Regional Cooperation for
Limited Area Modeling in Central Europe

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Mixing length developments in TOUCANS

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Outline

- The role of the mixing length in TOUCANS
- Development of TKE-based mixing length in TOUCANS
  - Departure from the original proposal
  - Further improvement of the new proposal
  - Do we have a final solution?
- The impact of the PBL height on mixing length
- Towards the 3D turbulence and grey zone
- Conclusion
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The role of the mixing length in TOUCANS

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- Mixing length (ML) is a crucial quantity in ”TKE/TTE - L” type of closure - dimension of the most energetic turbulence eddies

- TOUCANS distinguishes several turbulence length scales (TLS), i.e. eddy diffusivity scales for momentum (L_K), heat/moisture (L_H) and molecular dissipation scale (L_ϵ), which are related through the main/master length scale (L_n)

- The choice of the ML/TLS formulation is more or less independent of the scheme being used (surface layer is a bit specific)

- Currently in the code there are three different formulations (EL0 - L_{gc}, EL1 - L_{BS} and EL2 - L_{BS} + $L_{BS_{loc}}$)

The role of the mixing length in TOUCANS

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- The relationship between L_K, L_ϵ and L_n is stability-dependent and given in Mašek et. al. (2021):

$$L_K = L_n F_\epsilon^{\frac{1}{3}}, \qquad L_\epsilon = \frac{L_n}{F_\epsilon}, \qquad F_\epsilon = \left[\frac{1 - Ri_f}{\chi_3(Ri_f)}\right]^{\frac{3}{4}} \tag{1}$$

- Additionally, the direct relationship between L_K, L_ϵ and L_n can be made:

$$L_n = \left(L_K^3 \cdot L_\epsilon\right)^{\frac{1}{4}} \quad \longrightarrow \quad L = \frac{C_\epsilon}{\nu^3} l_m, \qquad \nu = \left(C_K C_\epsilon\right)^{\frac{1}{4}} \tag{2}$$

l_m is Prandtl type mixing length (from MOST) - important for scaling issue (later)

- When stability-dependence between ML/TLS is included:

$$K_M = C_K L_K \chi_3 \sqrt{e_k}, \qquad K_H = C_K L_H \phi_3 \sqrt{e_k}, \qquad L_H = C_3 L_K \tag{3}$$

The role of the mixing length in TOUCANS

- Computation of turbulent fluxes above the surface layer:

$$\overline{u'w'} = -K_M \frac{\partial u}{\partial z}, \qquad \overline{v'w'} = -K_M \frac{\partial v}{\partial z} \tag{4}$$

$$\overline{s'_L w'} = -K_H \frac{\partial s_L}{\partial z} + TOMs, \qquad \overline{q'_t w'} = -K_H \frac{\partial q_t}{\partial z} + TOMs \tag{5}$$

- Computation of turbulent fluxes in the surface layer:

$$\overline{(w'\phi')}_s = C_\phi \sqrt{(u^2+v^2)}[\phi(z) - \phi_s] \tag{6}$$

$$C_M = C_{MN} F_M(Ri), \qquad C_H = C_{HN} F_H(Ri) \tag{7}$$

$$C_{MN} = \left[\frac{\kappa}{ln\left(1+\frac{z}{z_0 m}\right)}\right]^2, \qquad C_{HN} = \left[\frac{\kappa^2}{ln\left(1+\frac{z}{z_0 m}\right) ln\left(1+\frac{z}{z_0 h}\right)}\right] \tag{8}$$

Departure from the original proposal (EL1)

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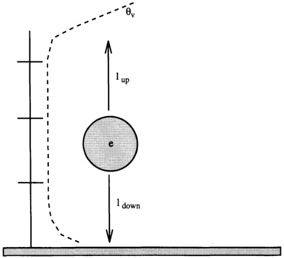
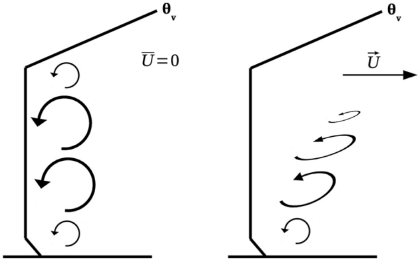
- Generalized BL89 formulation - Rodier et al. (2017):

$$\int_{z}^{z+L_{up}} \left[\frac{g}{\theta_v(z')}(\theta_v(z') - \theta_v(z)) + c_0\sqrt{e(z')}S(z')\right]dz' = e(z) \quad (9)$$

$$\int_{z-L_{down}}^{z} \left[\frac{g}{\theta_v(z')}(\theta_v(z) - \theta_v(z')) + c_0\sqrt{e(z')}S(z')\right]dz' = e(z) \quad (10)$$





Departure from the original proposal (EL1)

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- Imposing the κz limit in the surface layer (old way):

$$l_m = min(\kappa z, \frac{\nu^3}{C_\epsilon} L_{TKE}), \qquad L_{TKE} = \sqrt{L_{up} \cdot L_{down}} \tag{11}$$

- Smooth transition from the near-surface κz layer to the aloft layer where pure L_{TKE} solution prevails (new way):

$$l_m = f_w \cdot \kappa z + (1 - f_w) \cdot \kappa L_{TKE} \tag{12}$$

$$f_w = 3 \cdot f_w'^2 - 2 \cdot f_w'^3, \qquad f_w' = max\left[0, min\left(1, \frac{c_2 - \frac{z_H}{H_{PBL}}}{c_2 - c_1}\right)\right], \qquad c_2 > c_1 \tag{13}$$

scaling L_{TKE} with κ is in agreement with LES diagnostics - Reilly et al. (2022)

Departure from the original proposal (EL1)

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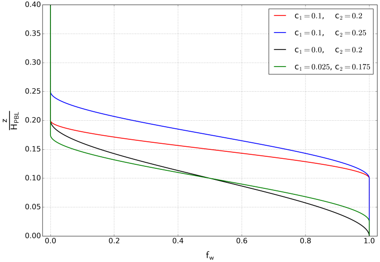
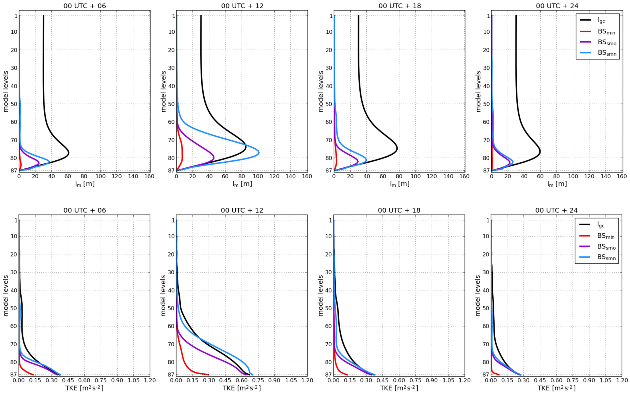


f_w function



$\mathrm{BS_{min}}$ - eq.(11); $\mathrm{BS_{smo}}$ - eq.(12) with $\frac{\nu^3}{C_\epsilon} L_{\mathrm{TKE}}$; $\mathrm{BS_{smn}}$ - eq.(12)

Further improvement of the new proposal

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- TKE-based formulation given by eq. (9)-(10) and (12)-(13) suffers from insufficient mixing near the PBL top (improvement is necessary)

- The crossing parcels (CP) treatment - Golaz et al. (2002):

$$L_{up} = max[L_{up}(i), L_{up}(i+1) - \Delta z] \quad (14)$$

$$L_{down} = max[L_{down}(i), L_{down}(i-1) - \Delta z] \quad (15)$$

- Variable upper-air asymptotic limit (VUAL):

$$L_{TKE} = max(L'_{TKE}, c_3 \cdot L_{TKE-max}) \quad (16)$$

L_{TKE}' - non-corrected L_{TKE} ; $L_{TKE-max}$ - column max. value; c_3 - tunable parameter

Further improvement of the new proposal

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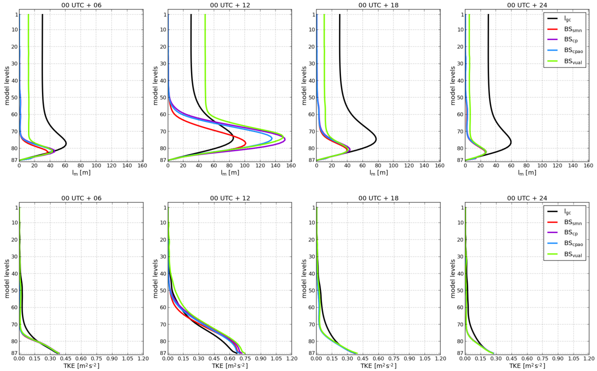
- The impact of CP and VUAL:



- The new averaging operator:

$$L_{TKE} = \left(\frac{L_{up}^{-\frac{5}{4}} + L_{down}^{-\frac{5}{4}}}{2} \right)^{-\frac{4}{5}} \qquad (17)$$

BS_{smn} - as previous

BS_{cp} - as BS_{smn} + CP - eq. (14)-(15)

BS_{cpao} - as BS_{cp} + new AO - eq. (17)

BS_{vual} - as BS_{cpao} + VUAL (c_3=0.225)

Do we have a final solution?

- We have all the necessary ingredients, but need to find an optimal ”internal” tuning (c_0, ~~c_1~~, c_2, c_3, AO and H_{PBL} computation)

- Verification scores for TKE-based ML formulation with global κ scaling are comparable to the reference (EL0) - some tuning of: 1) convection and cloud schemes and 2) land-surface to atmosphere coupling might be necessary

- Can we improve something else?

  1) The H_{PBL} computation (significantly affects all ML/TLS formulations)

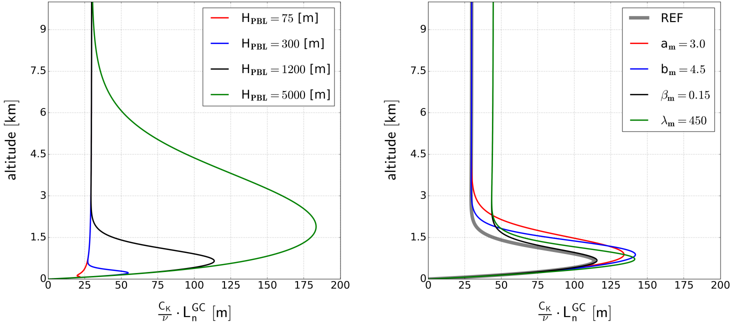
  2) Utilize the work of Bašták Ďurán et al. (2022.) to modify the upper asymptotic value of L_{TKE} (based on the bulk $\Delta\theta_s$) and represent the top PBL entrainment

  3) Consider inhomogeneous grid-box and two parcels ascending/descending in different environments

The impact of the PBL height on ML

- Geleyn-Cedilnik formulation (more sensitive than TKE-based):

$$L_n^{GC} = \frac{\nu}{C_K} \frac{\kappa z}{1 + \frac{\kappa z}{\lambda_m}\left[\frac{1+exp(-a_m\sqrt{\frac{z}{H_{PBL}}}+b_m)}{\beta_m+exp(-a_m\sqrt{\frac{z}{H_{PBL}}}+b_m)}\right]} \quad (18)$$



Computation of the PBL height

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- There is no such method that estimates the H_{PBL} accurately enough for different stability conditions

The weak-capping-inversion method:

$$\langle\theta(z)\rangle_L \geq \frac{1}{z}\int_0^{z_i}\langle\theta(z)\rangle dz + 0.25 \qquad (19)$$

convective and near-neutral PBL

Ayotte et al. (1996)

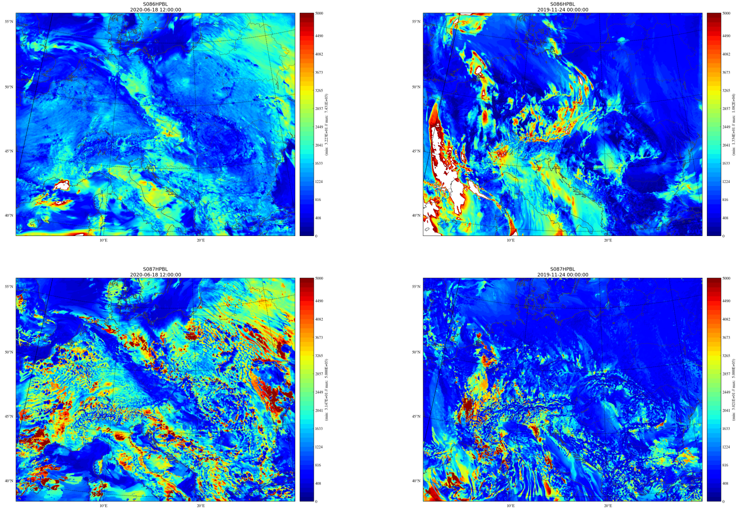
TKE-based method:

$$H_{PBL} = \frac{z_{05}}{0.95} \qquad (20)$$

more general (suitable for statically stable cond.)

Kosović and Curry (2000)

Computation of the PBL height



WCIM method

TKE-based method

Computation of the PBL height

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- WCIM provides unrealistic values in statically stable conditions, while TKE-based method is characterized by grainy patterns
- Can a combination of two methods work?
- We need a more robust method, e.g. Bastak et al. (2022):

$$H_{PBL} = c_{pblh} \cdot \sqrt{\int_{z=0}^{z_{top}} L_{up} \cdot dz}, \qquad c_{pblh} = 1.75 \tag{21}$$

L_{up} depends on stratification and turbulence within entire model column

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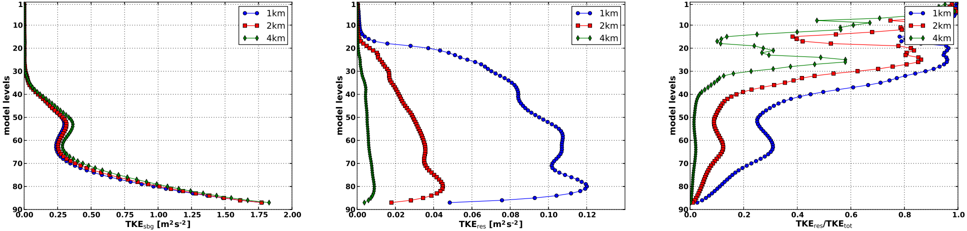
Towards the 3D turbulence and grey zone

- The existing turbulence schemes in NWP models are intended for use in horizontally homogeneous and flat terrain (1D)
- At $\Delta x \approx 1$ km and in complex (mountainous) terrain the turbulence intensity is typically underestimated - need for 3D effects
- Furthermore, at $\Delta x \approx 1$ km we are within the gray zone (turb. is partly resolved) - need to take care of partitioning between TKE_{res} and TKE_{sbg}



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Towards the 3D turbulence and grey zone

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- The hybrid turbulence scheme (quasi-3D) - Goger et al. (2018, 2019):

$$\frac{de_k}{dt} = -g\frac{\partial}{\partial p}\left(\rho K_{e_k}\frac{\partial e_k}{\partial z}\right) + I + II - \frac{e_k^{\frac{3}{2}}}{\tau_k} \quad (22)$$

$$\frac{de_t}{dt} = -g\frac{\partial}{\partial p}\left(\rho K_{e_t}\frac{\partial e_t}{\partial z}\right) + I - \frac{e_t^{\frac{3}{2}}}{\tau_t} \quad (23)$$

$$I = -\overline{u'w'}\frac{\partial\overline{u}}{\partial z} - \overline{v'w'}\frac{\partial\overline{v}}{\partial z} - \overline{u'u'}\frac{\partial\overline{u}}{\partial x} - \overline{u'v'}\frac{\partial\overline{u}}{\partial y} - \overline{u'v'}\frac{\partial\overline{v}}{\partial x} - \overline{v'v'}\frac{\partial\overline{v}}{\partial y} \quad (24)$$

$$II = E_{s_{sL}}\overline{s_{sL}'w'} + E_{q_t}\overline{q_t'w'} \quad (25)$$

Towards the 3D turbulence and grey zone

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- The hybrid turbulence scheme (quasi-3D) - Goger et al. (2018):

$$\frac{\partial}{\partial t}\left(e_{k,t}\right)_{hshear} = \left(c_s \Delta x\right)^2 \cdot \left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2\right]^{\frac{3}{2}} \tag{26}$$

$$L_H = c_s \Delta x, \quad c_s = 0.2 \tag{27}$$

Smagorinsky (1963)

- Variable horizontal length scale (L_H) - Goger et al. (2019):

$$L_H = W \cdot T_{L,u,v}, \qquad T_{L,u,v} = 0.15 \frac{H_{PBL}}{\sigma_{u,v}} \tag{28}$$

W - mean wind speed, $T_{L,u,v}$ - Lagrangian integral time scale

Towards the 3D turbulence and grey zone

- Variable horizontal length scale (L_H) - Goger et al. (2019):

$$\sigma_u^2 = u_*^2 \left[\left(5 - 4\frac{z}{H_{PBL}}\right) + 0.35 \left(\frac{H_{PBL}}{\kappa L}\right)^{\frac{2}{3}} \right], \quad \sigma_v^2 = 2u_*^2 \left(1 - \frac{z}{H_{PBL}}\right) \qquad (29)$$

inapplicable above the PBL + based on specific dataset (alternative within TOUCANS)

* additional term in statically unstable conditions

- Variable horizontal length scale (L_H) - Wang et al. (2021):

$$L_{Hshr} = sW \left[\left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 \right]^{-\frac{1}{2}}, \quad L_{Hstr} = sW \left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 \right]^{-\frac{1}{2}} \qquad (30)$$

$$s = \left(\frac{\Delta_0}{\Delta}\right)^{\alpha}, \quad L_H = \sqrt{L_{Hshr} L_{Hstr}} \qquad (31)$$

Towards the 3D turbulence and grey zone

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- status:

1) Goger et al. (2018) proposal and Wang et al. (2021) additional option for L_H are coded in CY43t2_ag branch at CHMI (stable and gradually developing solution) - validation is ongoing

2) There is ongoing work on implementation of modified Goger et al. (2019) proposal, with $\sigma_{u,v}$ computed from TOUCANS

- future work:

1) Validation of hybrid turbulence scheme and seeking for optimal L_H option

2) Implementation of optimal L_H option into 1D+2D turbulence scheme based on SLHD and its validation

3) Adaptation of TOUCANS for the grey zone (scale-aware scheme) - following Boutle et al. (2014), Honnert et al. (2011, 2020) and Honnert (2019)

Conclusions

- TKE-based formulation (EL1) is now more or less comparable to the reference (EL0)
- There is a significant tuning potential within the existing EL1 code and still some room for development (some options might take more time)
- Improvement of H_{PBL} estimation is an important short/mid-term goal, with implication to all ML formulations
- Developments of TKE-based ML and H_{PBL} computation can be used for creation of horizontal ML/TLS to improve the model performance in the grey zone ("terra incognita")

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