Entropy-cycle-related shallow convection closure

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(with many thanks to D. Lewellen and P. Marquet)

A1WD, Vienna, Austria, 13/5/2014

The problem(s) (1/2)

- In ‘dry’ turbulence (even without water vapour), no problem for closing the equations (at whatever level of sophistication), since the potential temperature $\boldsymbol{\theta}$ is both the tracer of entropy and the way to compute the buoyancy flux (or conversion term).
- As soon as moisture starts to play a role, either by expansion ($\boldsymbol{R_v > R_d}$) or by latent heat release (resp. storage) ($\boldsymbol{L_{v/s}/(c_p T) >> 1}$) this ceases to be true and one needs additional parameterisation hypotheses.
- The basic computations are tractable in both extreme and homogeneous cases of ‘zero’ or ‘one’ cloud-cover $\boldsymbol{C}$.
- In between, the ‘classical’ method is to imagine a ***linear*** weighting between the two extremes and to parameterise the weighting factor $\hat{R}$ as a monotonous but usually non-identical function of $\boldsymbol{C}$.
- It is often claimed that Sommeria and Deardorff (1977) (***SD77***) showed that ‘neutrality’ (associated with no-skewness) means $\hat{R}=\boldsymbol{C}$.

The problem(s) (2/2)

- But:
  - (a) Is it a correct interpretation of the conservativity of moist entropy in mixing processes?
  - (b) Is it really what SD77 had in mind as prolongation of their study?
  - (c) Is there no better way to link cloud-cover, buoyancy and stability?
  - (d) Are the 'classical' predictors (for the needed additional information) necessarily linked to turbulence statisitics?
- Issue (a) was addressed in Marquet and Geleyn (2013) (***MG13***) on the basis of 'specific moist entropy'.
- Issue (d) is the topic of the proposal of Lewellen and Lewellen (2004) (***LL04***), see next slides.
- Our aim here is to treat primarily issues (b) and (c) and thus to come up with a complete proposal on the paper.

The LL04 proposal (1/3)

- The idea is to compute, for each layer, the $\hat{R}$ factor as if the moist turbulence was acting in a mass-flux way, with variables averaged below [‘***sc***’ index] representative of the updraft aspect and local averaged values across the grid box representing the downdraft aspect (local but ‘favouring mixing’).
- After some manipulations, this delivers;

$$\hat{R} \approx \frac{\overline{w'\theta_v'} - (1 + 0.61q_t)\overline{w'\theta_l'} - \alpha\overline{w'q_t'}}{\beta(a\overline{w'q_t'} - b\overline{w'\theta_l'})} \tag{10}$$

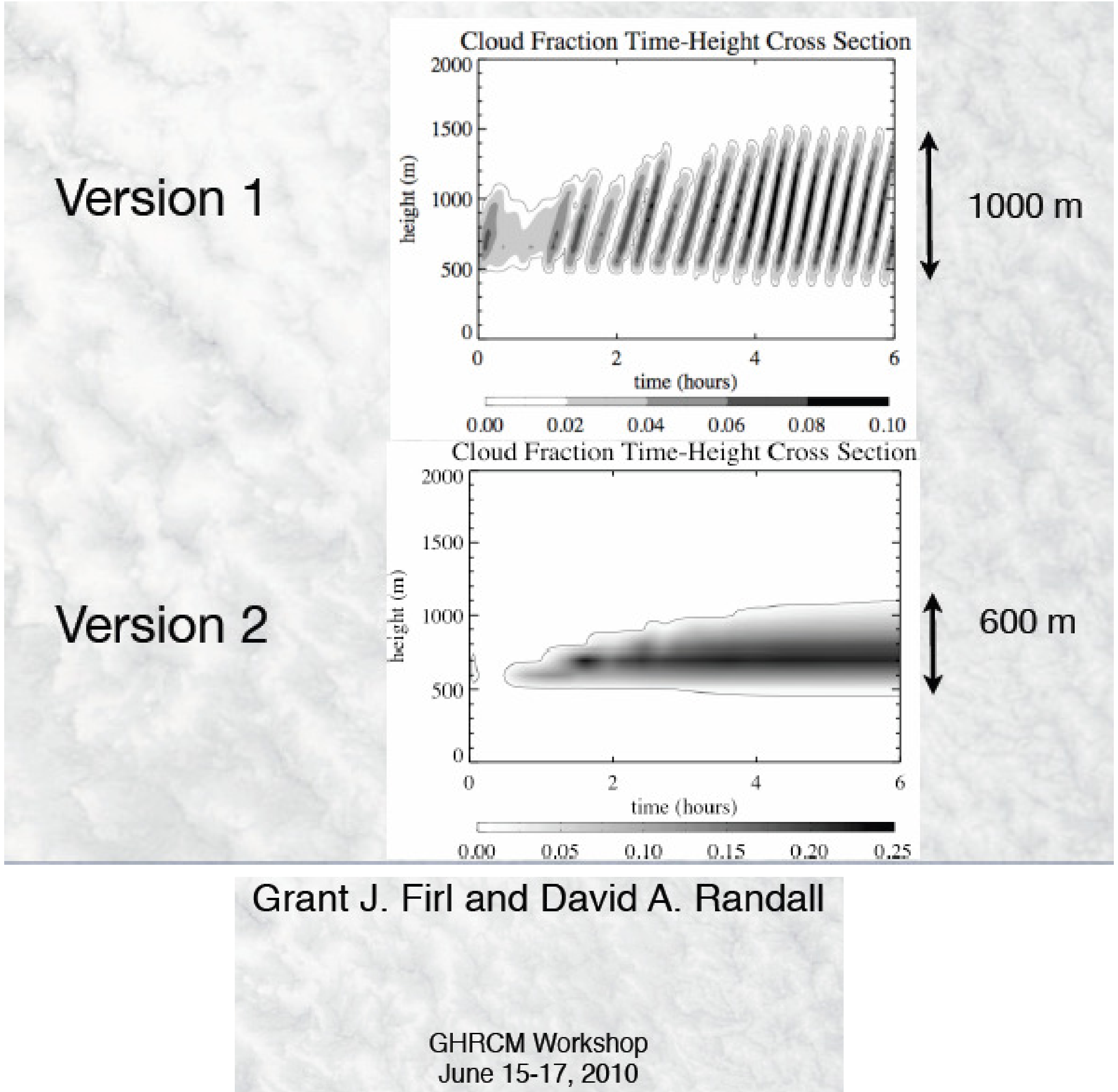
$$\approx \frac{(\theta_v^{sc} - \theta_v^{b}) - (1 + 0.61q_t)(\theta_l^{sc} - \overline{\theta}_l) - \alpha(q_t^{sc} - \overline{q}_t)}{\beta[a(q_t^{sc} - \overline{q}_t) - b(\theta_l^{sc} - \overline{\theta}_l)]}, \tag{11}$$

where a, b, α, β, and θ_v^b are all computed using $\overline{q}_t$ and $\overline{\theta}_l$.

- The first shape is in dependency on actual values (note: no higher-order terms, no saturation deficit information) for the parameterisation aspect. The second shape is in dependency on fluxes (based on many LES results) for diagnostic and, eventually, tuning purposes.

The LL04 proposal (2/3)

- Numerically it seems an important step forward:



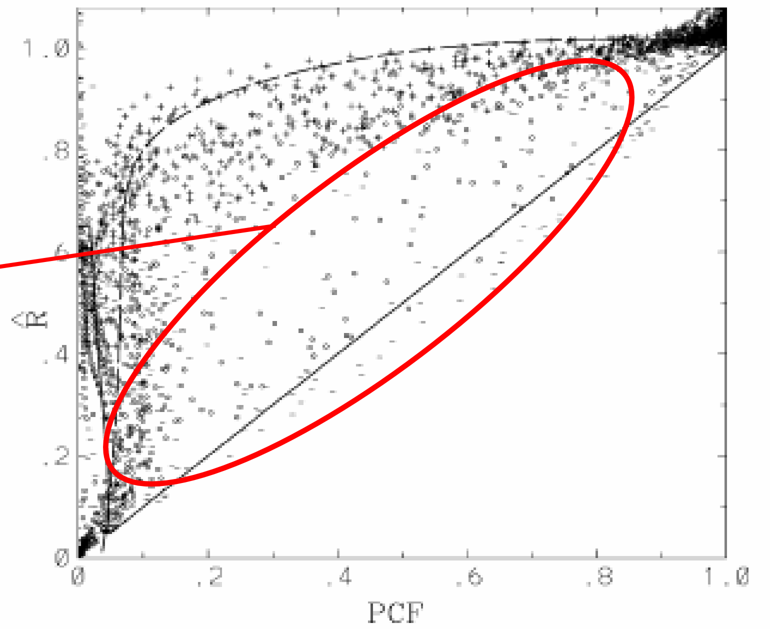
Cuijpers and Bechtold, 1995

LL04

The LL04 proposal (3/3)

- But:
  - The method still relies on the 'brute force' linear interpolation method, i.e. on the 'hear-say' from SD77.
  - Even if this would be OK, the question of how to link $\hat{R}$ and $\boldsymbol{C}$ for non neutral cases (correlation between upward motions and stonger moisture buoyancy impacts) is still treated heuristically:

Hint at some non-linearity?



  - Back now to the (a), (b) & (c) topics.

‘Classical turbulence’ interpretations (i.e. fully dry ones)

The prognostic TKE equation

$$\frac{\partial E}{\partial t} = A_{dv}(E) + \frac{1}{\rho}\frac{\partial}{\partial z}\rho K_E \frac{\partial E}{\partial z} + K_m\left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right] - \frac{g}{\theta}K_h\frac{\partial \theta}{\partial z} - \frac{C_\varepsilon E^{3/2}}{L}$$

Development of the terms of shear production and of production/destruction by buoyancy (‘conversion term’)

$$K_m\left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right] - \frac{g}{\theta}K_h\frac{\partial \theta}{\partial z} \approx K_m S^2\left[1 - \frac{K_h}{K_m}\frac{g}{\theta}\frac{\partial \theta}{\partial z} / S^2\right]$$

$$= K_m S^2\left[1 - \frac{K_h}{K_m}(N^2 / S^2)\right] = K_m S^2\left[1 - \frac{K_h}{K_m}R_i\right] = K_m S^2(1 - R_{if})$$

One thus establishes a direct link between the Richardson number, the Richardson-flux number, the conversion term (<w’.ρ’>) and the static stability (i.e. the squared BVF N^2). Should all this be reproduced identically in the ‘moist’ case? One has to realise that the above fully relies on a dual role of θ: conserved quantity AND stability parameter.

The moist entropic potential temperature θ_s within its related N^2 expansion (1/2)

- For homogenous (non-saturated and fully-saturated) situations, one can compute the 'squared' BVF by using the idea that density is a function of moist entropy 's', total water content 'q_t' and pressure 'p' only (MG13).
- Let us suppose that we know a 'transition parameter' ('C', which can be identified to Shallow Convection Cloud-cover) and let us define:

$$F(C) = 1 + C\left[\frac{L_v(T)}{C_p T}\,\frac{R}{R_v} - 1\right] \qquad M(C) = \frac{1 + D_C}{1 + D_C F(C)} \qquad D_C = \frac{T}{p - e_s(T)}\,\frac{de_s(T)}{dT}$$

- F(C) ensures the transition between the non-saturated case (***C=0***) where moisture acts only through expansion (R_v/R) and the fully-saturated one (***C=1***) where it acts only through latent heat release ($L_v(T)/(C_p.T)$).
- M(C) cares for the linked change of adiabatic gradient.

The moist entropic potential temperature θ_s within its related N^2 expansion (2/2)

- Then, for any atmospheric condition, one gets (MG13):

$$N^2(C)/g = \boxed{M(C)}\,\frac{1}{C_p}\frac{\partial s}{\partial z} + \boxed{\frac{\partial \ln(1-q_t)}{\partial z} + M(C)}\left[(1+r_v)\frac{R_v}{R}F(C) - \frac{C_{pd}}{C_p}\Lambda\right]\boxed{\frac{\partial q_t}{\partial z}}$$

- Interpretation (following Pauluis and Held (2002)):

'Classical' TKE ⇔ TPE conversion

Total water lifting effect (TKE ⇔PE)

Λ-scaled differential expansion and latent heat effects (TKE ⇔ ?)

A hint for a new way of looking at the *d(TKE)/dt* equation in order to account for the fact that, as soon as moisture appears, the dual role of *θ* is split between θ_s (conservation) and θ_v (conversion term)?

Novelty (1/3): a bit of algebraic manipulation

$$\frac{N^2(C)}{g.M(C)} = \left(\frac{c_{pd}}{c_p}\right)\frac{\partial ln(\theta_l)}{\partial z} + \left\{F(C)(1+r_v)\frac{R_v}{R} - \frac{1}{1-q_t}\left[\frac{1+F(C).D_C}{1+D_C}\right]\right\}\frac{\partial q_t}{\partial z}$$

$$\frac{N^2(C)}{g.M(C)} = \left(\frac{c_{pd}}{c_p}\right)\frac{\partial ln(\theta_l)}{\partial z} + \left\{F(C)\left[(1+r_v)\frac{R_v}{R} - \frac{1}{1-q_t}.\frac{D_C}{1+D_C}\right] - \frac{1}{1-q_t}.\frac{1}{1+D_C}\right\}\frac{\partial q_t}{\partial z}$$

$$\frac{N^2(C)}{g.M(C)} = \left(\frac{c_{pd}}{c_p}\right)\frac{\partial ln(\theta_l)}{\partial z} + \left[(1+r_v)\frac{R_v}{R} - \frac{1}{1-q_t}\right]\frac{\partial q_t}{\partial z}$$
$$+ C\left\{\left[\frac{L_v(T)}{c_p.T}.\frac{R}{R_v} - 1\right]\left[(1+r_v)\frac{R_v}{R} - \frac{1}{1-q_t}.\frac{D_C}{1+D_C}\right]\right\}\frac{\partial q_t}{\partial z}$$

$$\frac{N^2(C)}{g.M(C)} = \left(\frac{c_{pd}}{c_p}\right)\frac{\partial ln(\theta_l)}{\partial z}$$
$$+ \left\{\frac{R_v - R_d}{R} + C\left[\frac{L_v(T)}{c_p.T}.\frac{R}{R_v} - 1\right]\left[\frac{R_v - R_d}{R} + \frac{1}{1-q_t}.\frac{1}{1+D_C}\right]\right\}\frac{\partial q_t}{\partial z}$$

- For easier comparison with LL04, the equations have been 'classically' rewritten in $\boldsymbol{\theta_l}$ and $\boldsymbol{q_t}$.
- We now have a separation between two roles of $\boldsymbol{C}$:
  - Transversal effect of lowering of the resistance to vertical motions (via $\boldsymbol{M(C)}$);
  - (Correcly?) linearised impact in the sole $\boldsymbol{q_t}$ part, with $\boldsymbol{D_C}$ appearing only once.

Novelty (2/3): a rewriting of LL04 equations

- The first idea is to say that ***C*** will remain the true cloud-cover inside ***M(C)*** and will be replaced by an equivalent $\hat{R}^*$ to $\hat{R}$ (also noted ***Q*** in the numerical results). After some lengthy computations, both LL04 variants are replaced by:

$$\hat{R}^*(C) = \frac{\dfrac{\theta_v^{sc} - \theta_v^b}{M(C).\theta} - \dfrac{c_{pd}}{c_p}.\dfrac{\theta_l^{sc} - \overline{\theta_l}}{\overline{\theta_l}} - \dfrac{R_v - R_d}{R}(q_t^{sc} - \overline{q_t})}{\left(\dfrac{L_v(T)}{c_p T}.\dfrac{R}{R_v} - 1\right)\left(\dfrac{R_v - R_d}{R} + \dfrac{1}{1 - q_t}.\dfrac{1}{1 + D_C}\right)(q_t^{sc} - \overline{q_t})}$$

$$\hat{R}^*(C) = \frac{\dfrac{\overline{w'\theta'_v}}{M(C).\theta} - \dfrac{c_{pd}}{c_p}\dfrac{\overline{w'\theta'_l}}{\overline{\theta_l}} - \dfrac{R_v - R_d}{R}\overline{w'q'_t}}{\left(\dfrac{L_v(T)}{c_p T}.\dfrac{R}{R_v} - 1\right)\left(\dfrac{R_v - R_d}{R} + \dfrac{1}{1 - q_t}.\dfrac{1}{1 + D_C}\right)\overline{w'q'_t}}$$

- Now ***M(C)*** appears only as a modulator of the density aspect and $\boldsymbol{\theta_l}$ logically disappeared from the lower case (entropic interpretation). The linear weighting has been replaced by something more ‘physical’, but the problem of how to link $\hat{R}^*$, ***C*** and stability is surely still there.

Novelty (3/3): a new indicator of stability (or skewness)

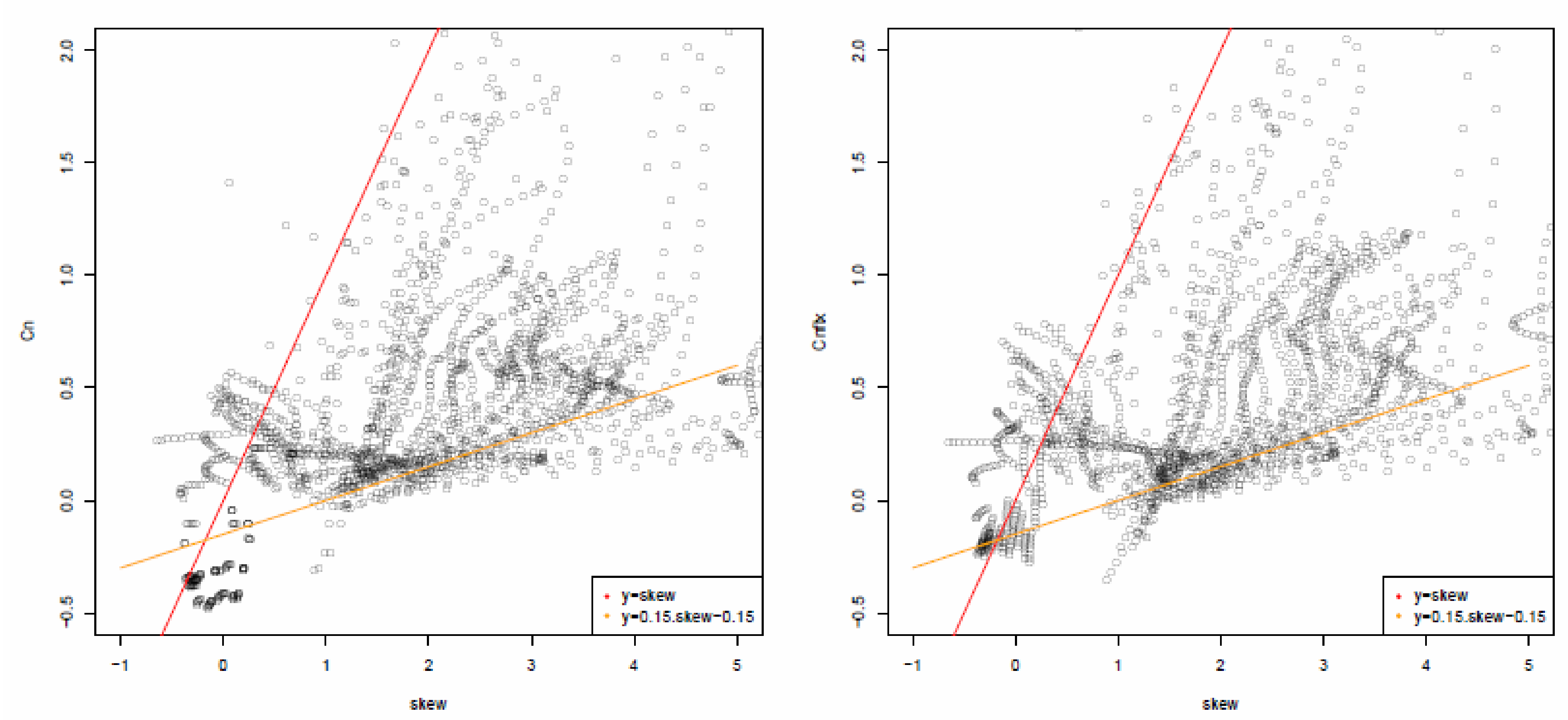
- In the new way of writing the LL04-type equations, we (implicitly) see a new quantity appear, i.e. what would be $\hat{R}^*$ in case of zero $\boldsymbol{\theta_v}$ up-down difference (resp. of zero buoyancy flux). Even if this is just an anticipated interpretation, let us call this $\boldsymbol{C_n}$, i.e. cloud-cover at neutrality (for $\boldsymbol{C_n}=\boldsymbol{C}=\hat{R}^*$ then). Note however that for instance it is not bound to [0,1].

$$C_n = \frac{-\frac{c_{pd}}{c_p}\cdot\frac{\theta_l^{sc}-\overline{\theta_l}}{\overline{\theta_l}} - \frac{R_v - R_d}{R}(q_t^{sc}-\overline{q_t})}{\left(\frac{L_v(T)}{c_pT}\cdot\frac{R}{R_v}-1\right)\left(\frac{R_v-R_d}{R}+\frac{1}{1-q_t}\cdot\frac{1}{1+D_C}\right)(q_t^{sc}-\overline{q_t})}$$

$$C_n = \frac{-\frac{c_{pd}}{c_p}\frac{\overline{w'\theta'_l}}{\overline{\theta_l}} - \frac{R_v - R_d}{R}\overline{w'q'_t}}{\left(\frac{L_v(T)}{c_pT}\cdot\frac{R}{R_v}-1\right)\left(\frac{R_v-R_d}{R}+\frac{1}{1-q_t}\cdot\frac{1}{1+D_C}\right)\overline{w'q'_t}}$$

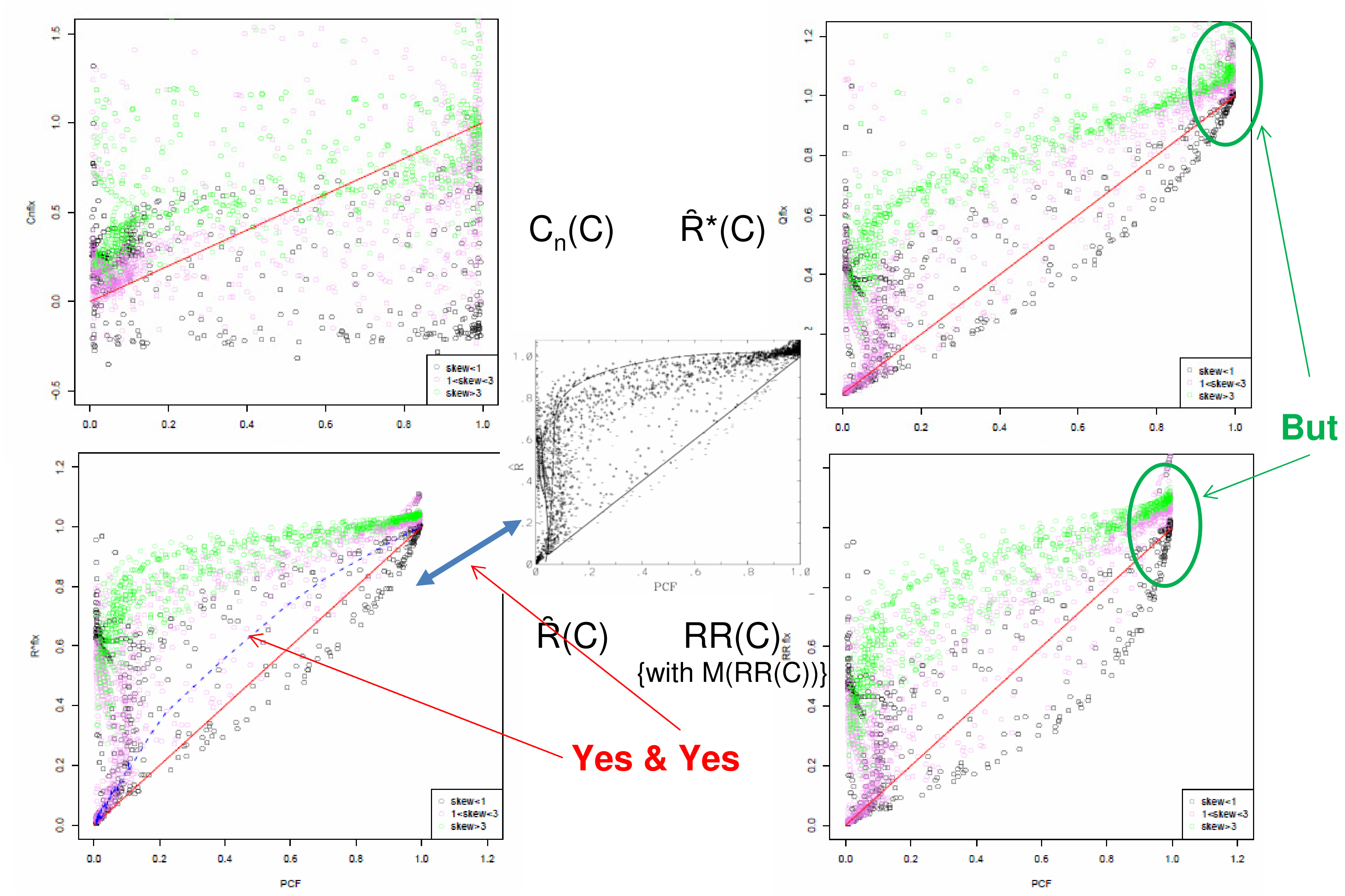
- Now, on the basis of the LL04 (LES-originating) data, we can try to link $\hat{R}^*$, $\boldsymbol{C}$ and $\boldsymbol{C_n}$.

Results with LL04 data (1): is C_n a good equivalent to skewness?



Answer: not so bad (the correlation is better than between skewness and buoyancy flux, not shown), and independently of either using the 'gradient'- or the 'flux' equation

Results with LL04 data (2): do we retrieve LL04 diagram and is there hope to do better?



Back to SD77!

- Reading them carefully, they never wrote that the linear interpolation was fully compatible with their choice. They just said that the picture below hinted at such an heuristic application.

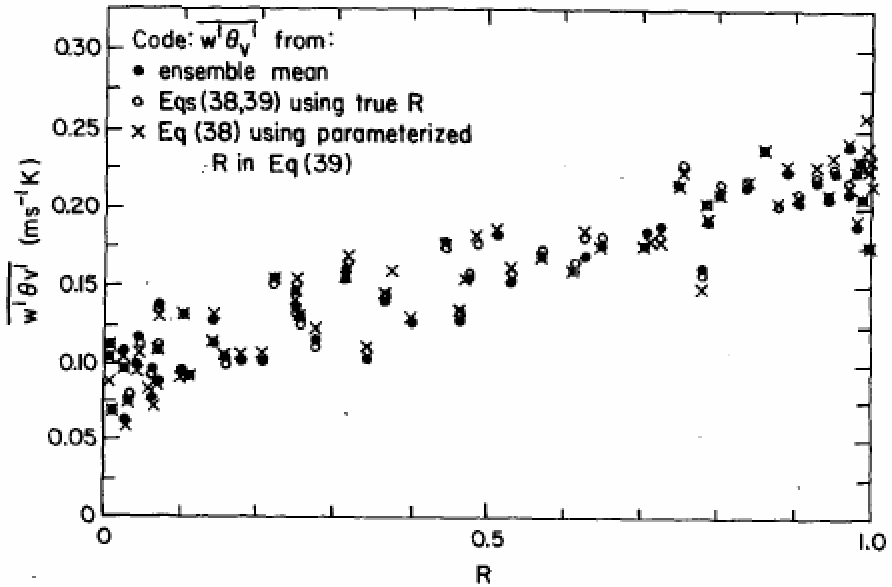


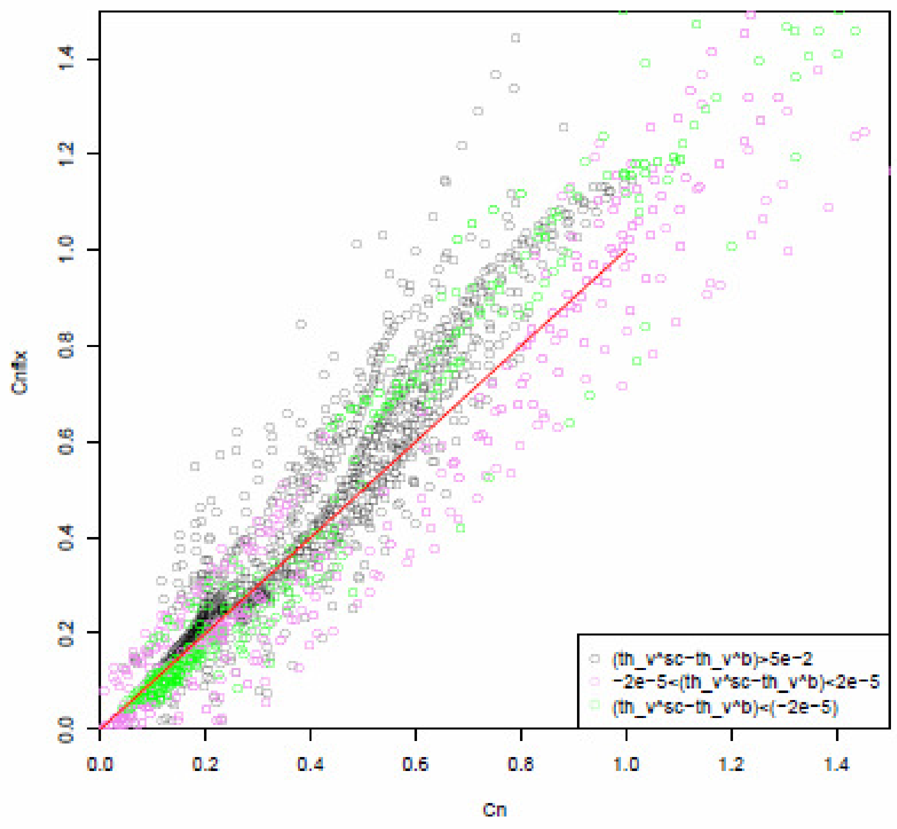
FIG. 5. Relationship between the virtual potential temperature heat flux and the subgrid cloud fraction.

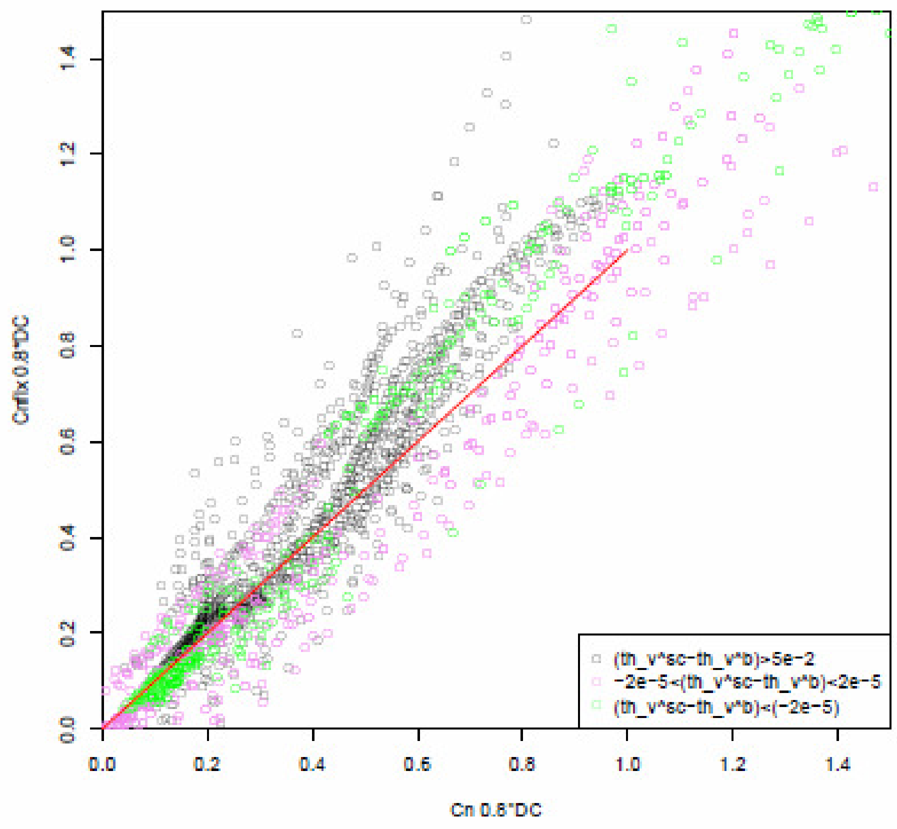
- Yet one may equally well draw a straight line or the curve like the one marking the difference between $\hat{R}^*$ and ***RR*** in the previous diagram.
- So we shall stick to the MG13 proposal and its interpretation in the ‘novelty’ slides hereabove.

About the 'but'!

- We did not verify it, but the issue is most likely linked to differing moist thermodynamic hypotheses between LL04 and our computations.
- Empirically, the diference can be minimised by multiplying DC by 0.8. We shall use this alternative on top of the 'flux' vs. 'gradient' one. Basically the more consistent results are obtained for no correction on 'fluxes' and correction on 'gradients'.
- Once this is admitted, the nice aggregation of green dots in the $\hat{R}^*(C)$ diagram can be approximated by $\hat{R}^* \sim C^{(1/4)}$.

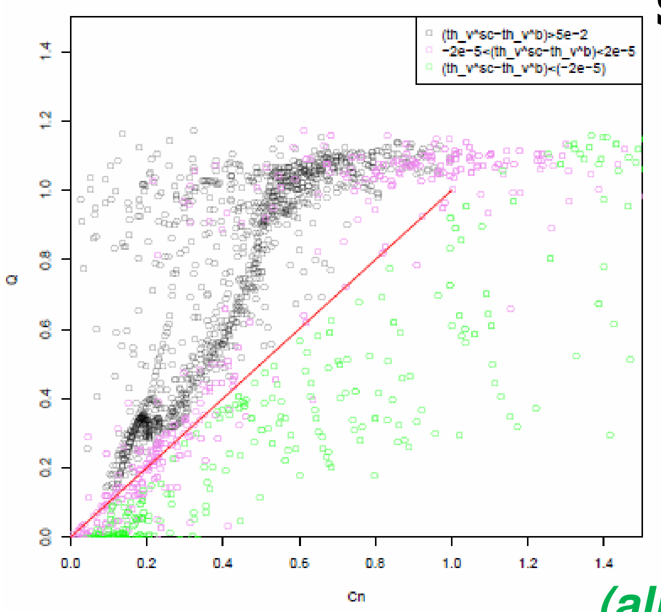
Results with LL04 data (3): is there no bad interference between the two choices?



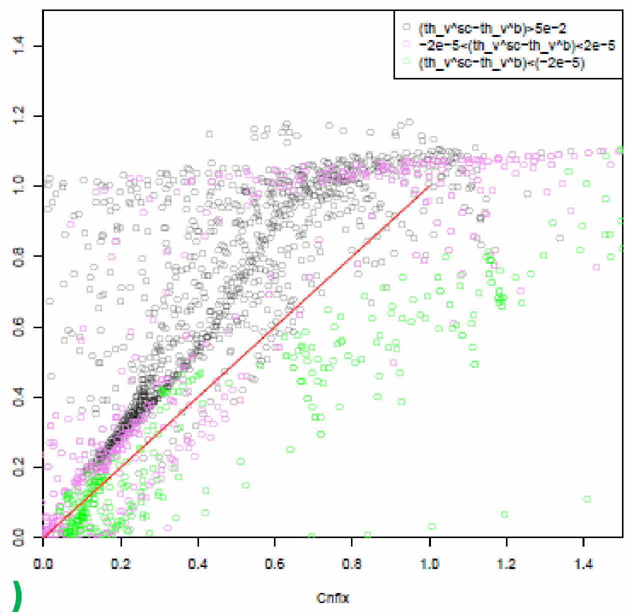


Answer : No

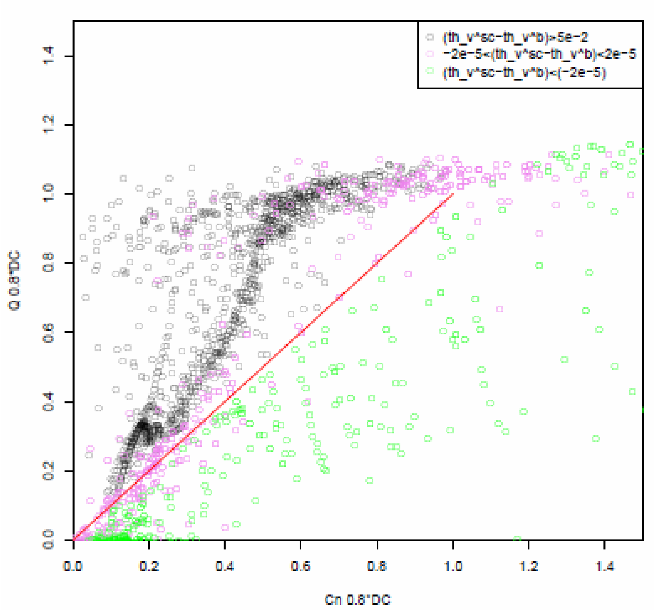
Results with LL04 data (4): is C_n dependency structured?



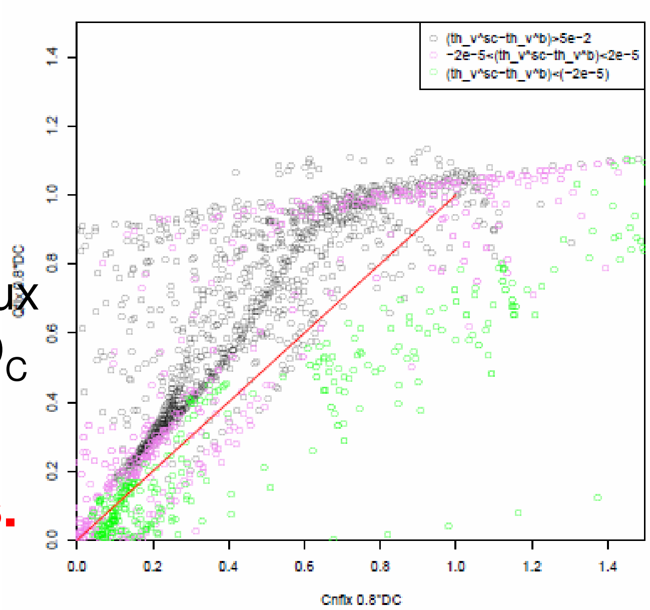
R̂*_grad R̂*_flux



(all as function of C_n)

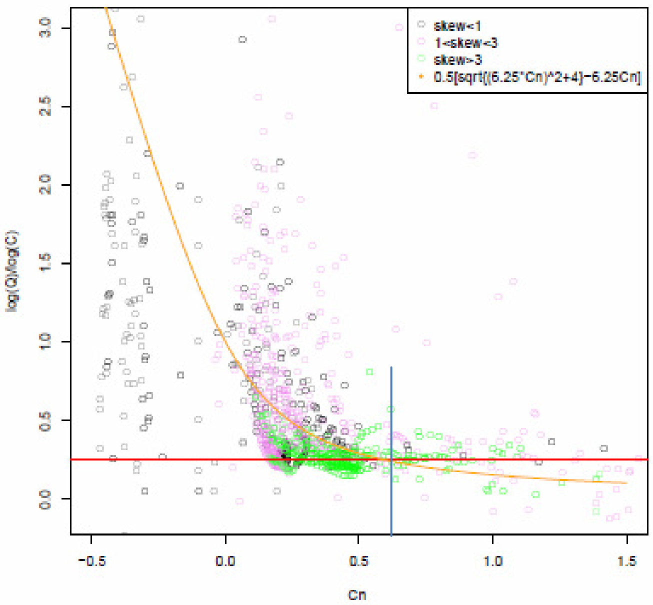


R̂*_grad 0.8*D_C R̂*_flux 0.8*D_C



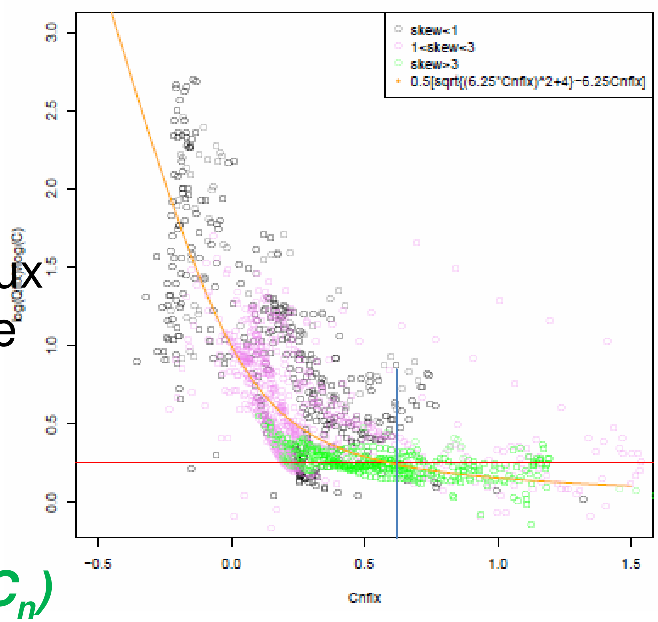
Answer : Yes. Critical C_n value = 0.6

Results with LL04 data (5): can we fit the link between the exponent of C and C_n?

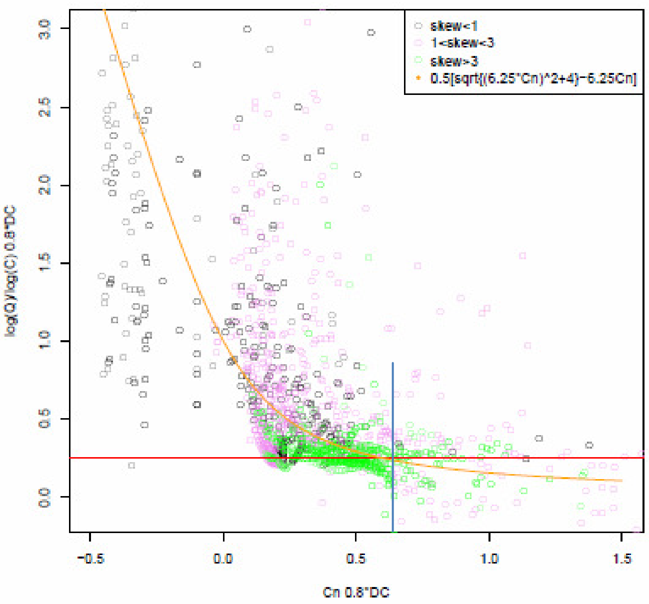


$\hat{R}$*_grad case

$\hat{R}$*_flux case



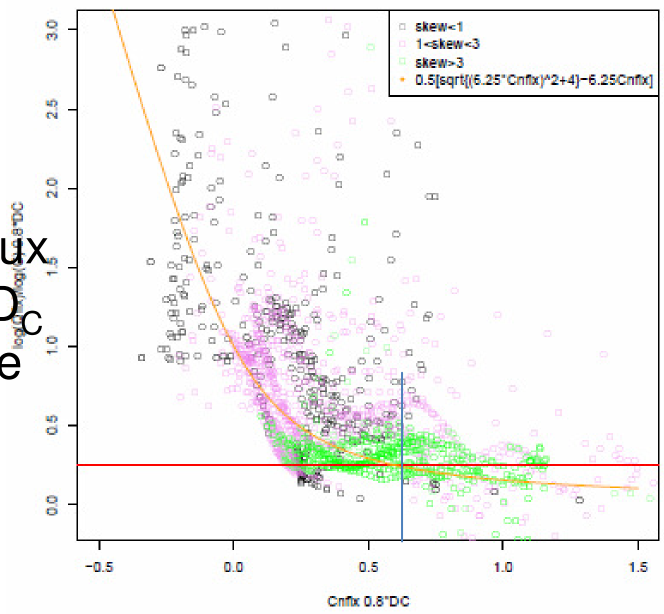
(all as function of C_n)



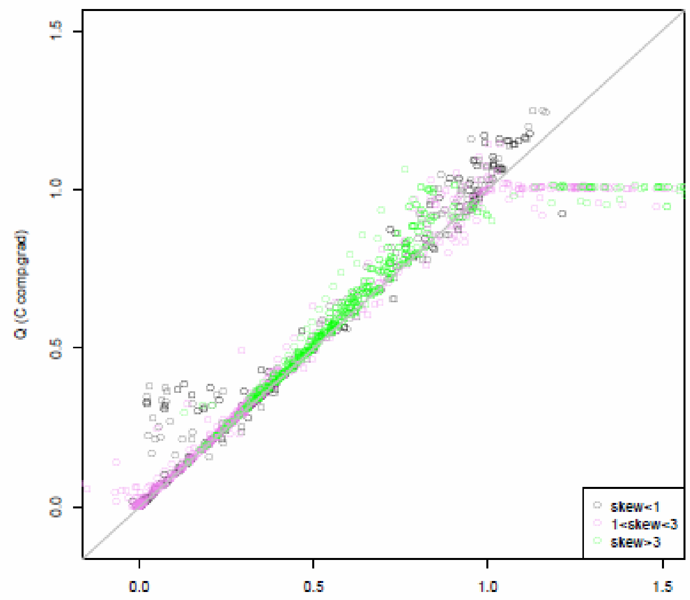
$\hat{R}$*_grad 0.8*D_C case

$\hat{R}$*_flux 0.8*D_C case

Answer : Yes and No

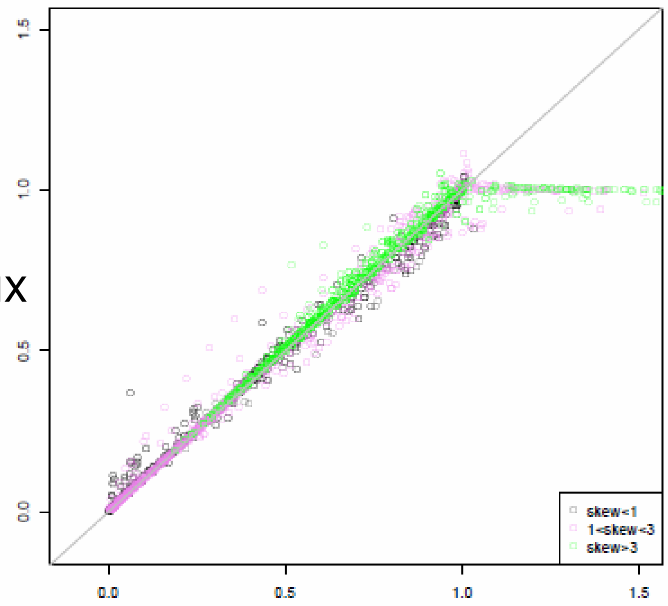


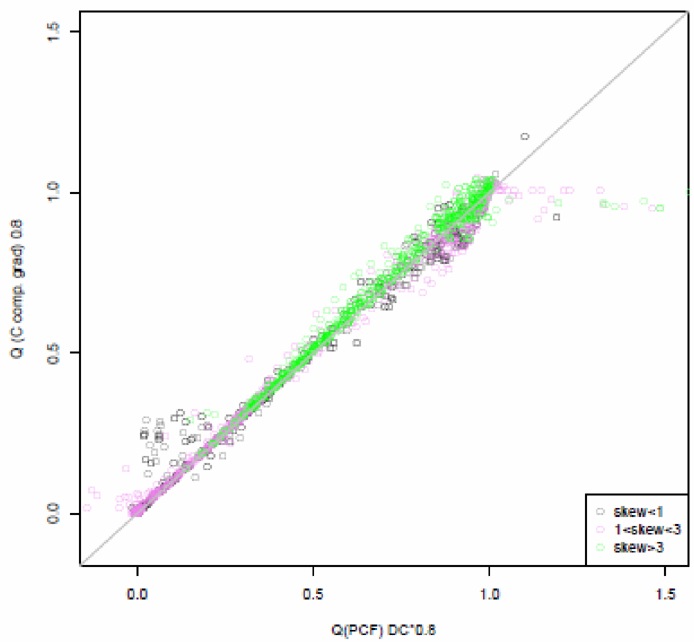
Results with LL04 data (6): *half full glass*, the $\hat{R}^*$ scatter diagram



$\hat{R}^*$_grad
case

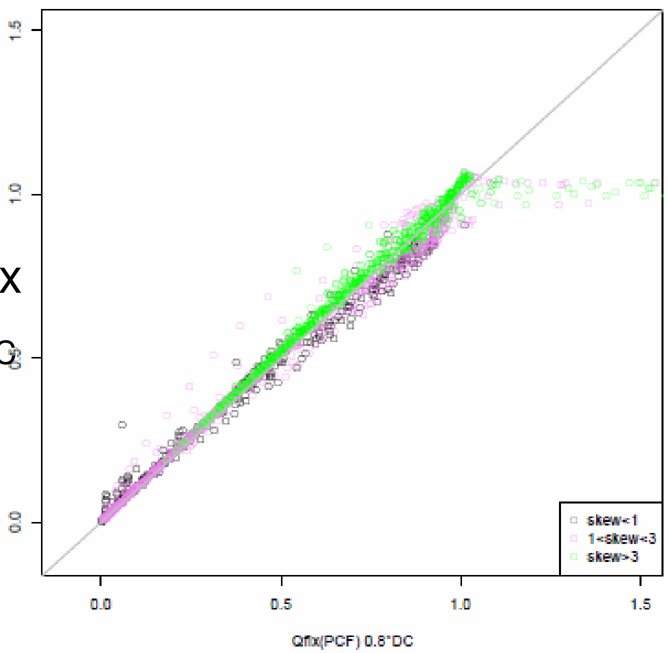
$\hat{R}^*$_flux
case



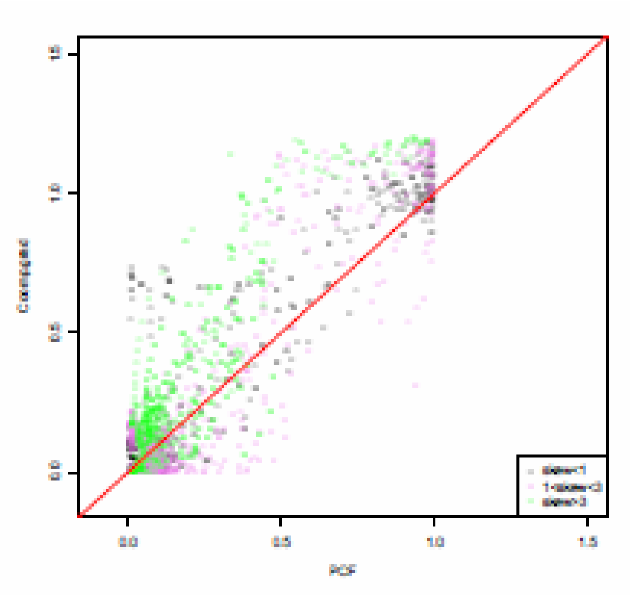


$\hat{R}^*$_grad
$0.8{*}D_C$
case

$\hat{R}^*$_flux
$0.8{*}D_C$
case

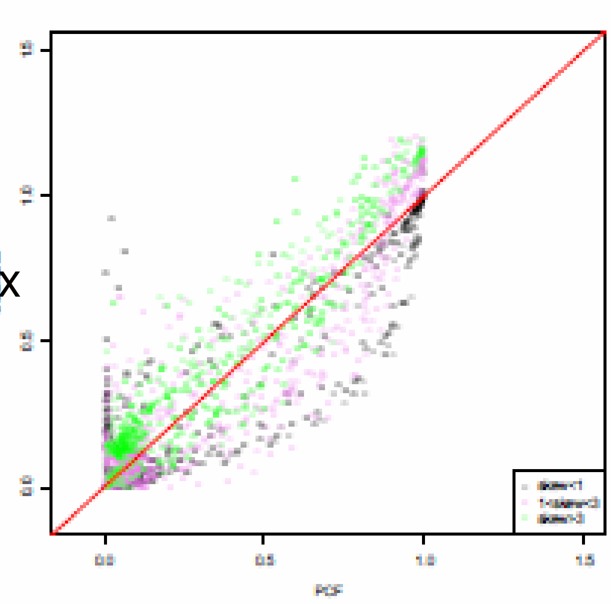


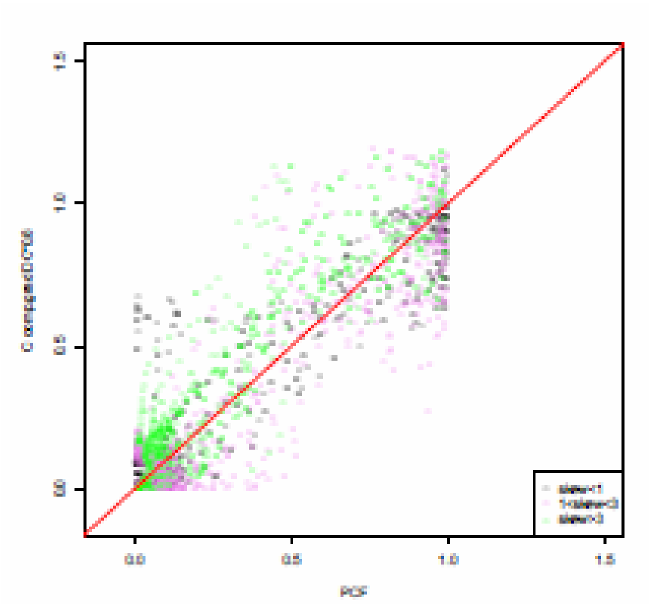
Results with LL04 data (7): *half empty glass*, the C scatter diagram



$\hat{R}$*_grad case

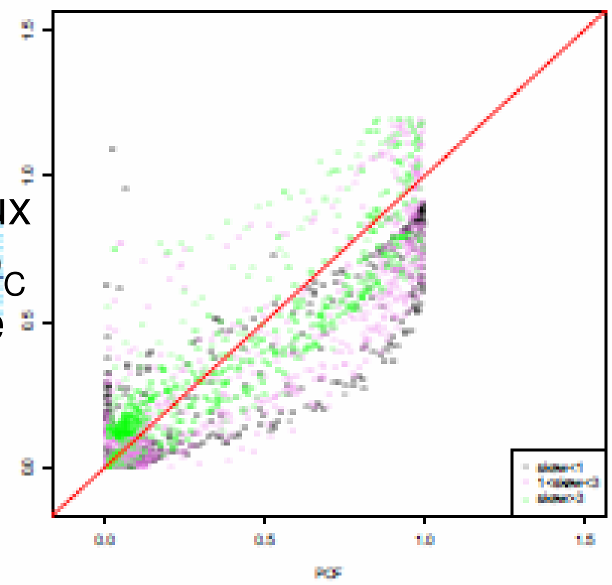
$\hat{R}$*_flux case



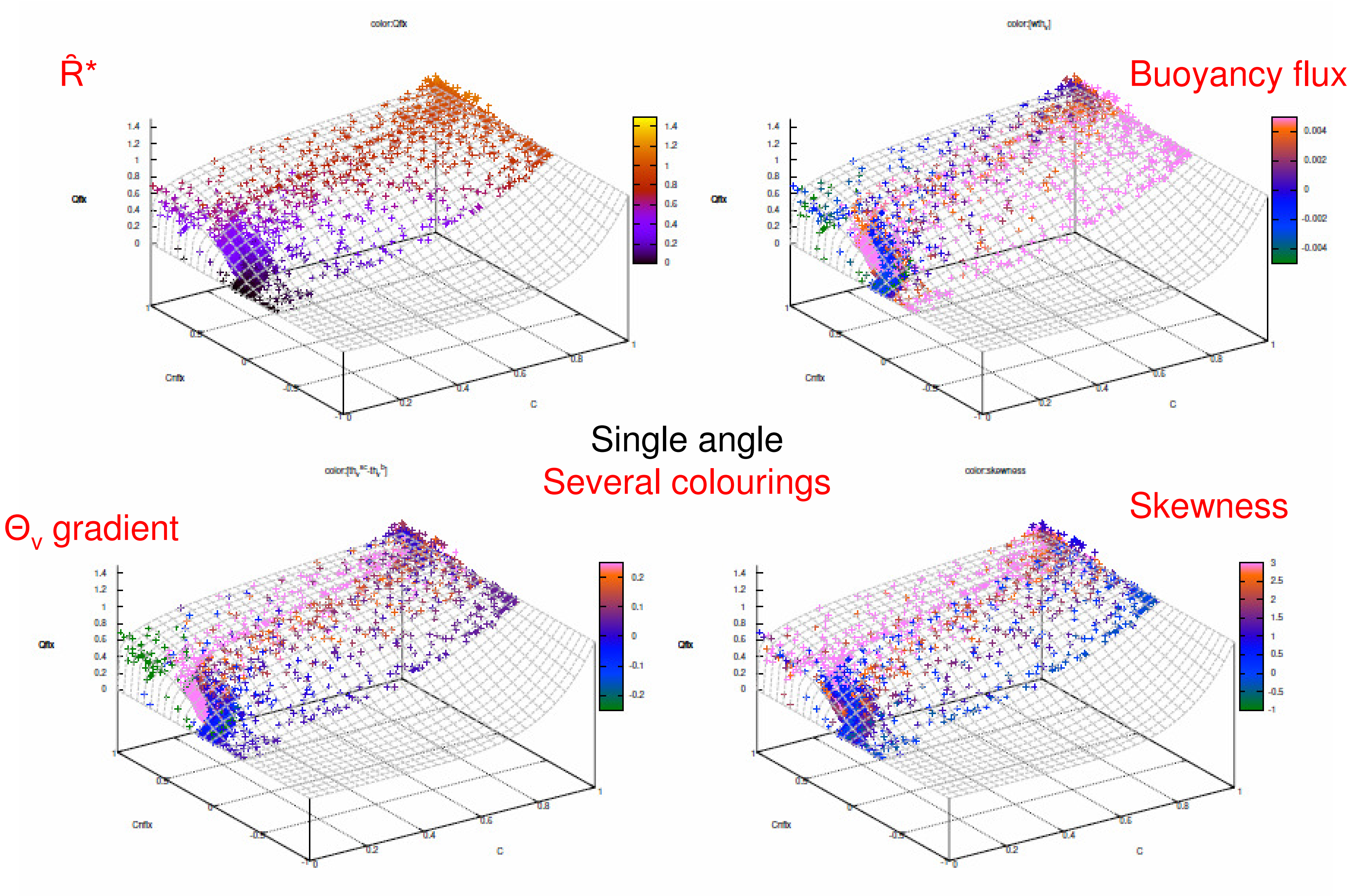


$\hat{R}$*_grad $0.8 \cdot D_C$ case

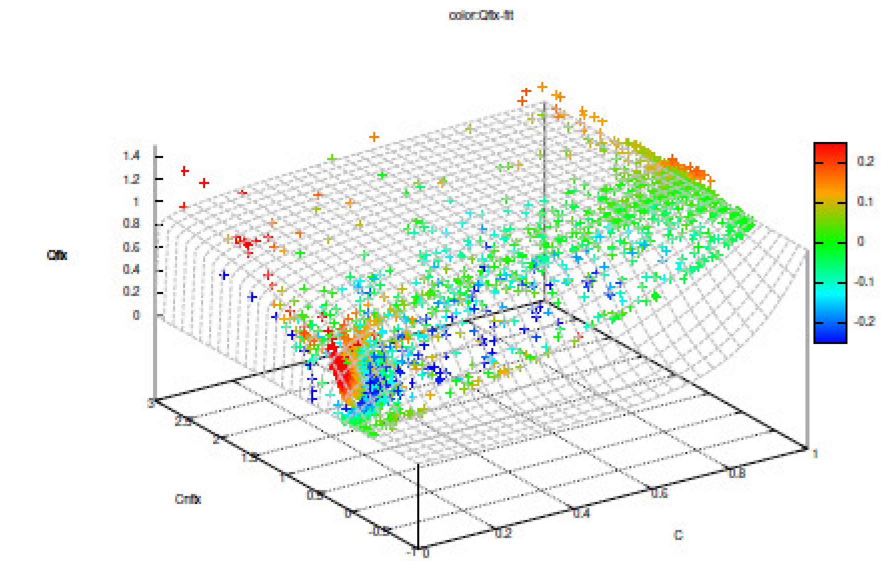
$\hat{R}$*_flux $0.8 \cdot D_C$ case

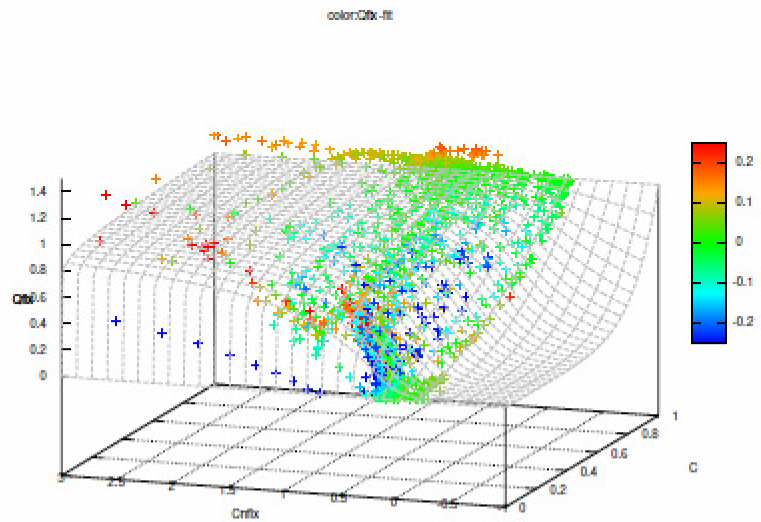


Results with LL04 data (8a): pseudo-3D fits

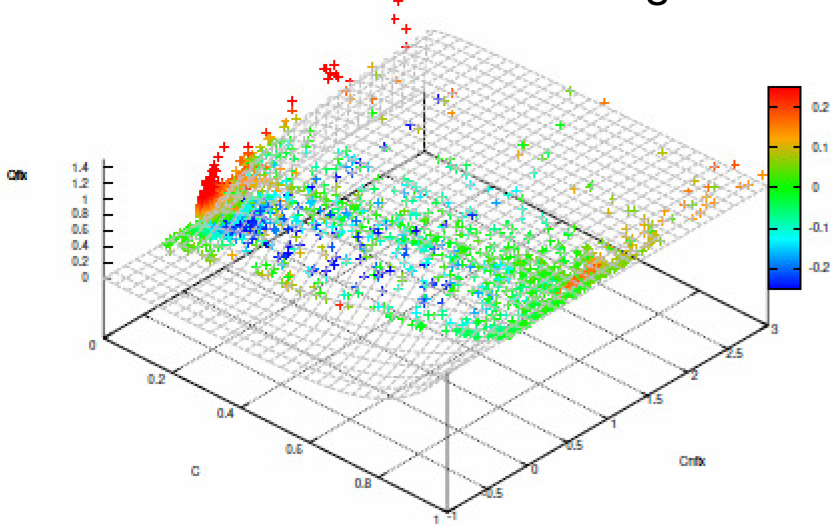


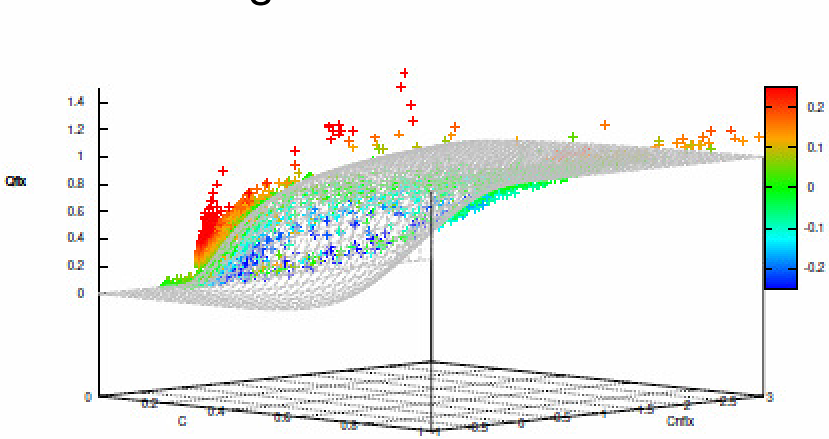
Results with LL04 data (8b): pseudo-3D fits



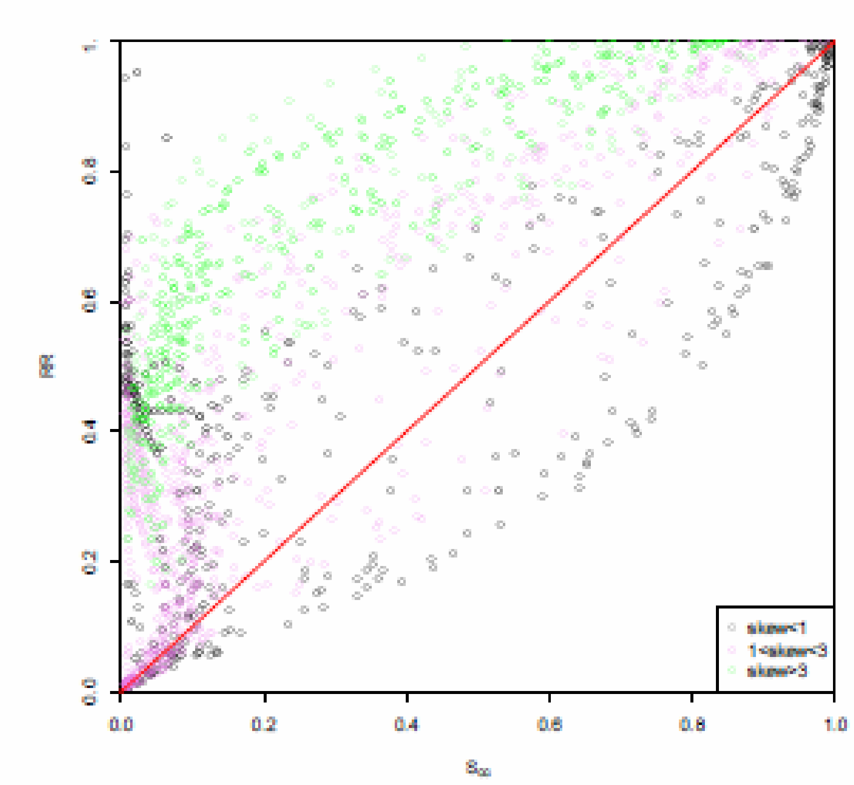


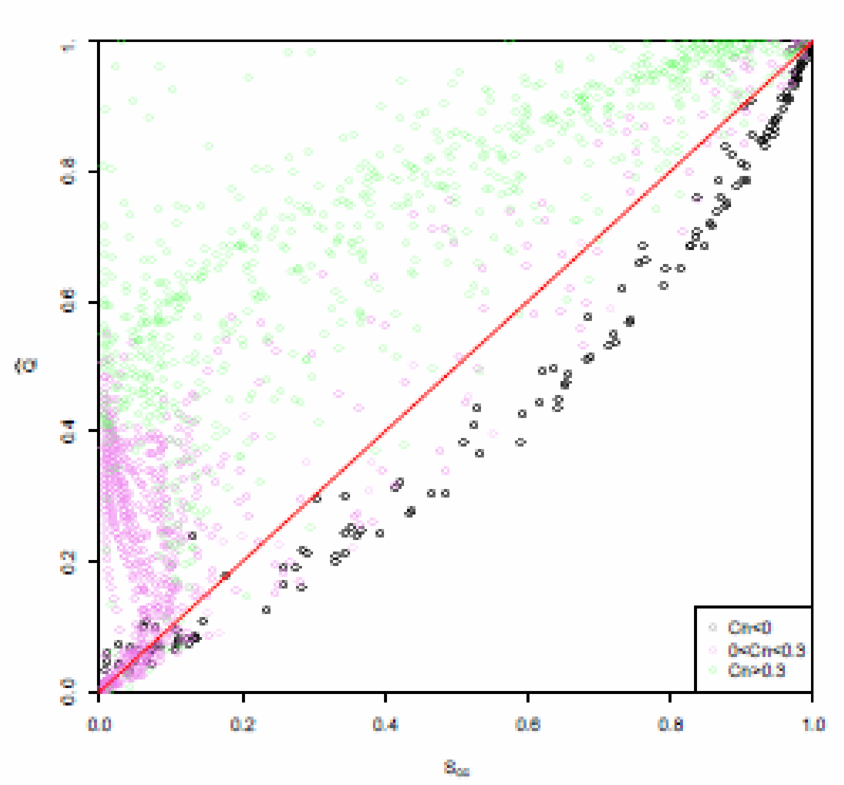
Several angles
Colouring= distance to the fitting surface





Results with LL04 data (8c): key result





Equivalent of the Fig. 9 of LL04 [$\boldsymbol{RR(C)}$] with the colour scaling indicative of the skewness of the statistical distribution of vertical velocities

The same but with our proposed method [$\hat{R}^*(\boldsymbol{C})$], the colour scaling this time indicative of $\boldsymbol{C_n}$. Less 'below-diagonal' solutions, the whole dispersion seriously reduced and the scaling far more pertinent. $\hat{R}^*(\boldsymbol{C},\boldsymbol{C_n})$ looks very promising!

Conclusions and Outlook

- It seems that the claim of SD77 supporting a straight linear interpolation between extreme cases, is neither correct nor supported by the tests we did.
- Rewriting the MG13 proposal in a slightly different way allows to separate two effects of partial cloud-cover, more in line with the entropic thinking in fact.
- This also allows to estimate a so-called ‘cloud-cover at neutrality’, which can most nicely replace skewness as second parameter (on top of $\boldsymbol{C}$) for determining $\hat{R}^*$.
- The translation to a parameterisation step is still to come. In principle it should not necessarily be bound to the sole LL04 method. Linked to an inversion of the Geleyn 1987 Ri*-type proposal, it already works fine. But this was of course not (yet) a tough test!