Radiative transfer problem in NWP context

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Introduction

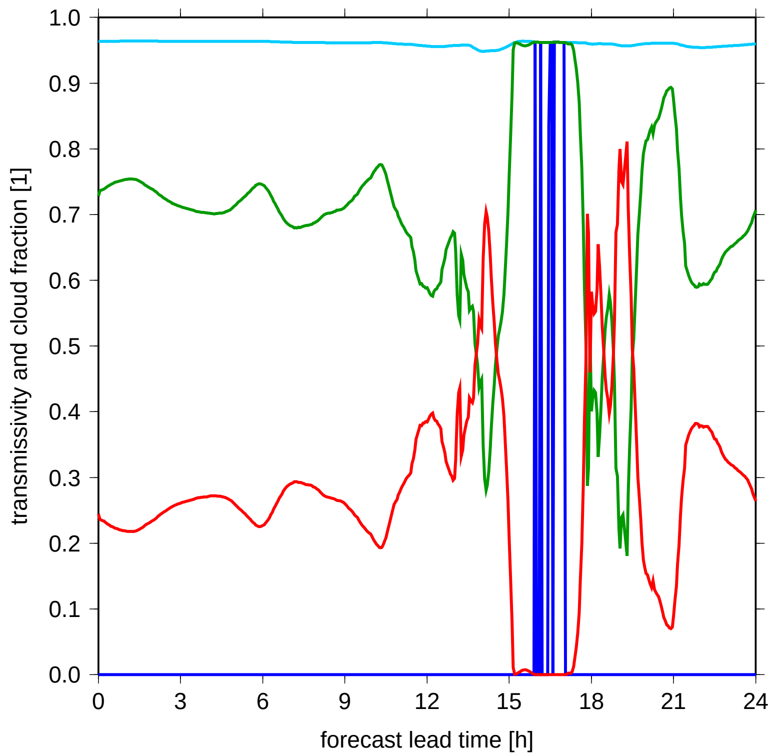
- solar heating is one of the key atmospheric forcings
- interaction of radiation with earth's surface, atmospheric gases, aerosols and clouds significantly influences energy budget and is thus important for determining weather and climate
- even if fundamental principles of radiative transfer are well known, its straightforward numerical computation is not feasible in NWP context
- main complicating factors are multiple scattering, number of thermal exchanges growing quadratically with number of layers and strong spectral variation of absorption coefficient
- set of approximations must be used to make the numerical computation reasonably expensive, on the other hand one wishes to keep as much realism as possible
- there is thus neverending cost versus accuracy dilemma

Two ways to reduce CPU cost

- radiative transfer computations are expensive part of model physics
- one possibility how to reduce CPU cost is to perform radiative computations on coarser grid than the rest of model physics
- main drawback of such approach is smoothed cloudiness field entering radiation, as well as lower resolution outcome
- another possibility is to update optical properties of radiatively active species not in every model timestep, but on time scales sufficient to resolve their temporal evolution
- such intermittent approach can work only if amount of stored global fields is reasonable

Clouds as main intermittency limitation

24 hour point evolution of model fields at level 65 ($\sim 850\,$hPa), summer convection case



cloud fraction n

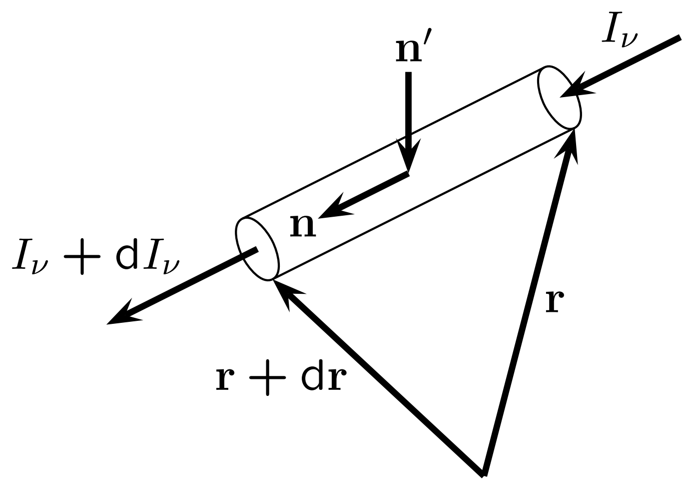
longwave transmissivity of clearsky part $a_{4\mathrm{C}}$

longwave transmissivity of cloudy part $a_{4\mathrm{N}}$

longwave transmissivity of the layer $a_4 = (1-n) \cdot a_{4\mathrm{C}} + n \cdot a_{4\mathrm{N}}$

Problem I – multiple scattering

Radiative transfer equation



$$
I_\nu(\mathbf{n}, \mathbf{r} + \mathrm{d}\mathbf{r}) = I_\nu(\mathbf{n}, \mathbf{r}) \overbrace{-k_\nu^{\mathrm{abs}} \rho \, |\mathrm{d}\mathbf{r}| \, I_\nu(\mathbf{n}, \mathbf{r})}^{\text{absorption}} + \overbrace{k_\nu^{\mathrm{abs}} \rho \, |\mathrm{d}\mathbf{r}| \, B_\nu(T(\mathbf{r}))}^{\text{emission}} +
$$
$$
+ \underbrace{k_\nu^{\mathrm{scat}} \rho \, |\mathrm{d}\mathbf{r}| \cdot \left[-I_\nu(\mathbf{n}, \mathbf{r}) + \frac{1}{4\pi} \oint_{4\pi} P_\nu(\mathbf{n} \cdot \mathbf{n}', \mathbf{r}) I_\nu(\mathbf{n}', \mathbf{r}) \, \mathrm{d}\Omega' \right]}_{\text{scattering}}
$$

$$
\frac{1}{2} \int_{-1}^{1} P_\nu(\mu, \mathbf{r}) \, \mathrm{d}\mu = 1 \qquad \mu \equiv \mathbf{n} \cdot \mathbf{n}' = \cos \Theta
$$

ρ – density, ν – wavenumber, $I_\nu(\mathbf{n}, \mathbf{r})$ – intensity (radiance),
$B_\nu(T)$ – blackbody radiance, $P_\nu(\mu, \mathbf{r})$ – scattering phase function

Radiative transfer equation for horizontally homogeneous plane parallel atmosphere

$$
\begin{aligned}
\mu \frac{\mathrm{d}I_\nu}{\mathrm{d}\delta_\nu}(\mu,\phi) = & -I_\nu(\mu,\phi) + (1-\varpi_\nu)B_\nu(T) + \\
& + \frac{\varpi_\nu}{4\pi}\Bigg[\int_0^{2\pi}\int_{-1}^{1} P_\nu(\mu,\phi,\mu',\phi') I_\nu(\mu',\phi')\,\mathrm{d}\mu'\,\mathrm{d}\phi' + \\
& + S_{0\nu}\exp\left(-\frac{\delta_\nu}{\mu_0}\right) P_\nu(\mu,\phi,\mu_0,\phi_0)\Bigg]
\end{aligned}
$$

$$
P_\nu(\mu,\phi,\mu',\phi') \equiv P_\nu\left(\mu\mu' + \sqrt{1-\mu^2}\sqrt{1-\mu'^2}\cos(\phi-\phi')\right)
$$

$$
\mathrm{d}\delta_\nu = (k_\nu^{\mathrm{abs}} + k_\nu^{\mathrm{scat}})\rho\,\mathrm{d}z \qquad \varpi_\nu = \frac{k_\nu^{\mathrm{scat}}}{k_\nu^{\mathrm{abs}} + k_\nu^{\mathrm{scat}}}
$$

I_ν – intensity of diffuse flux
$S_{0\nu}$ – direct solar flux at the top of atmosphere
μ – cosine of zenithal angle
ϕ – azimuthal angle
(μ_0,ϕ_0) – direction from the sun

Two stream approximation (solar band)

- in two stream approximation, angular distribution of intensity with just two degrees of freedom is assumed

- angular integration then yields linear system for fluxes:

$$\frac{\mathrm{d}F^{\uparrow}}{\mathrm{d}\delta} = \alpha_1 F^{\uparrow} - \alpha_2 F^{\downarrow} - \alpha_3 \frac{S}{\mu_0} \qquad \alpha_1 = 2[1 - \varpi(1-\bar{\beta})]$$

$$\frac{\mathrm{d}F^{\downarrow}}{\mathrm{d}\delta} = \alpha_2 F^{\uparrow} - \alpha_1 F^{\downarrow} + \alpha_4 \frac{S}{\mu_0} \qquad \alpha_2 = 2\varpi\bar{\beta}$$

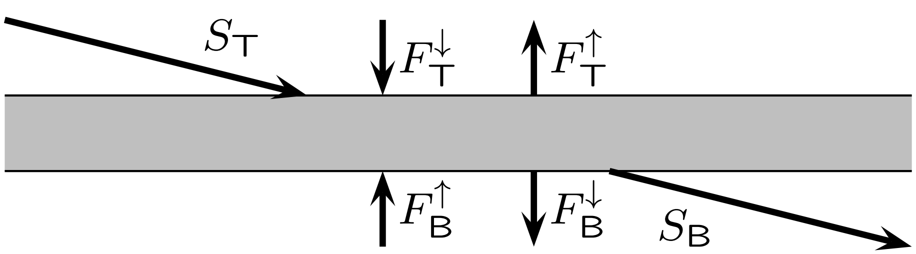
$$\frac{\mathrm{d}S}{\mathrm{d}\delta} = -\frac{S}{\mu_0} \qquad \alpha_3 = \varpi\beta(\mu_0)$$

$$\alpha_4 = \varpi[1-\beta(\mu_0)]$$

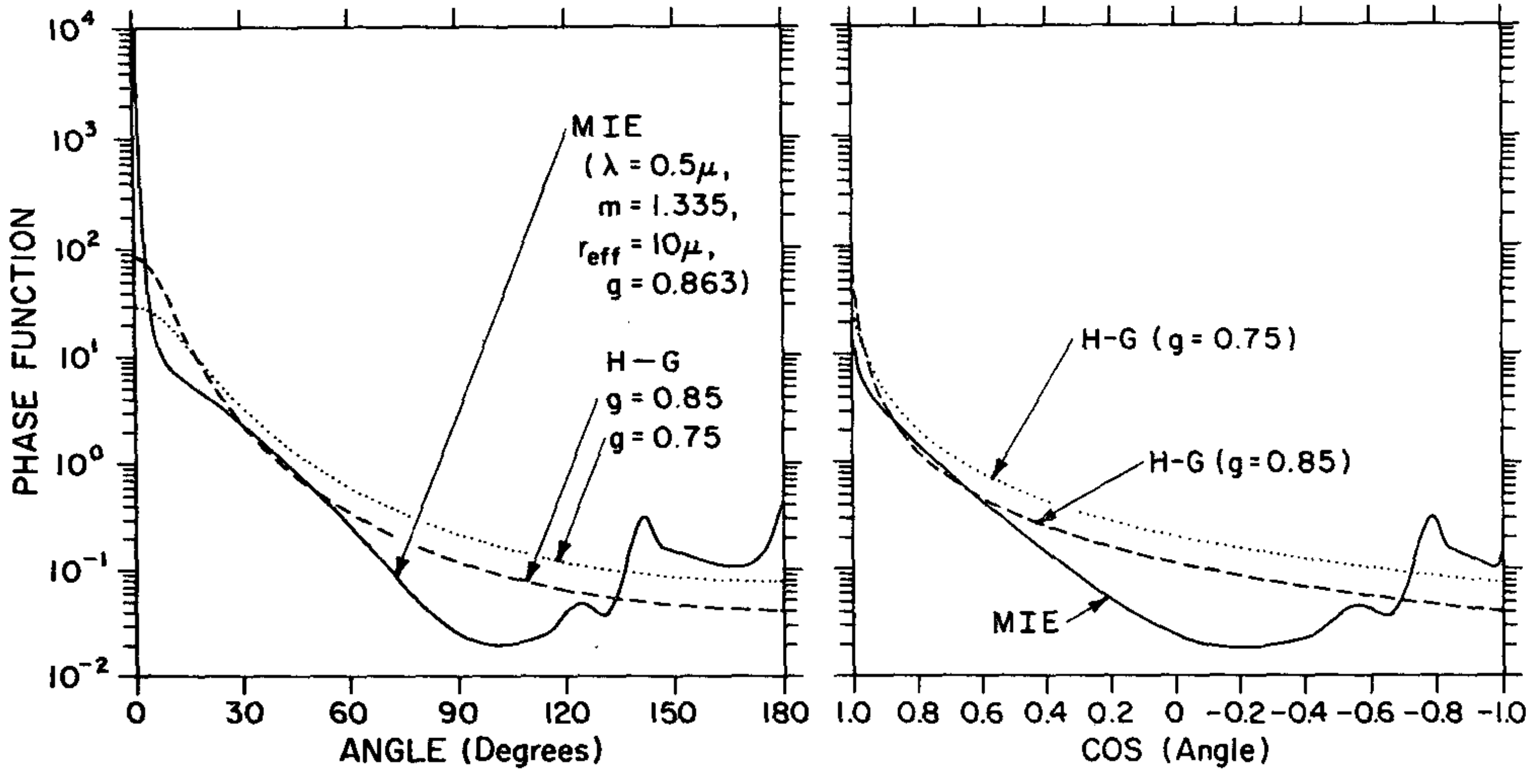
- shape of upscatter and backscatter fractions $\beta(\mu_0)$ and $\bar{\beta}$ is given by choice of phase function and angular dependency of intensity I

- for homogeneous layer, above system can be integrated analytically, providing linear relation between incoming and outgoing fluxes:

$$\begin{bmatrix} S_\mathrm{B} \\ F_\mathrm{B}^{\downarrow} \\ F_\mathrm{T}^{\uparrow} \end{bmatrix} = \begin{bmatrix} a_1 & 0 & 0 \\ a_2 & a_4 & a_5 \\ a_3 & a_5 & a_4 \end{bmatrix} \cdot \begin{bmatrix} S_\mathrm{T} \\ F_\mathrm{T}^{\downarrow} \\ F_\mathrm{B}^{\uparrow} \end{bmatrix}$$



Typical cloud phase function and its Henyey-Greenstein approximation



Mie scattering phase function for water cloud with droplet effective radius 10 μm at wavelength 500 nm, plus Henyey-Greenstein phase functions with asymmetry factors 0.85 and 0.75. Values at zero scattering angle are 9700, 86 and 28 respectively. (taken from Wiscombe 1977)

$$P_{\text{HG}}(\mu) = \frac{1 - g^2}{[1 + g^2 - 2g\mu]^{3/2}}$$

Delta scaling

- when the true phase function P is strongly asymmetric, it is advantageous to approximate it as a combination of forward δ-peak and less asymmetric scaled phase function P'

$$P(\mu) = 2f\delta(1-\mu) + (1-f)P'(\mu),$$

where δ is Dirac delta function and f is proportion of energy scattered in forward direction

- inserting above phase function into radiative transfer equation and repeating two stream developments leads to identical system (*) in **scaled variables**:

$$\begin{aligned} \delta' &= (1-\varpi f)\delta & \qquad k^{\mathsf{abs}'} &= k^{\mathsf{abs}} \\ \varpi' &= (1-f)\varpi/(1-\varpi f) & \qquad k^{\mathsf{scat}'} &= (1-f)k^{\mathsf{scat}} \\ g' &= (g-f)/(1-f) & & \end{aligned}$$

- scaled system of equations is referred to as δ-two stream system

(*) It is assumed that direct radiation scattered in forward direction remains direct. Exact identity holds only for μ_0 such that scaled upscatter and backscatter fractions fulfil relation $\beta(\mu_0) = 2\bar{\beta}\mu_0$.

Adding method

- for homogeneous layer, transmissivities and reflectivities a_1–a_5 can be expressed analytically via layer optical depth δ and coefficients of δ-two stream system α_1–α_4
- vertically non-homogeneous atmosphere can be represented as a set of homogeneous plane parallel layers, each divided to clearsky and cloudy parts
- fluxes leaving one layer are entering the next layer and are redistributed between its clearsky and cloudy parts according to cloud overlap assumption (this overlap is **geometric** and should not be confused with **spectral** one)
- inside layer, there is no lateral exchange between its clearsky and cloudy parts
- one ends up with linear system of equations for fluxes at layer interfaces
- after specifying boundary conditions, it can be solved easily by Gaussian elimination and back substitution

Problem II – thermal exchanges

(will be covered by Jean-François after explaining optical saturation)

Problem III – spectral integration

Optical saturation

- band transmission of non-scattering homogeneous layer is given as

$$\tau = \int_{\Delta\nu} \mathrm{exp}(-k_\nu u) w_\nu \, \mathrm{d}\nu,$$

where u is absorber amount and weight w_ν is proportional to intensity of incident radiation

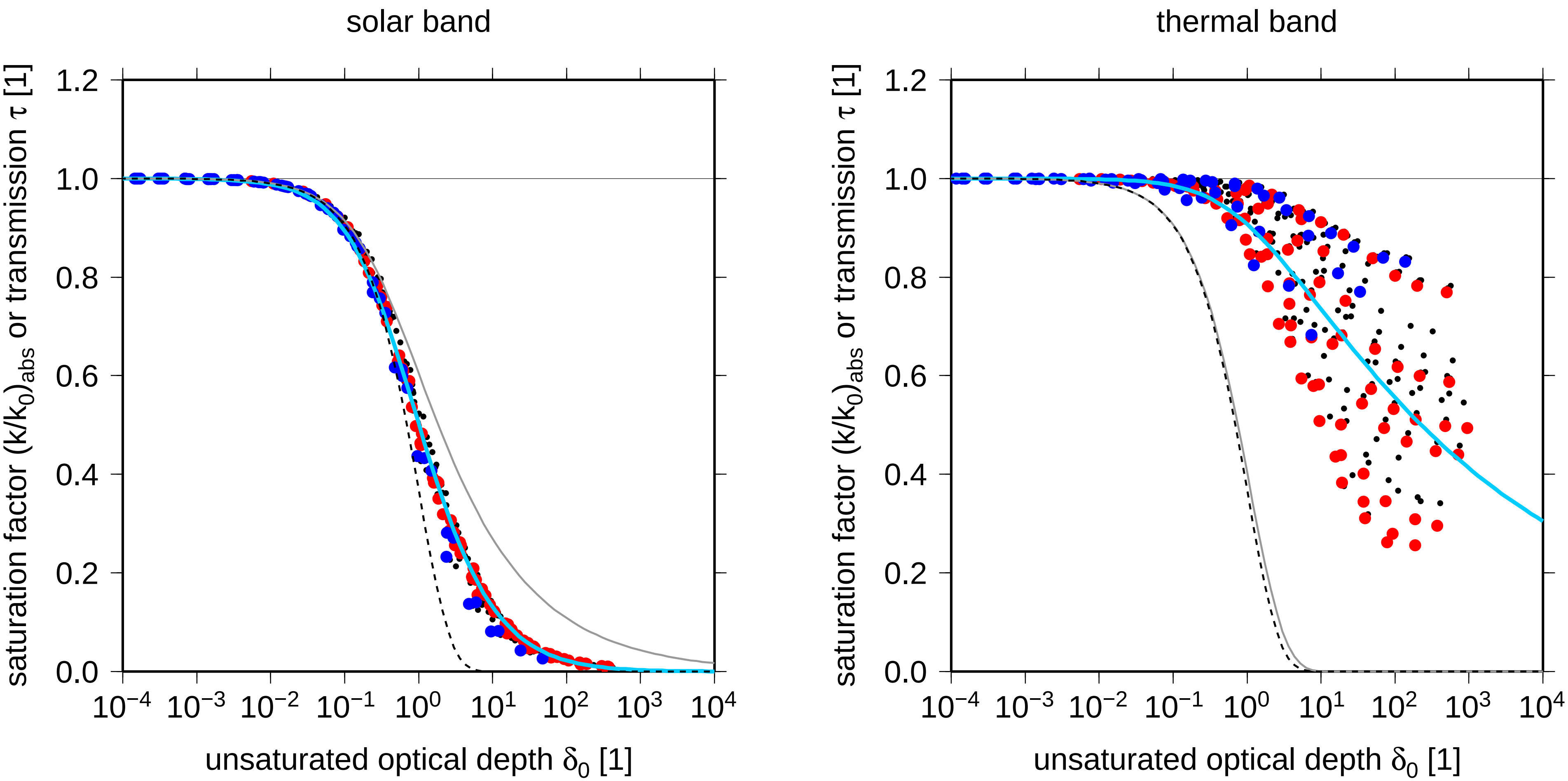
- from convex shape of exponential function it follows that:

$$\tau = \overline{\mathrm{exp}(-k_\nu u)} \geq \mathrm{exp}(-\overline{k_\nu} u)$$

- it means that band transmission is higher than transmission given by mean absorption coefficient, and equality is approached only in $k_\nu u \ll 1$ limit

- this effect is known as optical saturation and it is central problem of spectral integration

Saturation of cloud absorption



Broadband saturation factor $k^{\mathrm{abs}}/k_0^{\mathrm{abs}}$ as a function of unsaturated optical depth δ_0, computed from narrowband data: red – Stephens 1978 liquid clouds; blue – Rockel et al. 1991 ice clouds; black – mixed clouds. Thick cyan curve is fitted saturation factor, thin black dotted curve is unsaturated transmission and thin grey solid curve is saturated transmission.

The key challenge for thermal radiation computations

- Avoiding a CPU cost proportional to N^2 (i.e. what happens with emissivity methods) .
- Allowing at the same time the accounting of multiple scattering, while handling a spectral structure of absorption that favours saturation.
- Doing it as economically as possible, once in any system with N-proportionality.
- Caring for the cloud-gas combination in terms of intermittency, when full computations at each time-step and each grid-point are too expensive.

Two paths: Correlated k-distribution (CKD) & Net Exchanged Rates (NER)

«k-distribution» / «Exponential Sum Fitting Technique» / «picket-fence»

- For a given spectral interval, one goes back to a monochromatic framework for a series of «two-stream + adding» integrations where only the gaseous part will vary, the «grey» part of the computation remaining unchanged. The weighted average of intermediate results will eventually give directly the searched fluxes.
- One may consider this as a sampling of the (logarithmic) histogram of the absorption intensities in the chosen spectral domains, but the 'exponential sum fitting technique' (ESFT) formalism is more «didactic»:

$$\mathrm{Tr}(u, p, T) = \sum_{i=1,N} w_i . e^{-k_i . u . (p/p^*)^{a_i} . (T/T^*)^{b_i}}$$

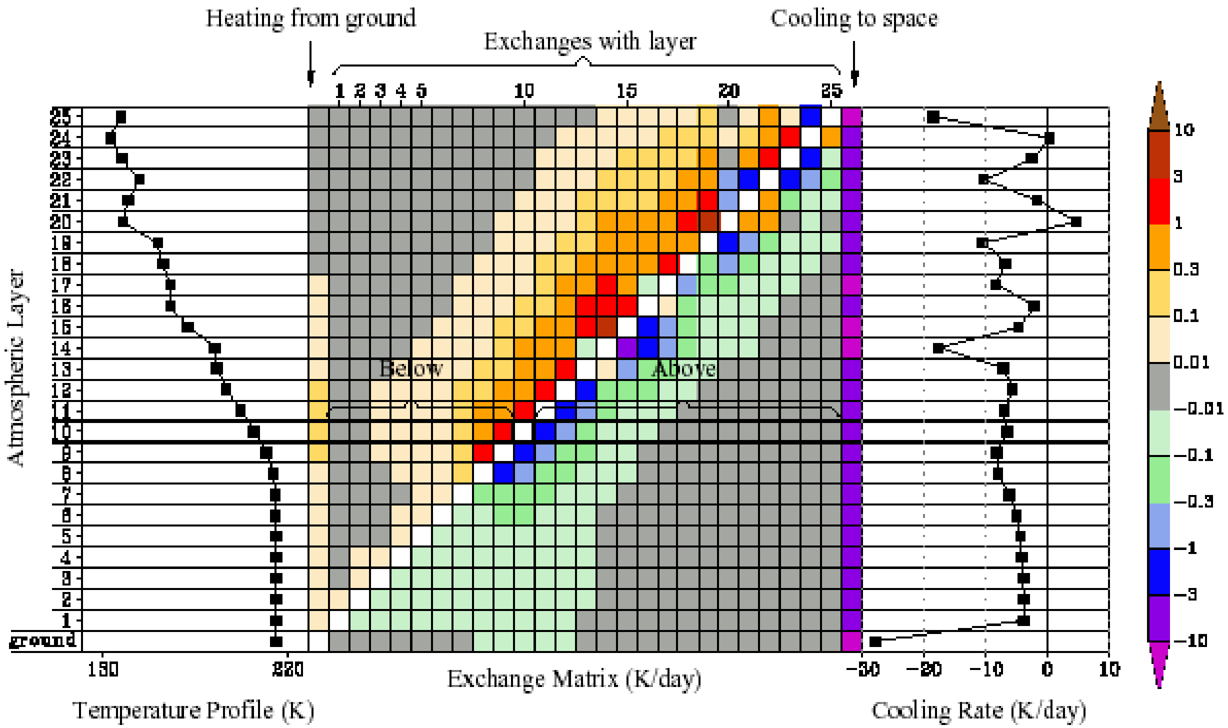
$$\text{with} \sum_{i=1,N} w_i = 1$$

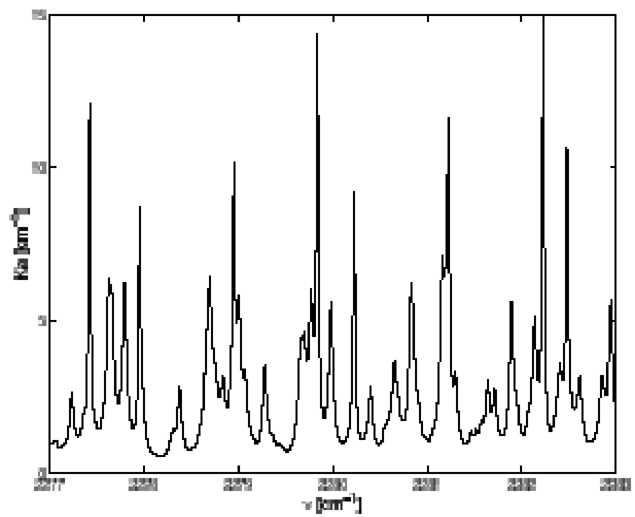
- The p,T dependency may become more complex (CKD)

The Net Exchange Rate formulation (NER) for the thermal case

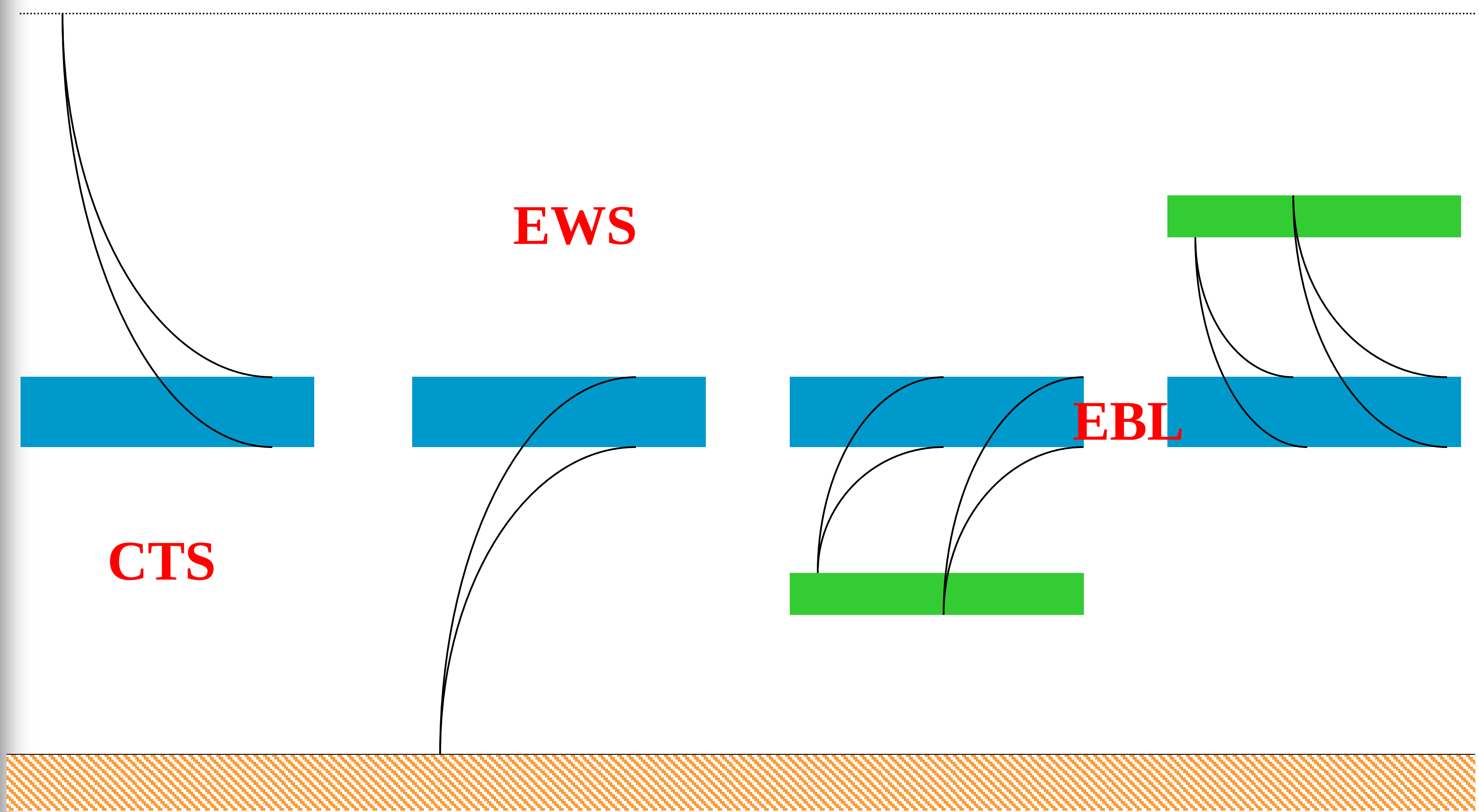
- One divides the atmosphere in ‘bodies’ (layers for us) and, considering each pair of them, one directly computes the net balance of exchanged photons.
- Contrary to all flux computation methods (CKD included), this allows to neglect a lot of symmetrically exchanged photons => **simplicity**.
- It also leads to a principle of reciprocity: the warmer body will always heat the colder one => **realism**.
- It ensures energy conservation => **accuracy**.

Link between NER and saturation





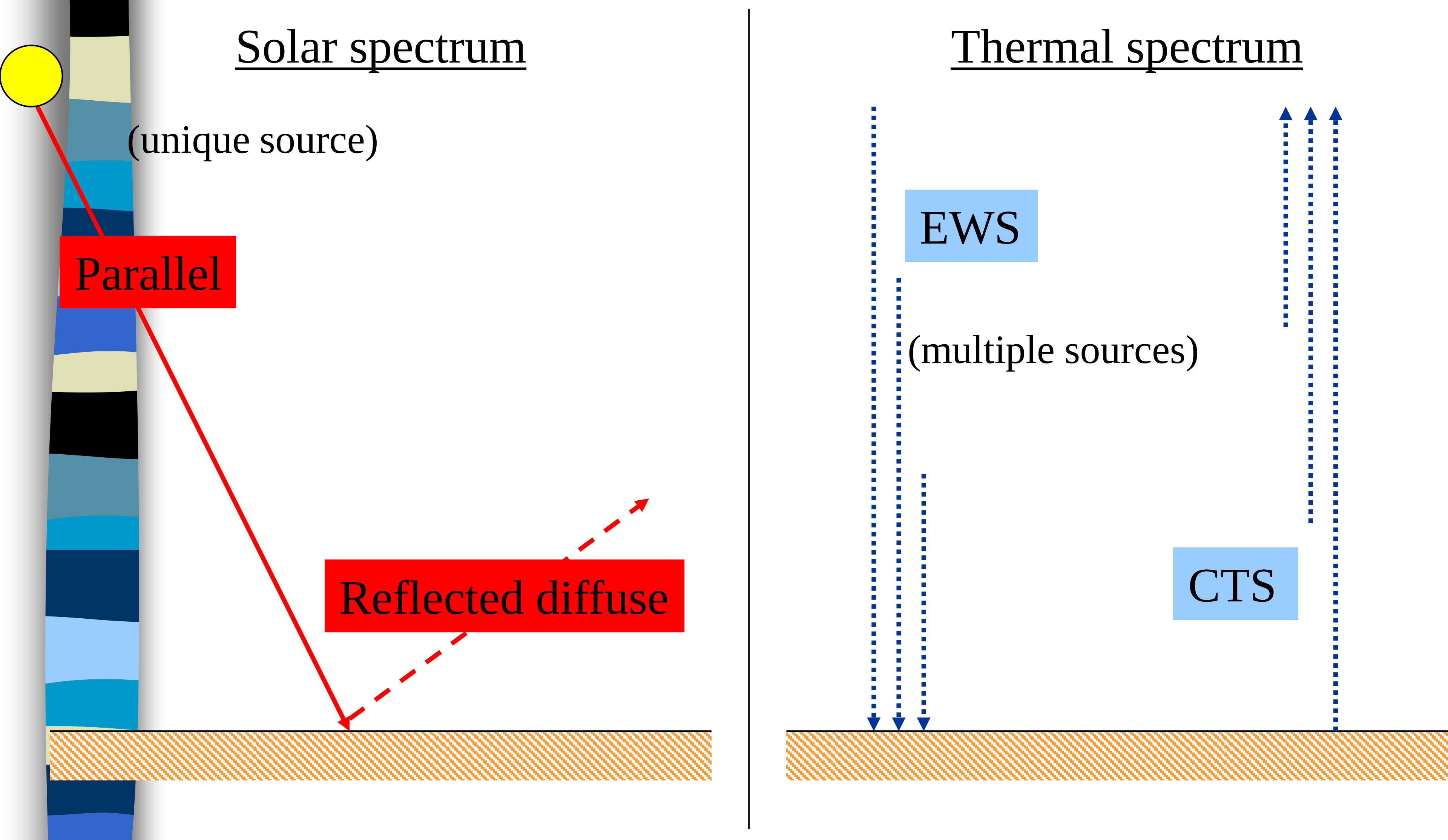
CTS+EWS+EBL decomposition of the thermal radiative exchange terms in absence of scattering



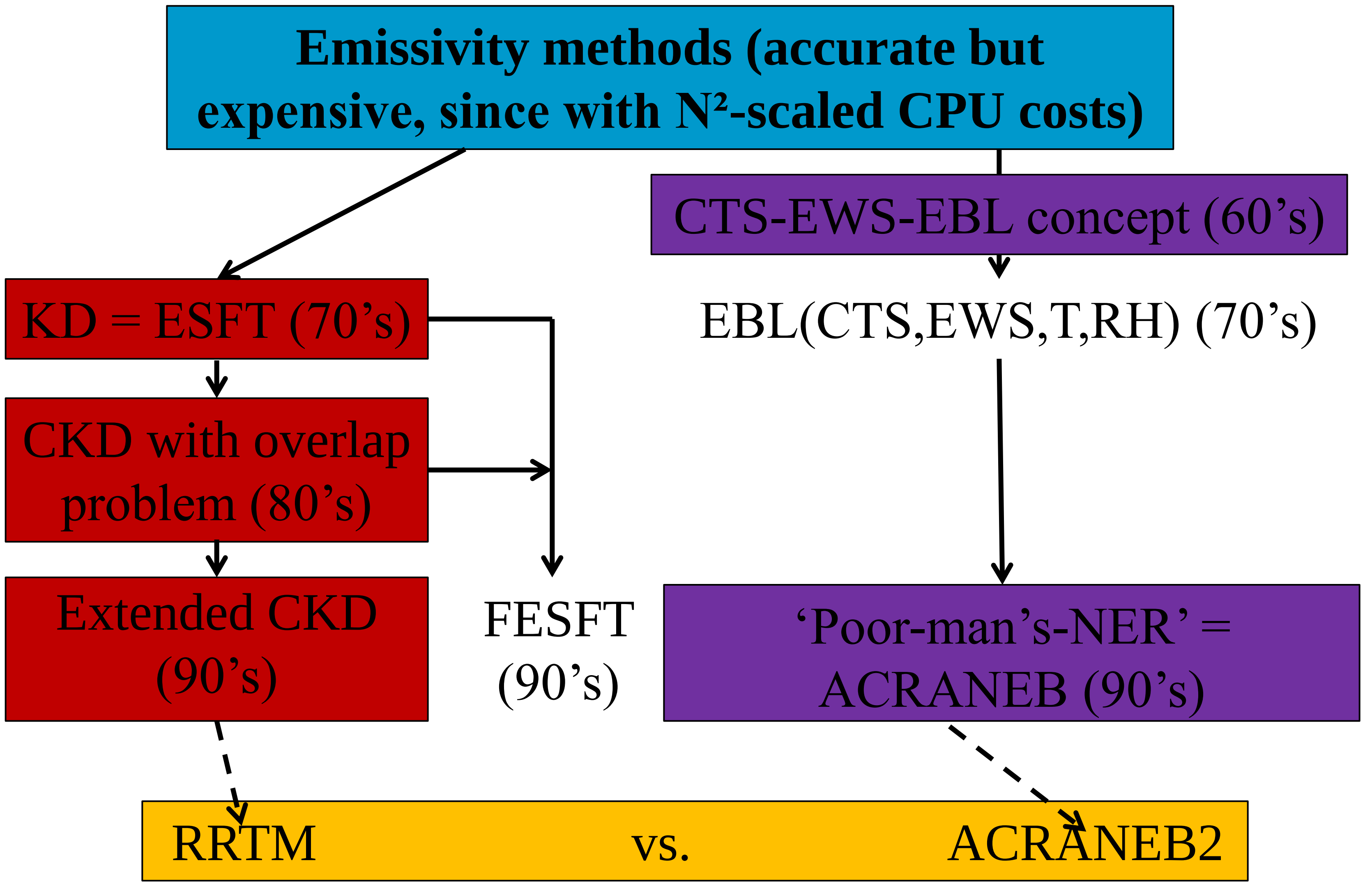
Method of idealised optical paths

- The basis of this method is very simple. One computes exactly the optical depths of gaseous absorption for every layer in a simplified geometry and one reinjects them as such in the «two-stream + adding» formalism, together with the ‘grey body’ effects.
- For the solar part, the computation for S is straightforward and that for F↓ and F↑ relies on the absorption during the return path of a photon reflected at the surface but never scattered.
- For the thermal part, the «CTS» and «EWS» computations rely on obvious direct optical paths. There remains, like always, the ‘CPU barrier’ for the «EBL» calculations, if no additional parameterisation exists.

Idealised optical paths



Historical perspective for the LW part of the RTE solution



Literature on the CKD formulation

- Arking, A. and K. Grossmann, J. Atmos. Sci., 29 (**1972**) 937-949.
  - **Application to the atmosphere of the (stellar) idea of Ambartzumian (1936): ESFT = k-distribution method**
- Lacis, A., W.C. Wang and J. Hansen, NASA Conf. Publ. 2076 (**1979**) 309-314.
  - **Proposal to extend ESFT to the non-homogeneous case via correlated k-*distribution (CKD), by introduction of a* 'T,p *dependency' for the* pseudo-monochromatic k coefficients of the exponential sum fitting**
- Ritter, B. and J.-F. Geleyn, Mon. Wea. Rev., 120 (**1992**) 303-325.
  - **Intermediate solution between KD and CKD as well as integrated proposal for the gaseous overlap (FESFT) via a reference non-gaseous computation => <u>no offspring</u>**
- Fu, Q. and K.N. Liou, J. Atmos. Sci., 49 (**1992**) 2139-2156.
  - **Solution of the CO2-H2O overlap problem by a targeted extension of the CKD logic (T,p,q dependency).**
- Mlawer, E.J., S.J. Taubman, P.D. Brown, M.J. Iacono and S.A. Clough, Journal of Geophysical Research (**1997**) 16663-16682.
  - ***Start of the very successful RRTM development …***

Literature on the NER formulation

- Green, J.S.A., Quart. J. Roy. Met. Soc., 93 (**1967**) 371-372.
  - **Principles of the CTS-EWS-EBL decomposition**
- Joseph, J.M. and R. Bursztyn, Journal of Applied Meteorology, 15 (**1976**) 319-325.
  - **Proposal to compute CTS-EWS exactly and to infer EBL heuristically from there (with T and RH as additional input information) => <u>no offspring</u>**
- Eymet, V. , J.-L. Dufresne, R. Ricchiazzi, R. Fournier and S. Blanco, Atmospheric Research, 72 (**2004**) 239-261.
  - **Revival (and in-depth analysis) of the NER partition (for Monte-Carlo coding, not NWP thus) after a long gap!**
- Masek, J. et al. (**2015**), in preparation.
  - **See the various presentations at A1WD14 on a new way to consider *the broadband NER approach for NWP …***

Band models

- computing band gaseous transmission directly by numerical quadrature would require spectral resolution high enough to resolve individual absorption lines

- such line by line approach is completely out of NWP scope and is used only as very accurate reference

- band models make assumptions about distribution of line positions and strengths and derive formula for band optical depth

- in case of Malkmus band model it has simple form

$$\delta = \frac{a}{2b}\left(\sqrt{1+4bu}-1\right),$$

  where coefficients $a(T)$ and $b(p,T)$ can be expressed via mean line strengths and halfwidths in assumed spectral interval

- width of spectral interval must be much larger than typical line halfwidth, but narrow enough so that weight function can be assumed constant $\Rightarrow$ narrowband approach

Scaling approximations

- for single Lorentz line, non-homogeneous optical paths can be treated by Curtis-Godson approximation
- it seeks equivalent homogeneous path giving the same weak and strong line limits as non-homogeneous path $\Rightarrow$ two-parametric scaling
- thanks to correspondence of band model parameters with mean line strengths and halfwidths, Curtis-Godson approximation can be combined with band model approach
- Curtis-Godson approximation introduces largest error in the region between weak and strong line limits
- it is least accurate for ozone, whose absorption increases with decreasing pressure
- accuracy could be improved by using some 3-parametric scaling

From narrowband to broadband

- in broad spectral band, assumptions of band model are not met and variation of spectral weights cannot be neglected

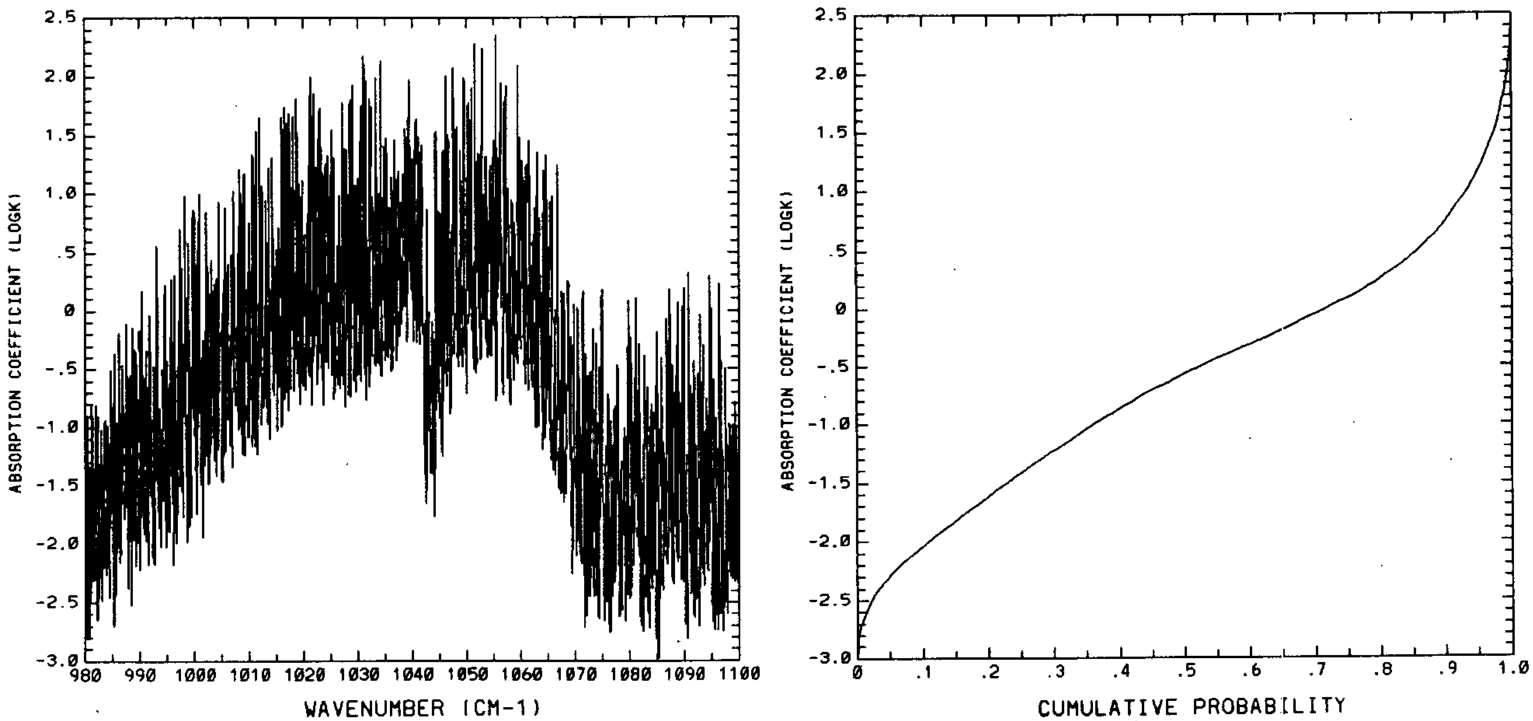
- band model can account for these departures by using a posteriori empirical corrections

- since the correspondence of band model parameters with mean line widths and strengths is preserved, non-homogeneous optical paths can still be treated by scaling approximation

- width of spectral intervals is then virtually unlimited (it is possible to have single shortwave and single longwave interval)

- for mixture of gases with varying composition, broadband model must be applied separately for each component and the effect of non-random spectral overlaps between them must be parameterized

Reordering of k values



Ozone absorption coefficient k [cm^{-1} atm^{-1}] as a function of wavenumber ν (left) and cumulative probability g (right) for pressure 25 hPa and temperature 220 K. (taken from Fu and Liou 1992)

k-distribution method

- fraction of spectral interval where $k_\nu < k$ is given by smoothly increasing function $g(k)$

- its inverse $k(g)$ can be viewed as reordering of k_ν values

- band transmission of homogeneous layer can then be reexpressed as:

$$\tau = \frac{1}{\Delta\nu}\int_{\Delta\nu} \exp(-k_\nu u)\,\mathrm{d}\nu = \int_0^1 \exp[-k(g)u]\,\mathrm{d}g$$

- strength of reordering comes from the fact that unlike k_ν, function $k(g)$ can be fitted easily, so the above integral can be computed numerically as a sum of few decaying exponentials weighted by increments Δg

- limitation is the assumption of constant spectral weights, without which reordering cannot work $\Rightarrow$ width of spectral band is limited

Correlated assumptions

- dependency of gaseous absorption on pressure and temperature causes problems when k-distribution method is applied to non-homogeneous optical paths

- band transmission of non-homogeneous layer is given as

$$\tau = \int_0^1 \exp\left[-\int_{z_1}^{z_2} k(g,p,T)\rho\,\mathrm{d}z\right]\,\mathrm{d}g$$

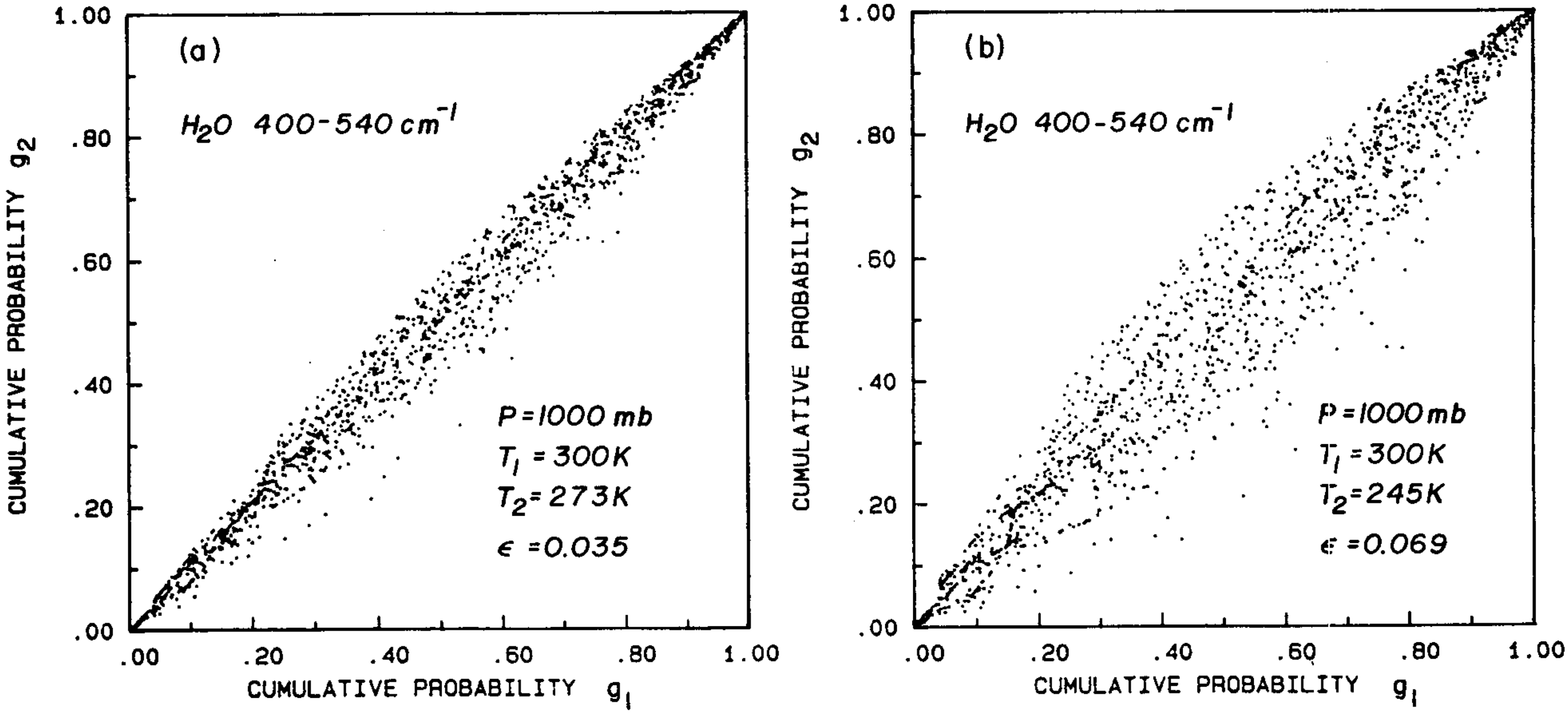
  only when orderings of k_ν values at different heights are the same

- this can be assured when **correlated assumptions** about $k_\nu(p,T)$ are made

- they are valid for some idealized cases (single line, periodic lines, multiple lines in weak and strong line limits), but not generally

- departure from correlated assumptions leads to **blurring**, with temperature variation having stronger effect than pressure variation

Blurring in correlated k-distribution method



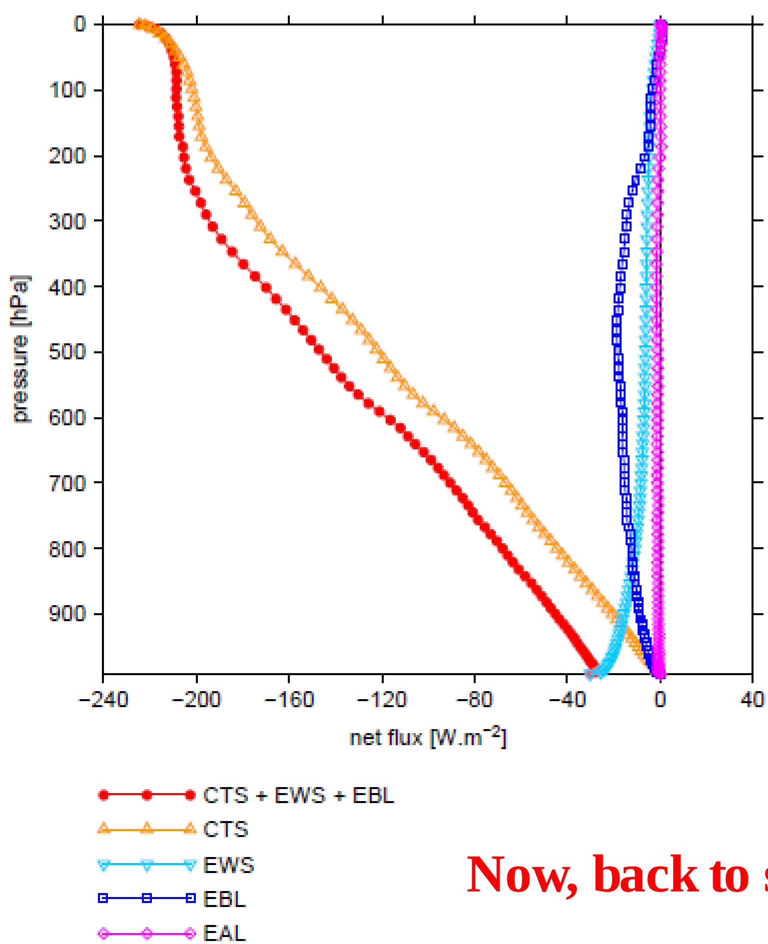
Blurring of correlated assumptions due to the temperature effect for H_2O in the spectral region 400–540 cm^{-1} at a pressure 1000 hPa. Layer temperatures are 300 K and 273 K (left), respectively 300 K and 245 K (right).
(taken from Fu and Liou 1992)

Back to CKD versus NER

(continuation by Jean-François)

Why all these efforts to 'merge' NER with the 'idealised path method'? (1/4)

Net fluxes (positive downwards)



- Main flavour of the NER interpretation in ACRANEB2: the dominant terms (**CTS** & **EWS**) must be computed exactly, while the more expensive and less important **EBL** terms may be approximated. The aim is the same as in Joseph and Bursztyn (1976), but the application far more rigorous …

Now, back to some look on the other side of the fence

Why all these efforts to 'merge' NER with the 'idealised path method'? (2/4)

-The adjointing of the Laplace transform (a third solution not developed here) does not work in a straightforward way with multiple sources.

-The ESFT has a drawback:

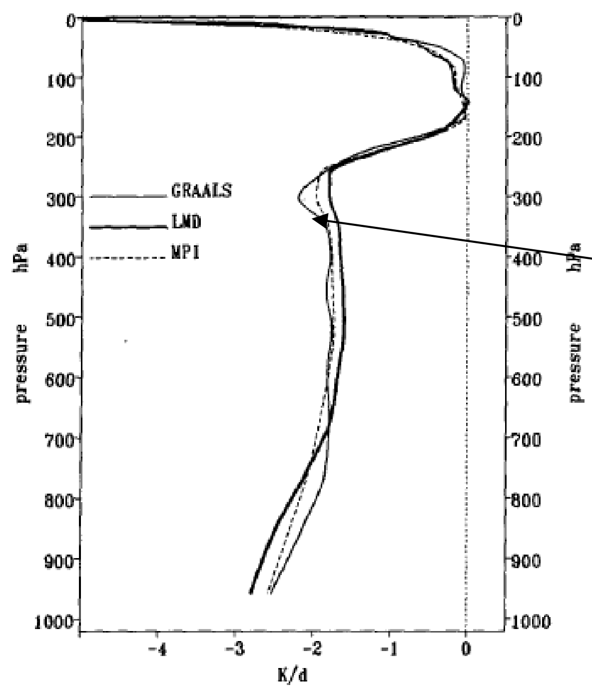


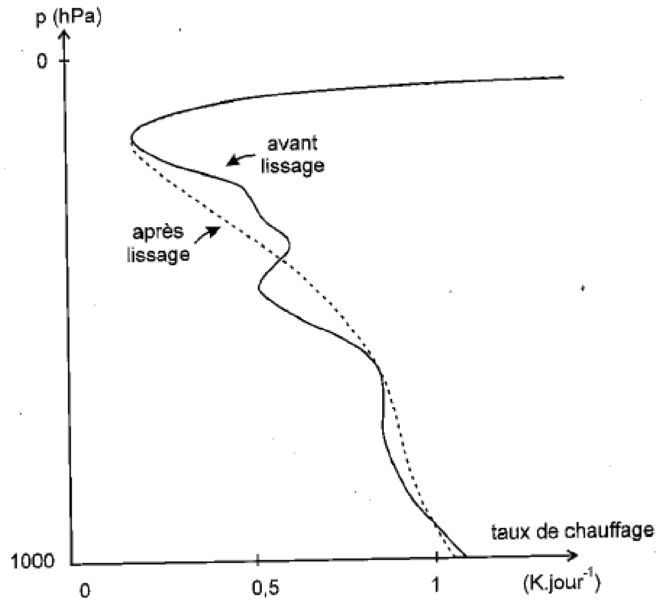
FIG. 8. Heating rates in a clear-sky midlatitude summer atmosphere due to thermal radiation. Comparison between line-by-line models (LMD, MPI) and GRAALS.

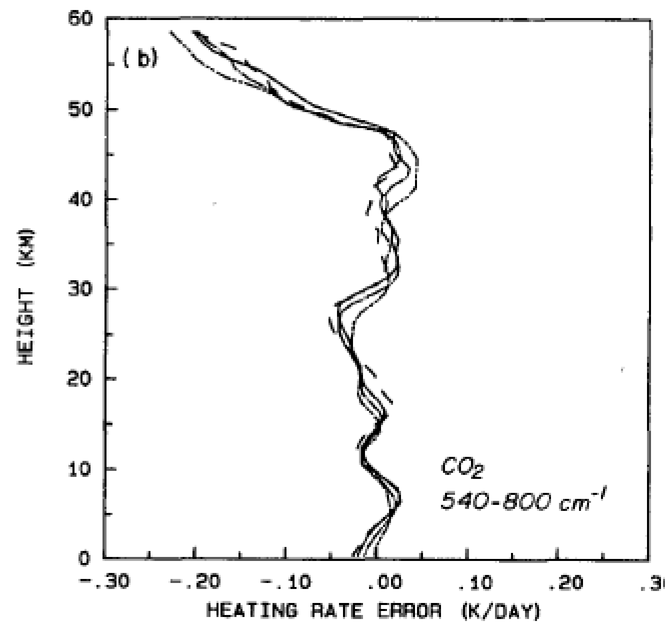
Consequences of numerical undersampling in the approximation of **Tr(u,p,T)**

Zoom (Ritter and Geleyn, 1992)

Why all these efforts to 'merge' NER with the 'idealised path method'? (3/4)

- The ESFT has a drawback (bis and ter):





Intentionally exaggerated view of the problem (very small number of terms, solar case)

Evaluation of the impact of a 'classical' reduction of the number of terms. Five standard atmospheres, Fu and Liou 1992

Why all these efforts to 'merge' NER with the 'idealised path method'? (4/4)

Furthermore, use of the NER technique nicely leads to the idea of selective intermittency for otherwise too expensive radiative computations. Results concerning gaseous transmission functions are 'compacted' by the NER 'hierarchisation', while the cloud aspects are reassessed at each time step (the combination then enters the 'eight solving steps', see elsewhere how …).

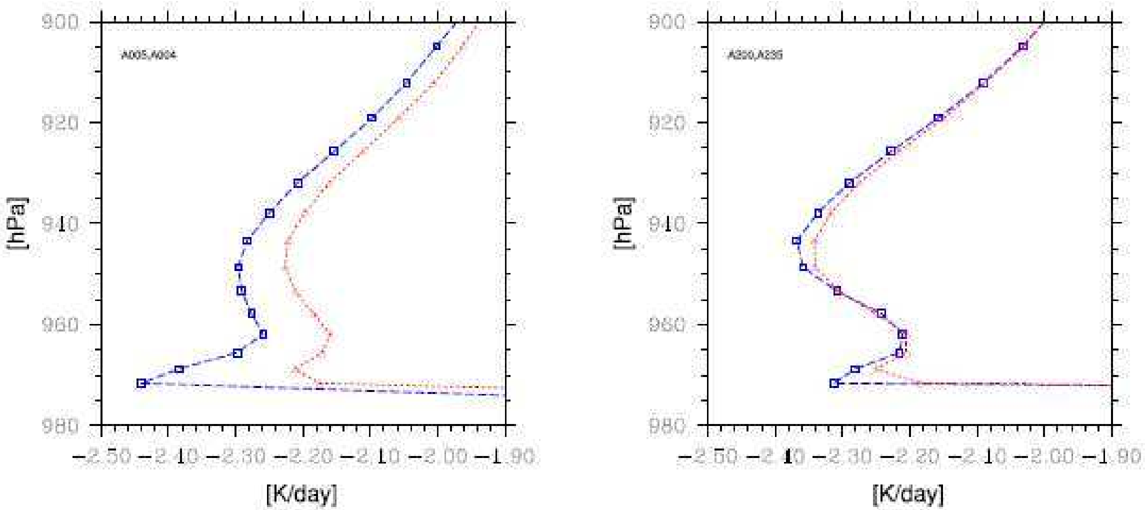


Figure 5: Impact of 1h intermittency in RRTM/FMR (left) and 1h/3h two level intermittency in ACRANEB2 (right), demonstrated on 12 hour domain averaged longwave heating rates: red – reference computation without any intermittency; blue – intermittent computation. Please keep in mind that RRTM/FMR intermittency applies also to clouds, while ACRANEB2 intermittency does not.

CKD vs. NER (+ idealised optical paths), summing up

- Both methods are deeply incompatible (using ESFT-type transmissivities in NER would be a nonsense => one is lead to the broadband approach for that case).
- Symmetrically, the reduction of the problem to a series of quasi-monochromatic computations makes any CTS-EWS-EBL decomposition superfluous in CKD => CPU savings cannot come from any hierarchisation of such terms.
- The pros and cons of each path are direct consequences of the above.

CKD (RRTM)

- Pros:
  - Clean interaction with multiple scattering (no need to evaluate the optical paths).
  - Quasi-infinite flexibility in the search for a cost/accuracy compromise.
  - Easy solution for the non-homogeneous paths (via the ‘correlated’ aspect).
- Cons:
  - Treating cheaply the gaseous spectral overlap problem requires compromises with respect to the ‘simplicity’ of the original method.
  - The numerical quadrature creates small but cumulative under-shoots / over-shoots.
  - There is a hidden redundancy in the computations: one solves n-times the same cloudy contribution; since it cannot be separated, this creates a problem when going to intermittency.

NER (ACRANEB2)

- Pros:
  - The gaseous spectral overlap problem can be cleanly distinguished from the fitting of the individual transmissivities.
  - The capacity to have multiple level intermittency (clouds at each time-step, gaseous transmissions less frequently, bracketing weights even less frequently) <u>is most probably a key asset</u>.
  - Even if not yet completely realised, the complementarity between two independent methods for evaluating the bracketing weights is potentially a strong asset for future steps.
- Cons:
  - All inaccuracies coming from the broad-band approach are combined, so that some ‘tuning’ is compulsory.
  - In particular, the consequences of the Curtis-Godson approximation for neither weak-line nor strong-line cases are directly felt.
  - The success for treating the Voigt case is welcome but unexplained.