\partial z} + g\,\frac{\partial \ln(\theta_s)}{\partial z} \\
&\quad + g\,M\,(\mathbf{C}) \left[F\,(\mathbf{C})\,(1 + r_v)\,\frac{R_v}{R} - \Lambda \right] \frac{\partial\,(q_t)}{\partial z}
\end{aligned}
$$

$M\,(C)$ and $F\,(C)$ - functions of C, $\Lambda = 5.87$ - closure constant, $R = R_d\,q_d + R_v\,q_v$, $c_p = c_{pd}\,q_d + c_{pv}\,q_v + c_l\,q_l$, $r_v = q_v/q_d$, q_d - specific content for dry air

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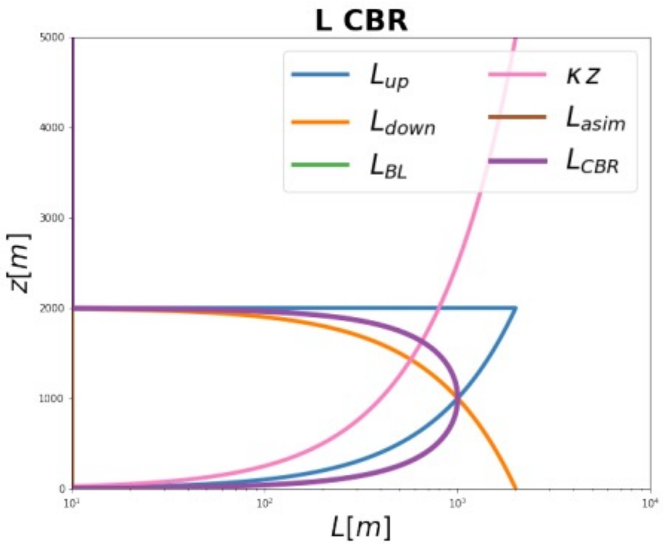
The turbulence length scale:

Length scale formulations in CBR

Bougeault and Lacarrere (1989) (L_{BL}):

$$
\begin{aligned}
L_{BL} &= \left(0.5\, L_{up}^{-\frac{2}{3}} + 0.5\, L_{down}^{-\frac{2}{3}}\right)^{-\frac{3}{2}}, \\
&\int_{z}^{z+L_{up}} \frac{g}{\theta_{vr}} (\theta_v(z) - \theta_v(z'))\, dz' = e_k(z), \\
&\int_{z-L_{down}}^{z} \frac{g}{\theta_{vr}} (\theta_v(z') - \theta_v(z))\, dz' = e_k(z), \\
L_{CBR} &= \max\left(\min\left(\kappa z, L_{asim}\right), L_{BL}\right)
\end{aligned}
$$



$\kappa = 0.4$ is the von Kármán constant, $L_{asim} = 10.0m$ - asimptotic length scale

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The turbulence length scale:

Wind shear contribution to L according to de Rodier et al. (2017)

- prevents over-estimation of L in stable stratification with strong shear
-

$$L = \frac{(C_K\, C_\epsilon)^{\frac{1}{4}}}{C_K}\left(\frac{L_{up}^{-\frac{2}{3}} + L_{down}^{-\frac{2}{3}}}{2}\right)^{-\frac{3}{2}},$$

$$\int_z^{z+L_{up}} \frac{g}{\theta_{vr}}\left(\theta_v(z) - \theta_v(z') + \mathbf{C_0}\cdot\sqrt{\mathbf{e_k}}\cdot\mathbf{S(z')}\right)\, dz' = e_k(z),$$

$$\int_{z-L_{down}}^{z} \frac{g}{\theta_{vr}}\left(\theta_v(z) - \theta_v(z') + \mathbf{C_0}\cdot\sqrt{\mathbf{e_k}}\cdot\mathbf{S(z')}\right)\, dz' = e_k(z),$$

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- The turbulence length sale update (2):

New calibration of the length scale formulations

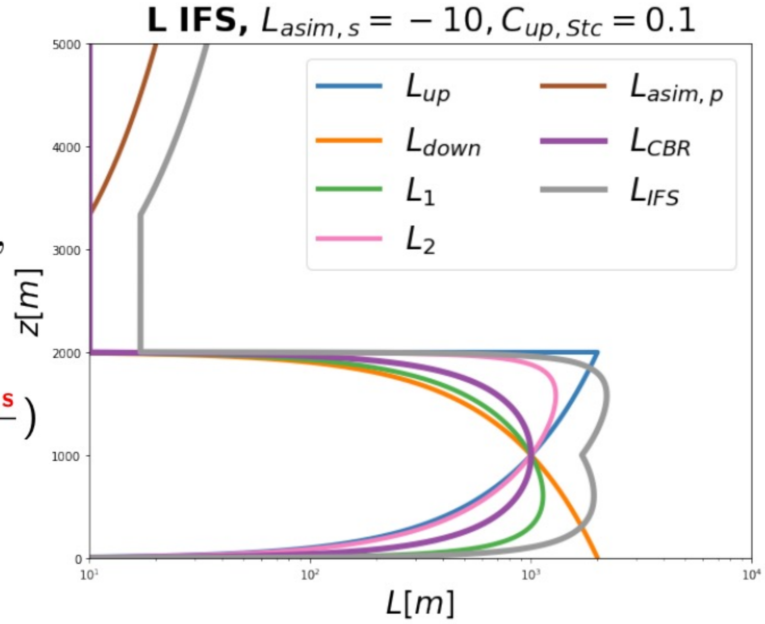
$$L_1 = \left(\mathbf{C_{up}}\, L_{up}^{-\frac{2}{3}} + (1 - \mathbf{C_{up}})\, L_{down}^{-\frac{2}{3}} \right)^{-\frac{3}{2}},$$

$$L_2 = \left(\mathbf{C_{up,Stc}}\, L_{up}^{-\frac{2}{3}} + (1 - \mathbf{C_{up,Stc}})\, L_{down}^{-\frac{2}{3}} \right)^{-\frac{3}{2}},$$

$$L_{1,2} = \max\left(L_1, L_2\right)$$

$$L_{asim,p} = L_{asim} - \max(0.0, (z_{Lm} - z)\, \frac{L_{asim} - \mathbf{L_{asim,s}}}{z_{Lm}})$$

$$L_{IFS} = \mathbf{C_{LSC}} \max\left(\min\left(\kappa z, L_{asim}\right), L_{1,2}, L_{asim,p}\right),$$



$\mathbf{L_{asim,s}}, \mathbf{C_{up,Stc}}$ - functions of EIS

$C_{LSC} = 1.7$, $z_{Lm} = 1e4m$, $C_{up} = 0.8$, $L_{asim} = 50.0m$ - calibration constants.

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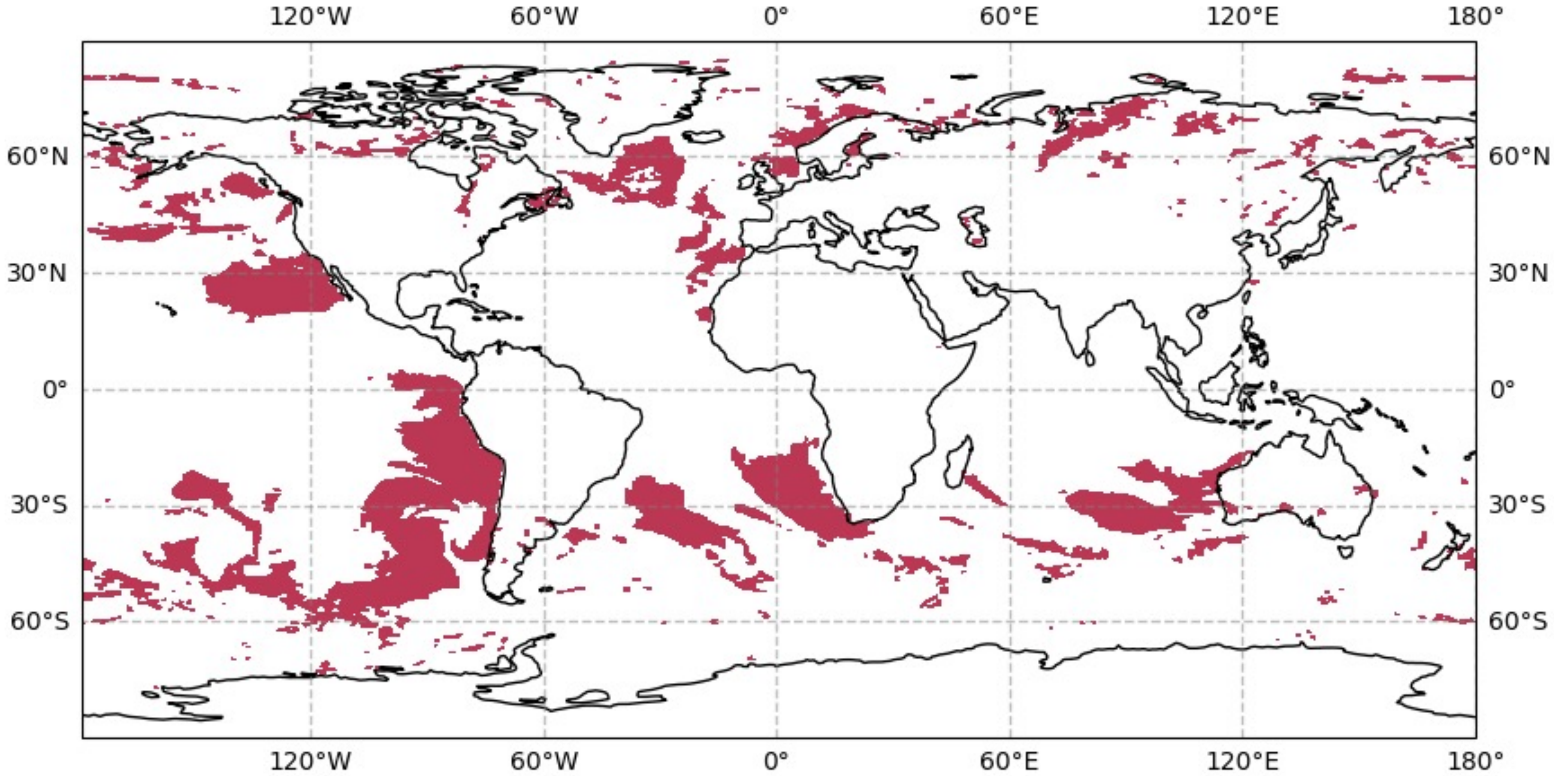
The turbulence length scale:

- Identification of stratocumulus regime via EIS>4, Bowen ratio<0.25, and negative sensible heat flux (snapshot):



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The turbulence length scale:

The EIS dependence:

$$
\begin{aligned}
EIS < EIS^{StCuL} \quad &: \quad L_{asim,s} = 30.0, \;\; C_{up,Stc} \equiv C_{up} = 0.8 \\
EIS^{StCuL} < EIS < EIS^{StCuH} \quad &: \quad \mathbf{L_{asim,s} = -10.0, \;\; C_{up,Stc} = 0.2} \\
EIS > EIS^{StCuH} \quad &: \quad L_{asim,s} = 10.0, \;\; C_{up,Stc} \equiv C_{up} = 0.8
\end{aligned}
$$

- linear intepolation of values between regimes on ΔEIS interval

$EIS^{StCuL} = 2.0$, $EIS^{StCuH} = 14.0$, $\Delta EIS = 1.0$ - calibration constants.

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Partial equilibrium computation of the TKE source terms:

Stable stratification with strong wind shear

- Under-estimation of turbulent mixing due to low value of TKE.
- Problem with **temporal discretization**: $e_k^{t-} \mathbf{=0} \wedge \mathbf{ST^{t0}} + \mathbf{BT^{t0}} \leq \mathbf{0} \Rightarrow e_k^{t0} = 0$
- TKE source terms are computed using TKE from previous time step:

$$
\begin{aligned}
ST^{t0} &= K_M^{t0} \left(S^2\right)^{t-} = C_K \, L^{t0} \, \chi_3^{t0} \sqrt{e_k^{t-}} \left(S^2\right)^{t-}, \\
BT^{t0} &= -K_H^{t0} \left(N^2\right)^{t-} = C_3 \, C_K \, L^{t0} \, \phi_3^{t0} \sqrt{e_k^{t-}} \left(N^2\right)^{t-},
\end{aligned}
$$

^{t0} marks the current time step and index $^{t-}$ marks the previous time step

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Partial equilibrium computation of the TKE source terms:

Stable stratification with strong wind shear

- Solution: the TKE source terms are partly computed from TKE equilibrium conditions:

  $ST = (R_{eq}\, K_M^{eq} + (1 - R_{eq})\, K_M)\, S^2,$

  $BT = (R_{eq}\, K_H^{eq} + (1 - R_{eq})\, K_H)\; N^2,$

- $K_M^{eq} and K_H^{eq}$ can be taken from current first order scheme, or they can be computed consistently from equilibrium TKE, $\tilde{e_k}$:
  - can be taken from **current first order scheme**
  - or they can be computed consistently from **equilibrium TKE**:

    $\mathbf{\tilde{e_k}} = \frac{C_K}{C_\epsilon} L^2\, (\chi_3 - C_3 \phi_3 Ri)\, S^2$

$R_{eq} = 0.5$ is a calibration constant

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Combined summer and winter one-month period TCO399:

RMSE TKE scheme versus operational

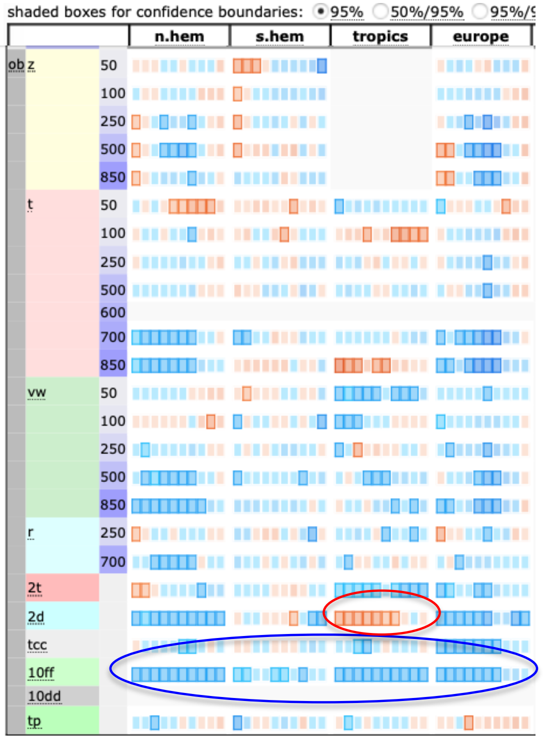
evaluated against their own analysis:

Spinup up to 2 days

Tropics: degradation of T100(3%) and VW100 (2%)

VW850 and T850 improvement

evaluated against observation:



2d degradation (1.5%) in Tropics

10ff improvement

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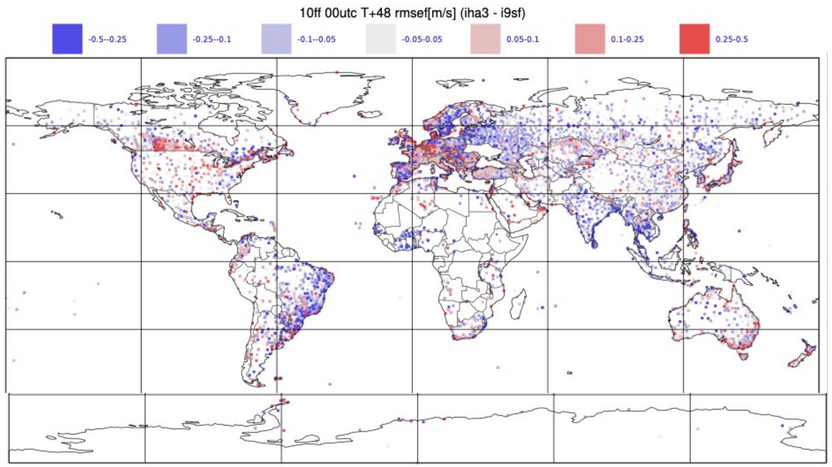
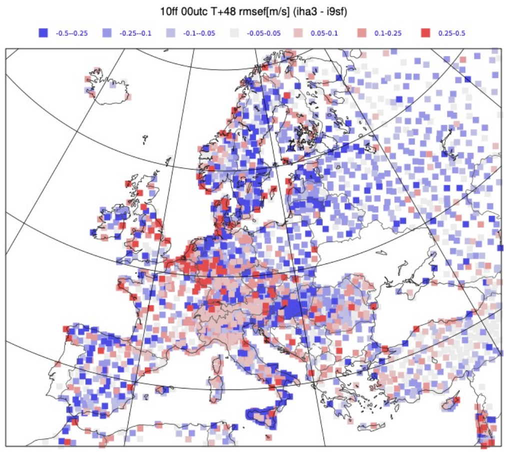
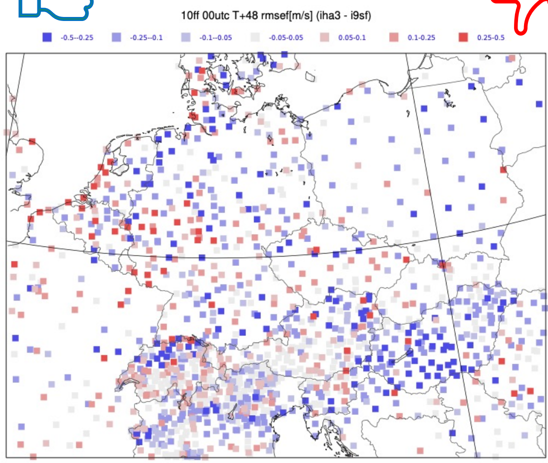
TKE scheme: PBL wind evaluation

- Surface RMSE vs obs. – TCO1279

  - improvement of 10ff (~1.5%), neutral scores for t2m, t2d, and tcc



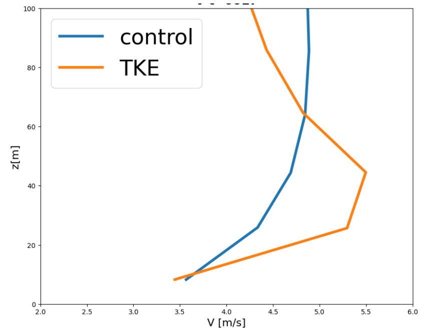




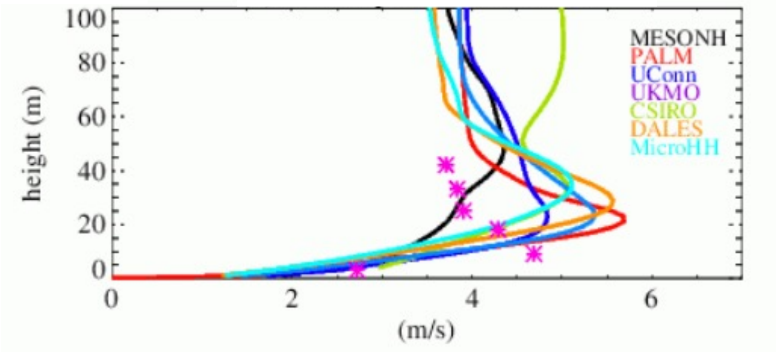
Winter. 10ff

- Low level jet in stably stratified GABLS 4 case

IFS Single column model



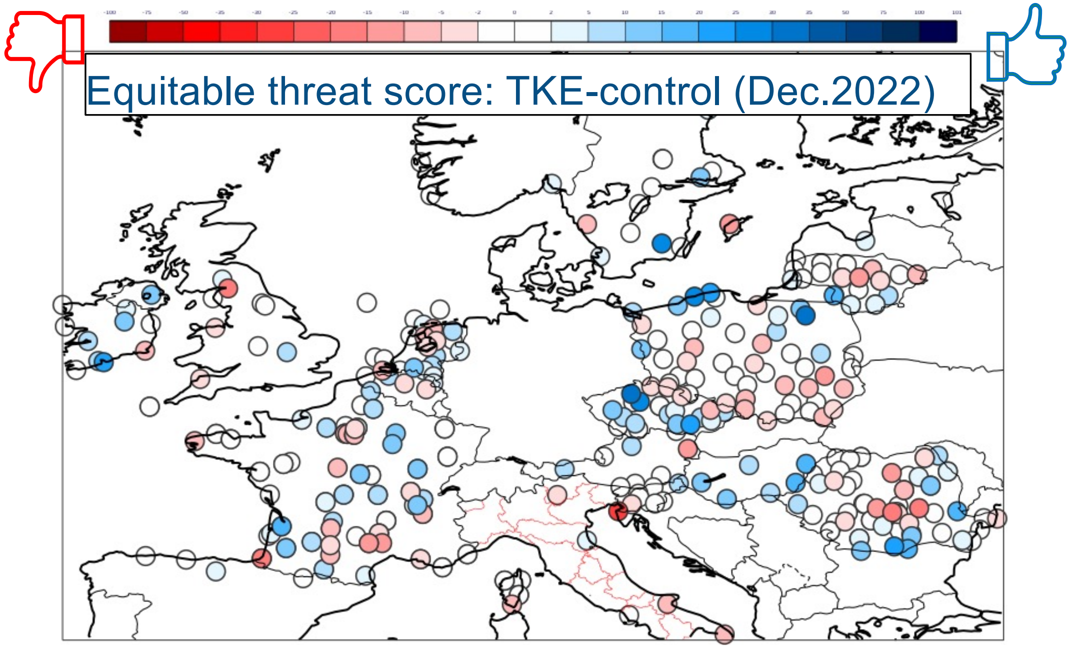
LES references



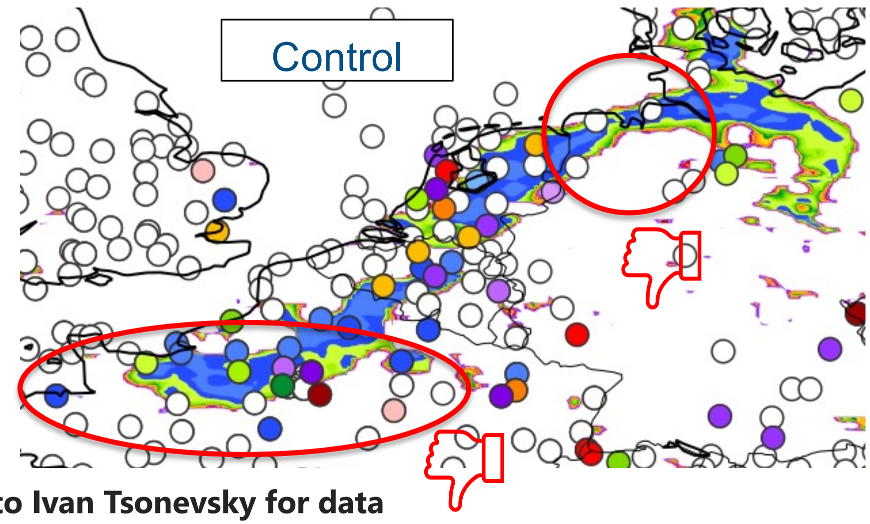
TKE scheme : fog evaluation on TCO1279 with hourly station data (Dec. 2022)

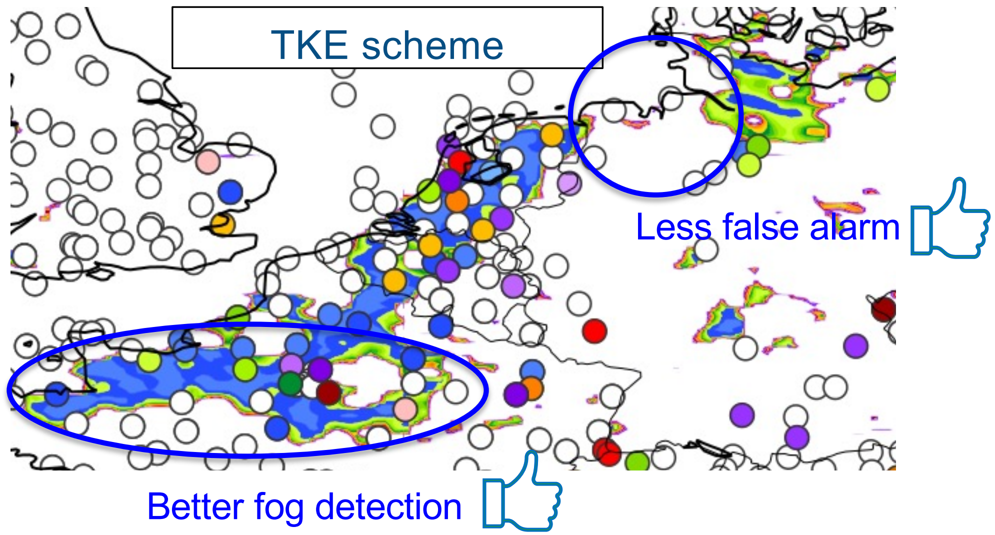
Overall improvement in fog (visibility <1km) detection:

	Control	TKE
POD	15.927	17.773
FAR	71.650	70.854
ETS	13.104	14.575



Visibility,
12.9.2022 6UTC





Thanks to Ivan Tsonevsky for data

- Evaluation:
  - Overall large-scale scores show slight improvement at TCO399 compared to the operational setup
  - Improvement in T, VW, and Z at 850 hPa and 700 hPa
  - Tropics: a slight degradation at 100hPa of T (3%) and VW (1%), and TD (1.5%)
  - Overall improvement in Europe
  - Acceptable representation of StCu regions
  - Low level jet (LLJ) representation improvement
  - Fog representation improvement
  - Interaction with deep convection

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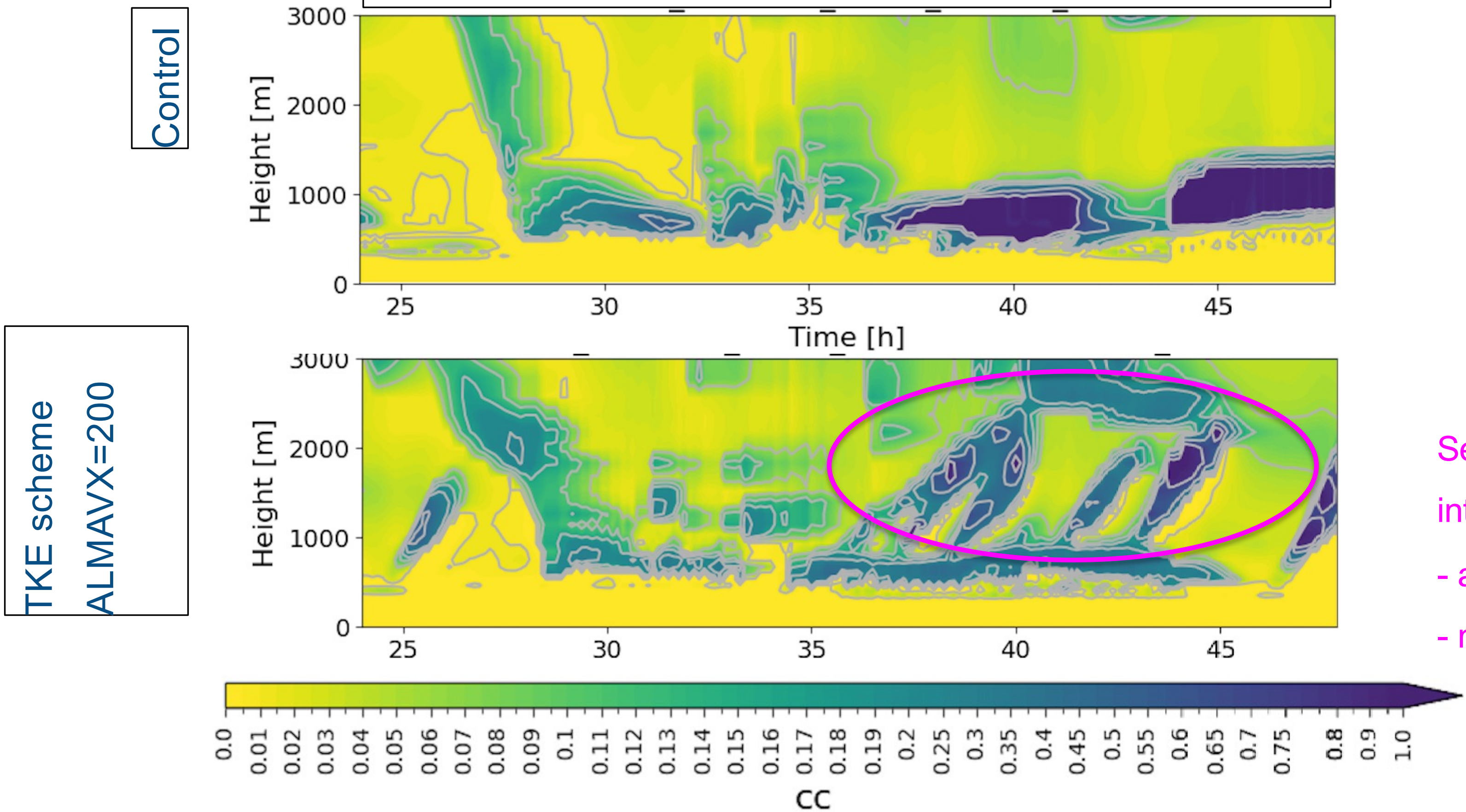
TKE scheme : Interaction with deep convection

- Increase of turbulence length scale (L) in deep convection cloud (ALMAVX – maximal L in cloud)

SCM case – TWPICE: cloud cover evolution:



Sensitivity to DC

interaction (ALMAVX)

- additional transport

- more organization?

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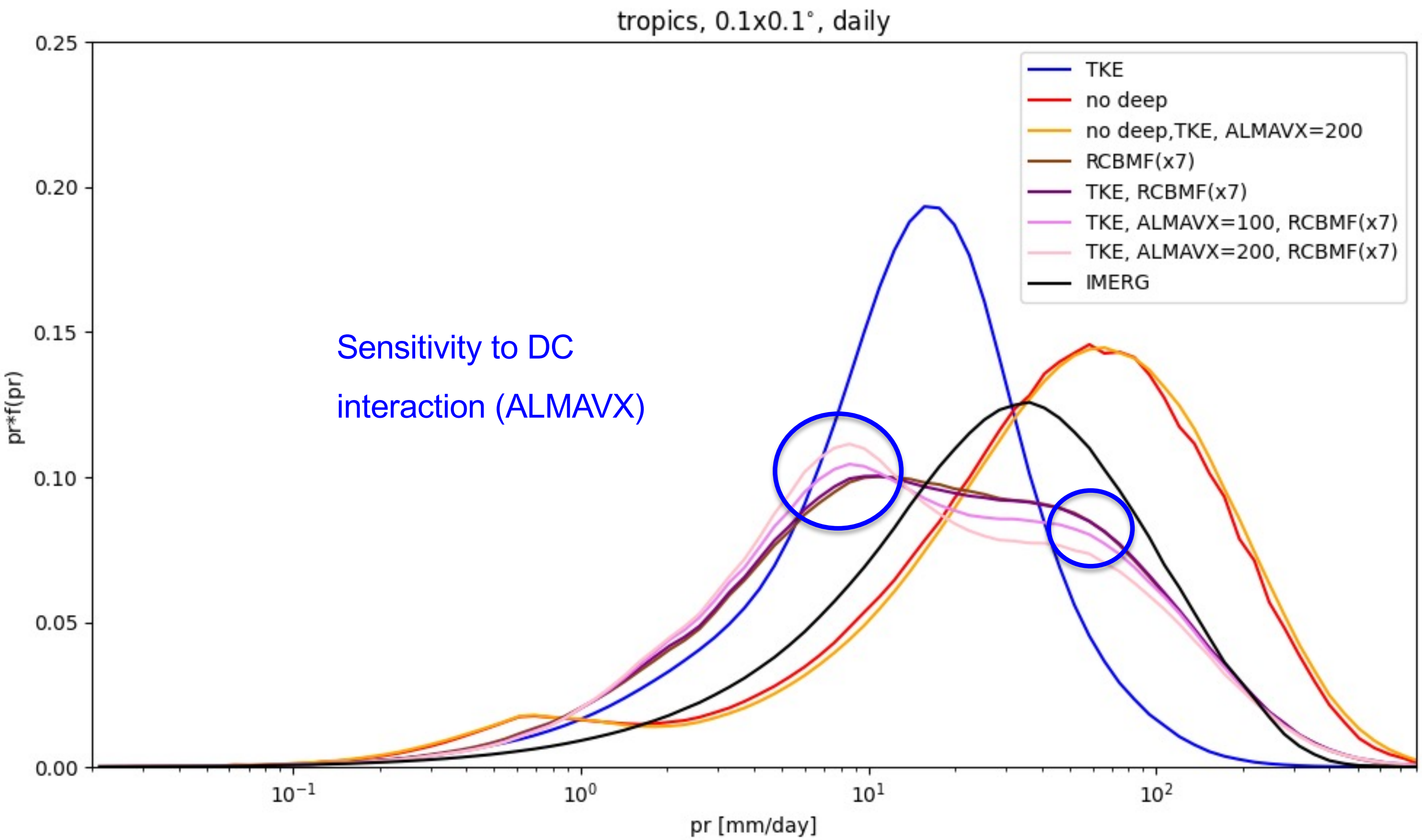
TKE scheme : precipitation in Tropics (TCO1279)

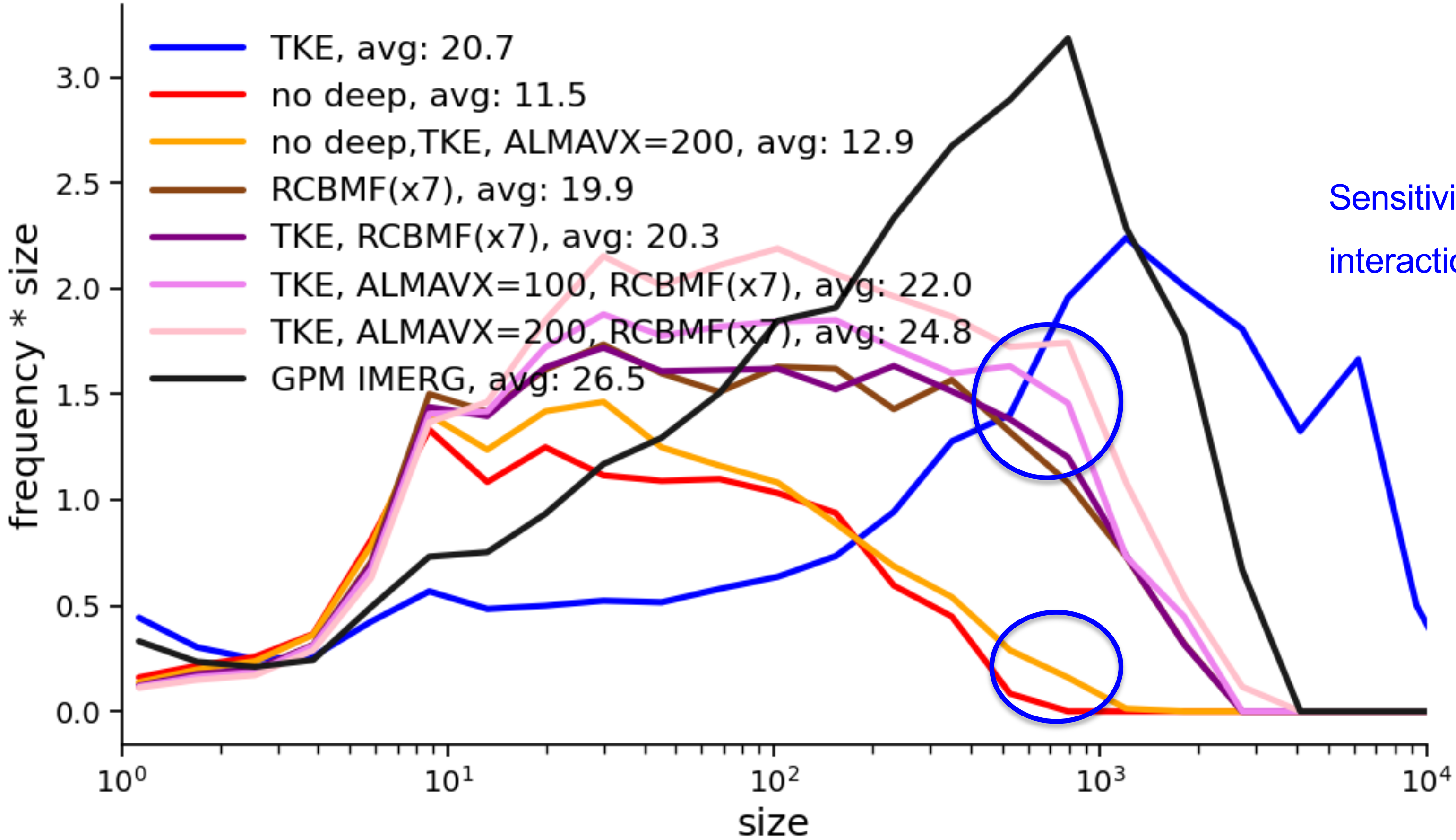
Precipitation distribution:



Thanks to Tobias Becker

TKE scheme : precipitation in Tropics (TCO1279)

Cluster size distribution:



Sensitivity to DC interaction (ALMAVX)

Thanks to Tobias Becker

Additional slides

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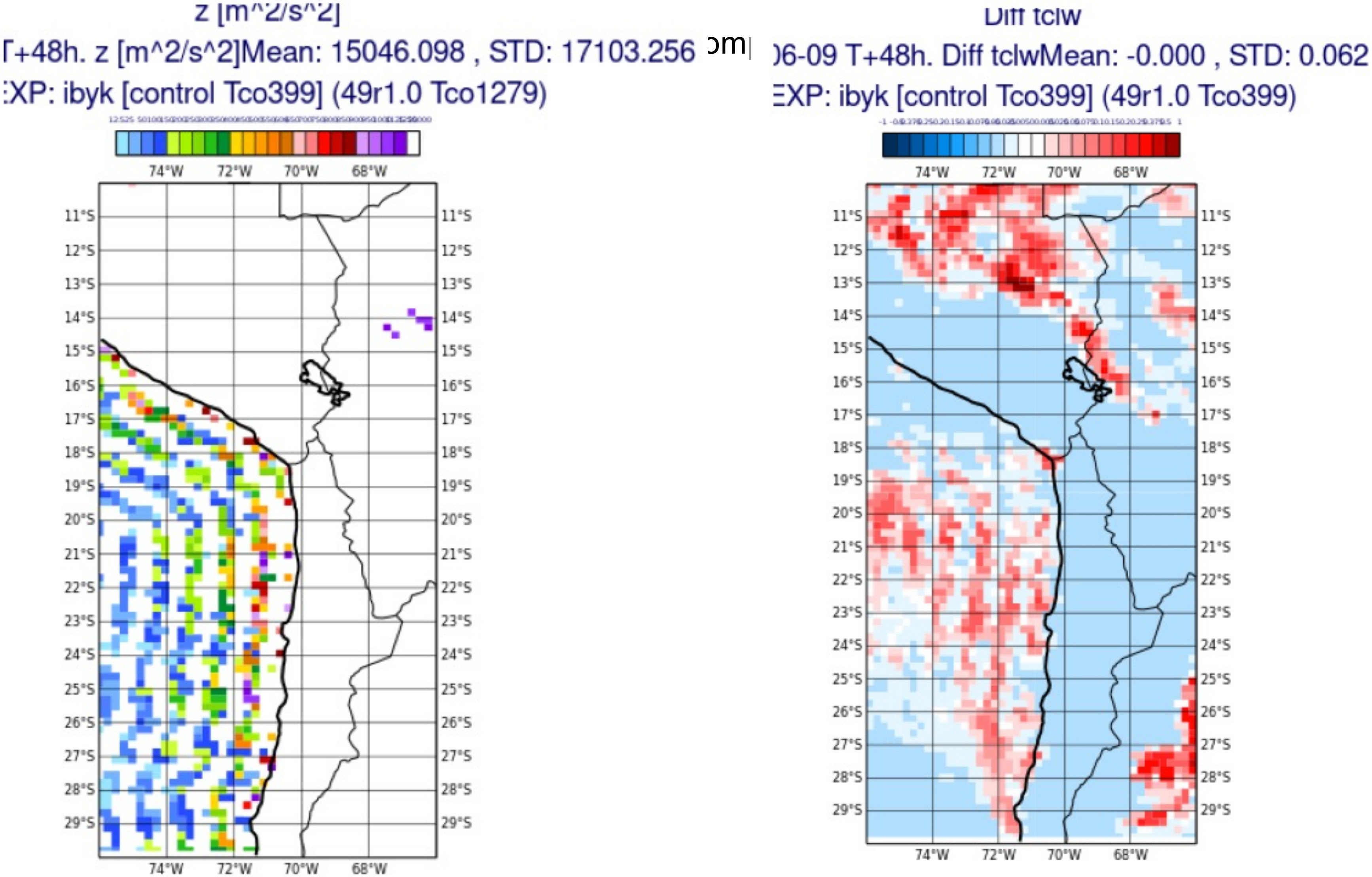
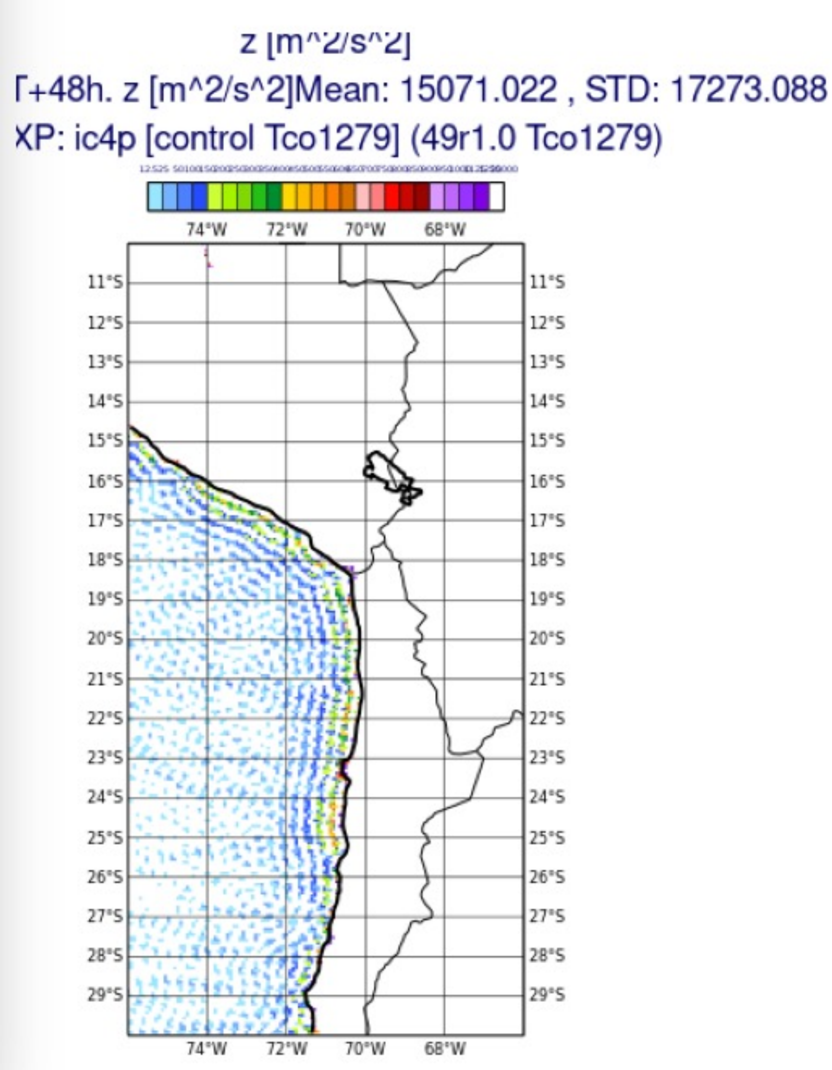
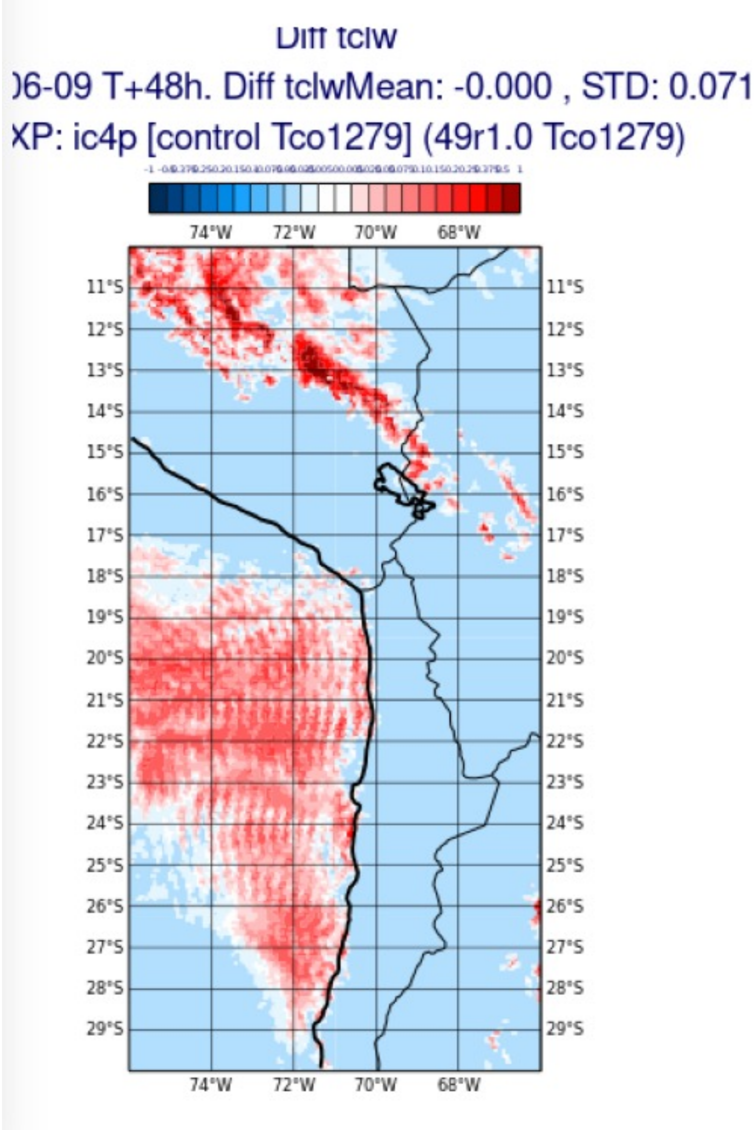
Waves in StCu region - Geopotential and relative tclw (compared to domain mean) – Tco399

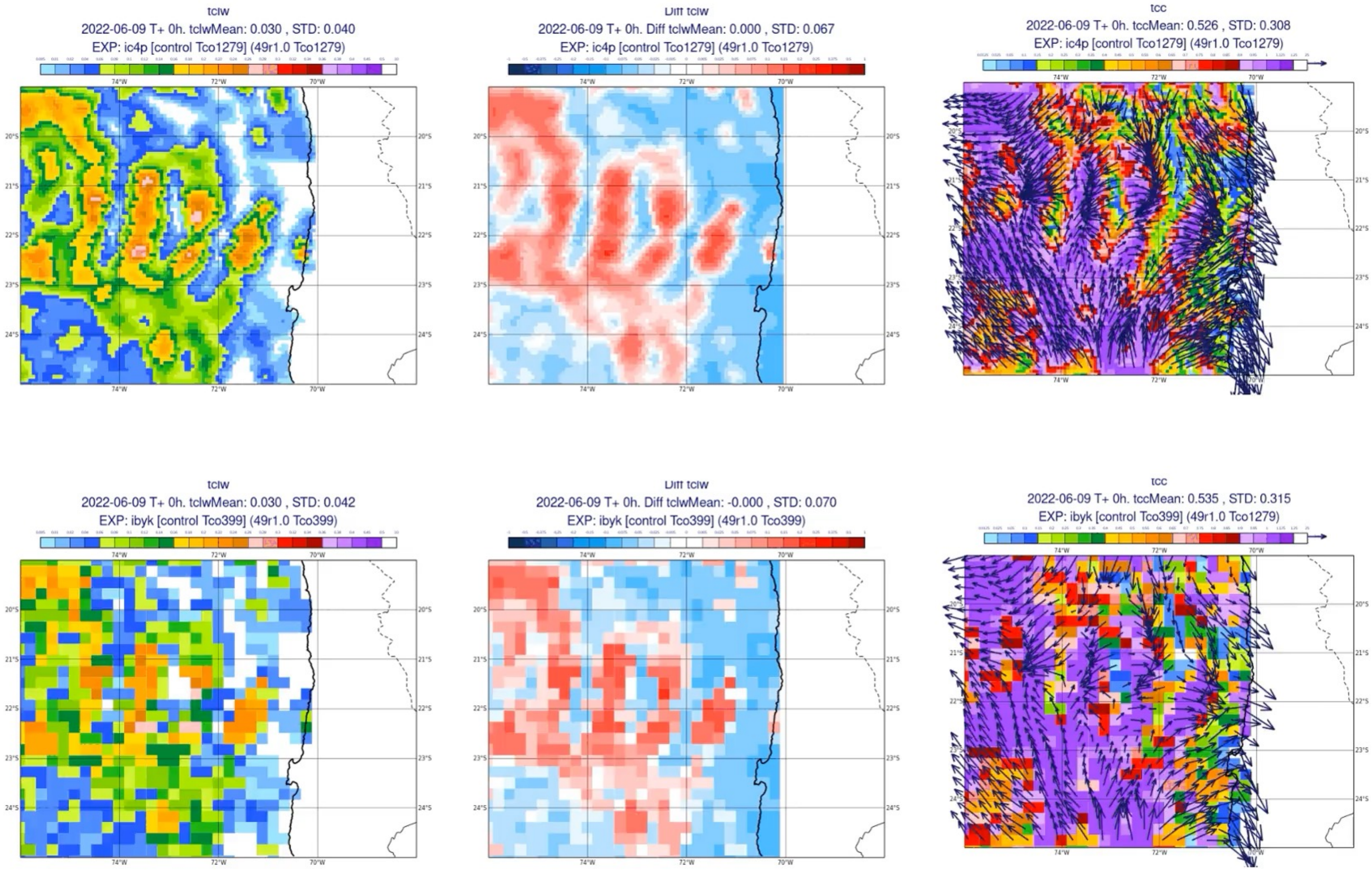


Waves in StCu region - Geopotential and relative tclw (compared to domain mean) – Tco1279

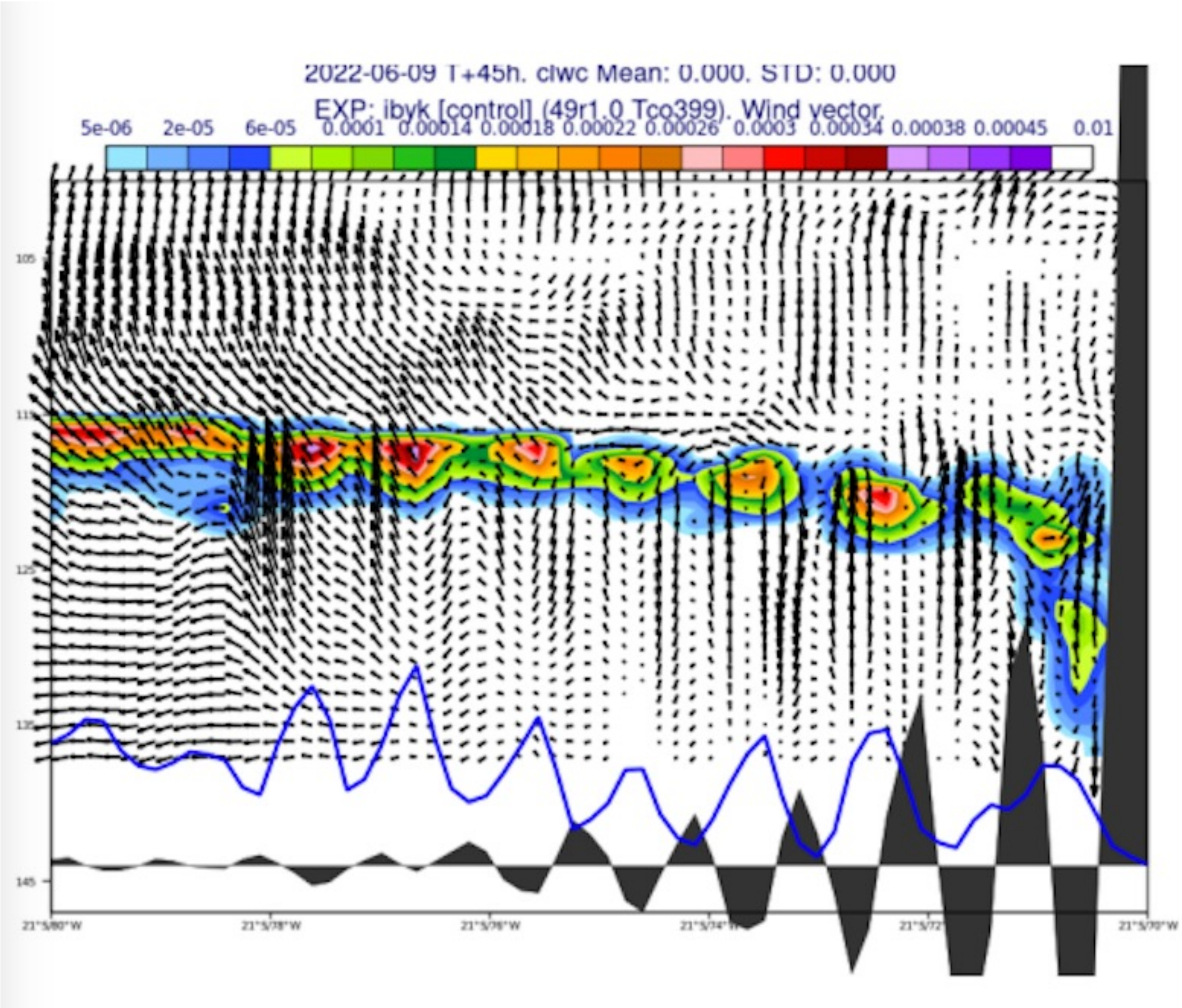




Animation (1 hour time step) of tclw, relative tclw, and tcc – Tco399 vs Tco1279



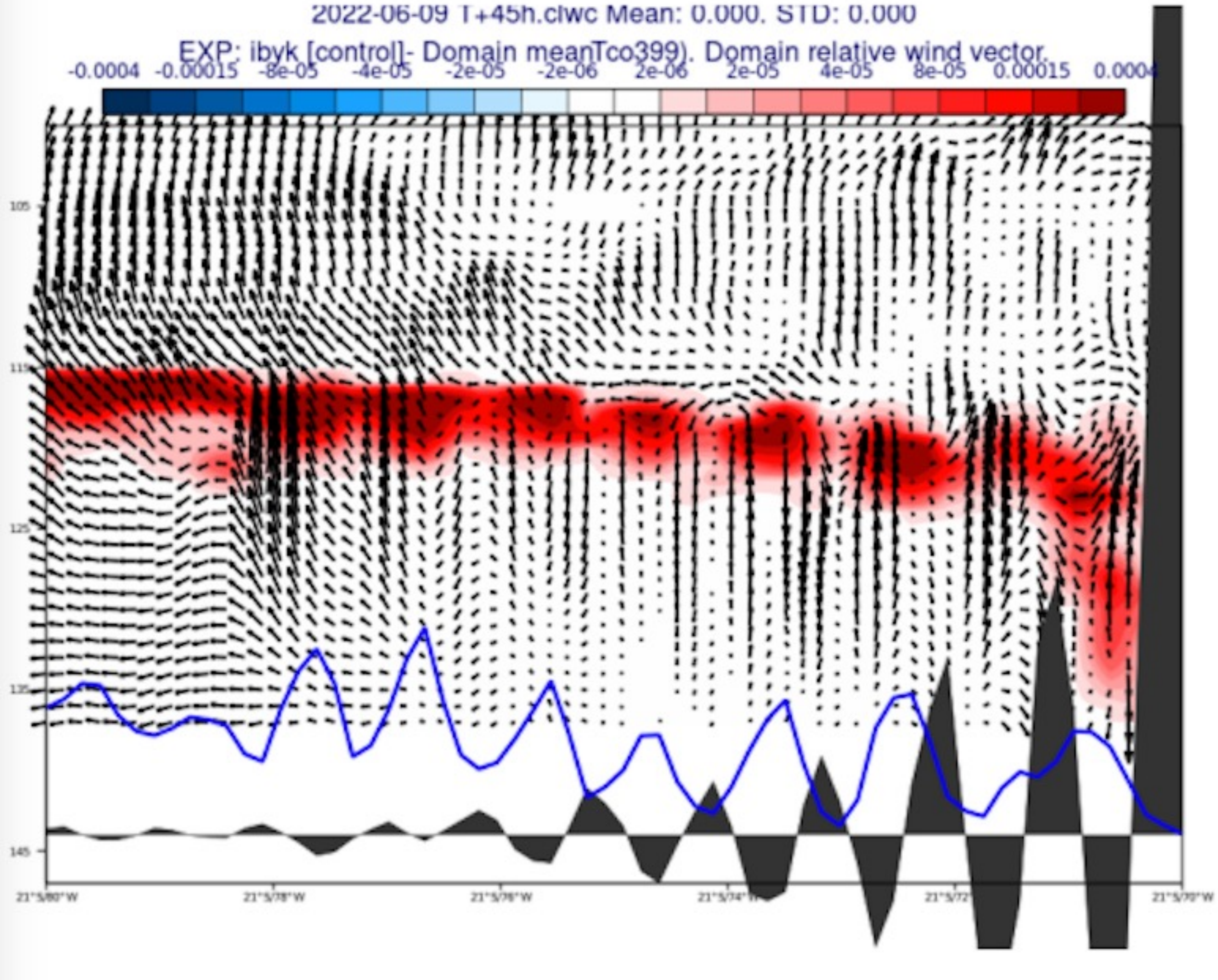
Vertical cross-section along –21 deg latitude



- A Tco399 control experiment
- Color shades show **clwc**
- **Black** shading is the geopotential
- Blue line is **tclw**
- Arrows are **wind vectors** parallel to the cross section (vertical velocity is scaled by 50)
- Vertical coordinate is the model level

Vertical cross-section along –21 deg latitude

Vertical cross-section along –21 deg latitude



- A Tco399 control experiment
- Color shades show the departure of **clwc** to cross-section mean
- **Black** shading is the geopotential
- Blue line is **tclw**
- Arrows are **relative wind vectors** parallel to the cross-section (vertical velocity is scaled by 50, reference is cross-section mean wind vector)
- Vertical coordinate is the model level