TKE-based mixing length in TOUCANS

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Outline

- Introduction
- Turbulence length scales in TOUCANS
- TKE-based mixing length formulations
  - Bougeault-Lacarrère (1989) - BL89
  - Generalization of BL89 by Rodier et al. (2017) - QR2017
- Getting towards the final solution
- Conclusion

- TOUCANS is TKE/TTE-l type of closure (2^{nd} order)
- Turbulent fluxes:

Above the surface layer (u_i=u,v and ψ=s_{sL},q_t):

$$\overline{u_i'w'} = -K_m \frac{\partial \overline{u_i}}{\partial z} \quad (1a) \qquad K_m = \frac{\nu^4}{c_\epsilon}\sqrt{\chi_3\sqrt{f(Ri)}}L\sqrt{e_k} \quad (1b)$$

$$\overline{\psi' w'} = -K_h \frac{\partial \overline{\psi}}{\partial z} + TOMs \quad (2a) \qquad K_h = C_3 \frac{\nu^4}{c_\epsilon}\frac{\phi_3}{\chi_3}\sqrt{\chi_3\sqrt{f(Ri)}}L\sqrt{e_k} \quad (2b)$$

Surface layer (ϕ=u, v, s_{sL} and q_t):

$$\overline{\phi' w'} = C_\phi \sqrt{u^2 + v^2}(\phi - \phi_s) \quad (3)$$

Introduction - TKE prognostic equation

- TKE prognostic equation:

$$\frac{de_k}{dt} = -g\frac{\partial}{\partial p}\left(\rho K_{e_k}\frac{\partial e_k}{\partial z}\right) + I + II - C_\epsilon \frac{e^{3/2}}{L_\epsilon} \tag{4}$$

$$I = -\overline{u'w'}\frac{\partial \overline{u}}{\partial z} - \overline{v'w'}\frac{\partial \overline{v}}{\partial z} \tag{5}$$

$$II_d = \frac{g}{\theta}\overline{\theta' w'} \tag{6}$$

$$II_m = E_{s_{sL}}\overline{{s_{sL}}'w'} + E_{q_t}\overline{{q_t}'w'} \tag{7}$$

Turbulence length scales in TOUCANS

- TOUCANS differentiates following length scales: L_K, L_ϵ, l_m and L
- Following Redelsperger et al. (2001), the relationship between turbulence length scales is stability-dependent:

$$L_\epsilon = \frac{C_\epsilon}{\nu^3} \frac{{\chi_3}^{\frac{3}{2}}}{f(Ri)^{\frac{3}{4}}} l_m \qquad (8a)$$

$$L_K = \frac{C_\epsilon}{\nu^3} \frac{f(Ri)^{\frac{1}{4}}}{{\chi_3}^{\frac{1}{2}}} l_m \qquad (8b)$$

$$L_K = \frac{f(Ri)}{{\chi_3}^2} L_\epsilon \qquad (9a)$$

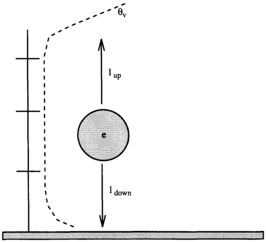
$$f(Ri) = \chi_3 - RiC_3\phi_3 \qquad (9b)$$

- Main computational length scale:

$$L = (L_K^3 L_\epsilon)^{\frac{1}{4}} = \frac{C_\epsilon}{\nu^3} l_m \qquad (10)$$

TKE-based mixing length formulations (BL89)

- Bougeault-Lacarrère (1989) - BL89:



$$\int_{z}^{z+L_{up}} N_v^2(z'-z)dz' = e(z) \qquad (11)$$

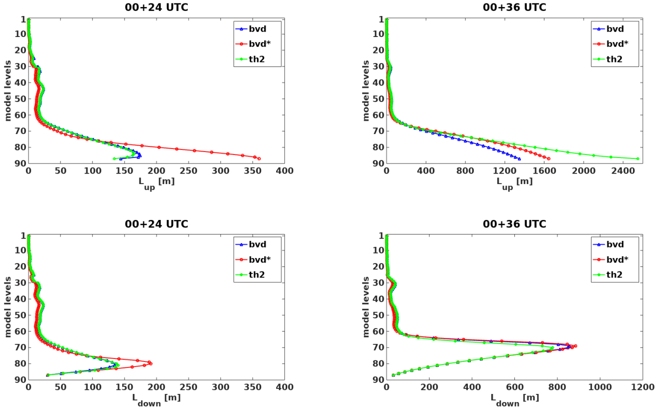
$$\int_{z-L_{down}}^{z} N_v^2(z-z')dz' = e(z) \qquad (12)$$

$$\int_{z}^{z+L_{up}} \frac{g}{\theta_v(z')}[\theta_v(z')-\theta_v(z)]dz' = e(z) \qquad (13)$$

$$\int_{z-L_{down}}^{z} \frac{g}{\theta_v(z')}[\theta_v(z)-\theta_v(z')]dz' = e(z) \qquad (14)$$

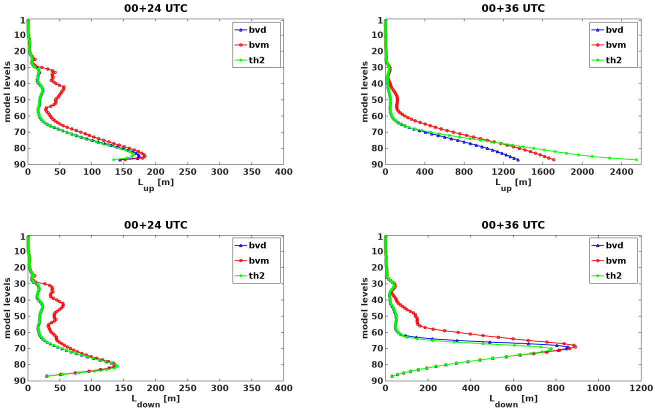
TKE-based mixing length formulations (BL89)

- Problems with discretization of the BL89 integral: i) 'dz' bug, ii) N_v approx. and iii) $\theta_{v,ref}$



TKE-based mixing length formulations (BL89)

- Inclusion of moist effects through N_{vm}:

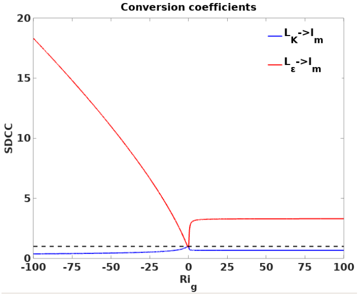


TKE-based mixing length formulations (BL89)

- Averaging operators for vertical displacements:

$$L_{TKE} = \sqrt{L_{up} \cdot L_{down}} \quad (15a) \qquad L_{TKE} = \frac{2L_{up} \cdot L_{down}}{L_{up} + L_{down}} \quad (15b)$$

- Key question: to which of TOUCANS scales should we assign L_{TKE}?



⋆ L option leads to weak mixing

⋆ L_ϵ option is unstable

⋆ L_K option violates MOST:

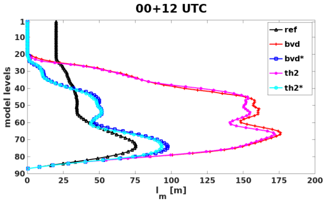
$$l_m \approx (1 - 0.18 Ri_f)\kappa z \qquad (16)$$

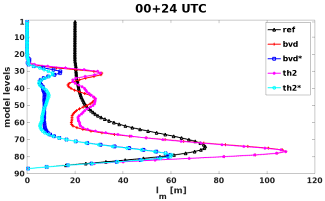
TKE-based mixing length formulations (BL89)

- Prandtl-type mixing length and near-surface κz limit:

$$l_m = min(\kappa z, L_{TKE}) \quad (17a) \qquad\qquad l_m = \kappa \cdot min(z, L_{TKE}) \quad (17b)$$

- To strong mixing with local κ-scaling:





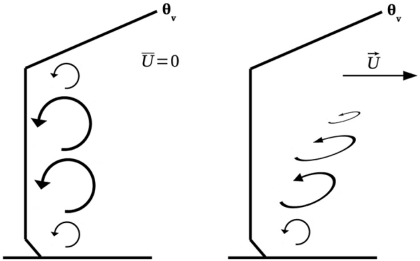
$$l_m^{gc} = \frac{\kappa z}{1 + \frac{\kappa z}{\lambda_m}\left[\frac{1+exp(-a_m\sqrt{\frac{z}{H_{PBL}}}+b_m)}{\beta_m+exp(-a_m\sqrt{\frac{z}{H_{PBL}}}+b_m)}\right]} \qquad (18)$$

TKE-based mixing length formulations (QR2017)

- Generalization following Rodier et al. (2017):

$$\int_{z}^{z+L_{up}} \left[\frac{g}{\theta_v(z')}(\theta_v(z') - \theta_v(z)) + c_0 \sqrt{e(z)} S(z') \right] dz' = e(z) \qquad (19)$$

$$\int_{z-L_{down}}^{z} \left[\frac{g}{\theta_v(z')}(\theta_v(z) - \theta_v(z')) + c_0 \sqrt{e(z)} S(z') \right] dz' = e(z) \qquad (20)$$

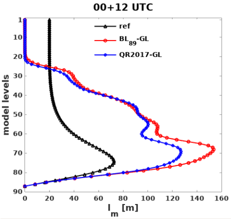


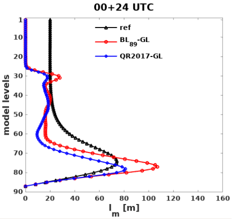
C_0=0.5

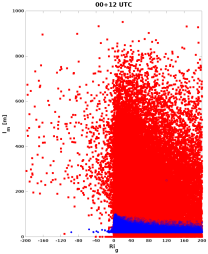
neutral exp.

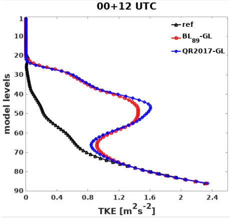
stability dependent C_0?

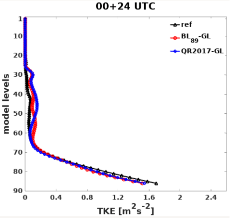
TKE-based mixing length formulations (QR2017)

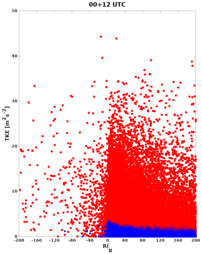




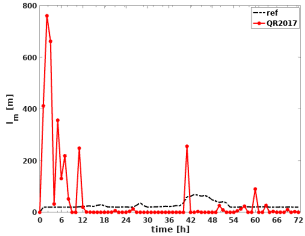








TKE-based mixing length formulations (QR2017)



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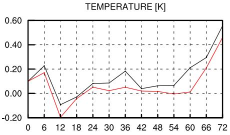

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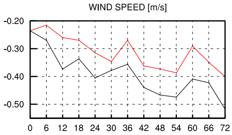

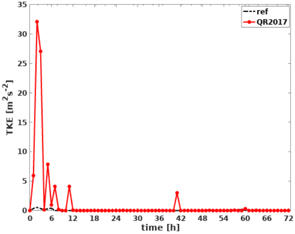

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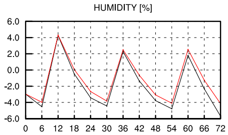

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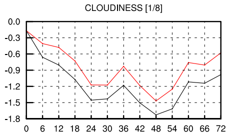


WINTER CASE

Random point at level 40

Getting towards the final solution

- Additional sensitivity tests showed that reduction of mixing above the PBL significantly improves upper-air verif. scores (λ_a=20 and 40 [m])

- Include third term into (19)-(20):

$$C_1 \cdot \frac{1}{e}\left|\frac{\partial e}{\partial z}\right| \cdot g \cdot f(N_v) \cdot f_w(z) \tag{21}$$

- Create turbulence-dependent asymptotic mixing length:

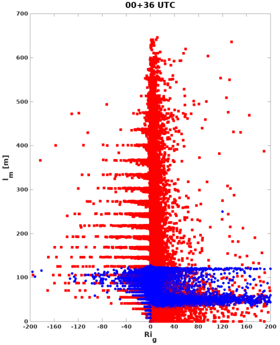
$$\lambda_a = C_2 \cdot \frac{\int \sqrt{e} z dz}{\int \sqrt{e} dz} \tag{22}$$

- Smoothing the transition from κz layer to the layer where pure L_{TKE} solution prevails and/or towards the upper asymptotic layer

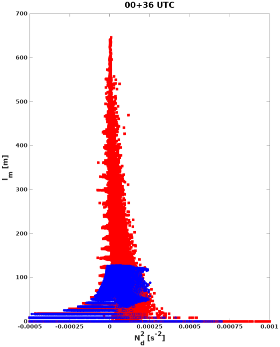
Conclusions

- l_m is indentified as a scale to which L_{TKE} should be assigned
- The impact of used averaging operators for L_{up} and L_{down} is relatively small
- Inclusion of shear effects into BL89 integral enables the use of desired global κ-scalling, but doesn't solve the problem of too strong mixing above the PBL
- Seeking for a solution to reduce mixing above the PBL should be done in some physical way, rather than imposing sharp cut-off
- More detailed analysis of the impact of changes in l_m on TKE is crucial for further work, either by using DDH or some other 'manual' way

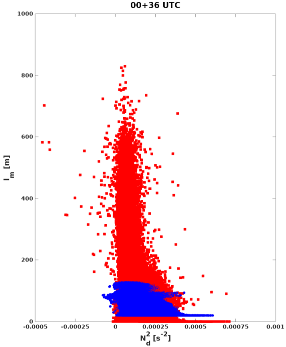
Extra slide



Inside PBL



Inside PBL



Above PBL