TOUCANS overview

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TOUCANS

- T - Third
- O - Order moments (TOMs)
- U - Unified
- C - Condensation
- A - Accounting and
- N - N-dependent
- S - Solver (for turbulence and diffusion)

Reynolds-averaged basic equations:

$$\frac{D\overline{u}}{\partial t} = S_u \boxed{-\frac{\partial \overline{u'w'}}{\partial z}}, \quad \frac{D\overline{v}}{\partial t} = S_v \boxed{-\frac{\partial \overline{v'w'}}{\partial z}},$$

$$\frac{D\overline{s_{sL}}}{\partial t} = S_{s_{sL}} \boxed{-\frac{\partial \overline{s'_{sL}w'}}{\partial z}}, \quad \frac{D\overline{q_t}}{\partial t} = S_{q_t} \boxed{-\frac{\partial \overline{q'_t w'}}{\partial z}}$$

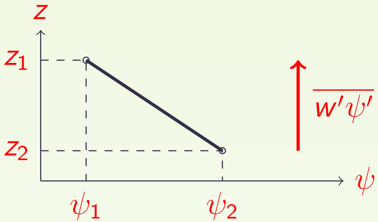
u, v, w -wind components, $s_{sL} = c_{pd}\left(1 + \left[\frac{c_{pv}}{c_{pd}} - 1\right] q_t\right) T + g\,z - (L_v\, q_l + L_s\, q_i)$ a diffused moist conservative variable, g gravitational acceleration, z height, c_{pd} and c_{pv} specific heat values for dry air and water vapour, L_v and L_s latent heats of vaporisation and sublimation, T temperature, q_t total specific water content, q_l and q_i specific contents for liquid and solid water, S_ψ - external source terms, t - time, $\frac{D()}{\partial t} = \frac{\partial ()}{\partial t} + \overline{u}\frac{\partial ()}{\partial x} + \overline{v}\frac{\partial ()}{\partial y}$, $\overline{()}$ - average, $()'$ - fluctuation

Modeling of moments

- modeling of Second Order Moments (SOMs) $\overline{\partial u'w'}$, $\overline{\partial u'w'}$, $\overline{\partial s'_{sL}w'}$, $\overline{\partial q'_t w'}$ require 15 prognostic equation (11 in dry case) with appearance of TOMs
- Third Order Moments (TOMs) additional 26 prognostic equation (16 in dry case) with appearance of FOMs
- Fourth Order Moments (FOMs) additional 9 prognostic equation (Cheng, Canuto, Howard (2005), with parameterisation of some FOMs as a function of SOMs)

Local turbulent diffusion

- reduction of the system to 0 prognostic equation for SOMs
- analogy with molecular diffusion
- depends only on local gradients
  - down-gradient transport
- $\overline{w'\psi'} = -K_\psi \frac{\partial \bar{\psi}}{\partial z}$



(K_ψ - coefficient of turbulent diffusion)

Turbulent diffusion - local transport

$$\frac{\partial s_{sL}}{\partial t} = g\frac{\partial \rho \overline{w' s'_{sL}}}{\partial p}$$

$$\frac{s^*_{sL} - s^-_{sL}}{\delta t} = \frac{\partial \left(-g \rho K_H \left(\frac{\partial \overline{s^*_{sL}}}{\partial z}\right)\right)}{\partial p}$$

$$\text{at surface: } \overline{w' s'_{sL}} = -C_H |V_L| \left([s^*_{sL}]_L - [s_{sL}]_s\right)$$

* marks variable at next time step after local diffusion, V -horizontal wind speed, $[]_L$-lowest model level, $[]_s$ -surface, δt - time step, K_H - exchange coefficient for heat and moisture, C_H - drag coefficient for heat and moisture, ρ - density, p - pressure

Exchange coefficients $K_{M/H}$ and drag coefficients $C_{M/H}$ in Louis scheme (LPTKE=.FALSE., LCOEFKTKE=.FALSE.):

$$K_{M/H} = l_{m/h} l_m \left[\left(\frac{\partial \bar{u}}{\partial z} \right)^2 + \left(\frac{\partial \bar{v}}{\partial z} \right)^2 \right]^{\frac{1}{2}} F_{M/H}(Ri)$$

$$C_{M/H} = C^N_{M/H}(z, z_0, \kappa).F_{m/h}(Ri)$$

$l_{m/h}$ - Prandtl mixing length for momentum and pot. temperature
$F_{M/H}(Ri)$ - stability functions, Ri - gradient Richardson number
z_0 - roughness, $C^N_{M/H}$ - drag coefficient at neutrality($Ri = 0$)
κ - von Karman constant

Louis stability functions F_M and F_H:

stable case:

$$F_M(Ri) = \frac{1}{1 + \frac{2bRi'_m}{\sqrt{1+\frac{d}{k}Ri'_m}}}, \quad Ri'_m = \frac{Ri}{1 + \frac{Ri}{Ri_{lim}}}$$

$$F_H(Ri) = \frac{1}{1 + 3bRi'_h\sqrt{1 + d\,k\,Ri'_h}}, \quad Ri'_h = \frac{Ri}{\left(1 + \alpha\frac{Ri}{Ri_{lim}}\right)^{\frac{1}{\alpha}}}$$

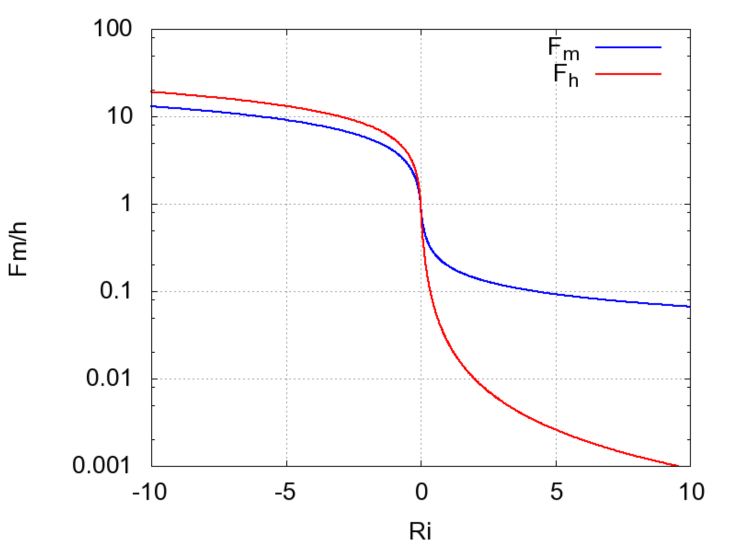
unstable case:

$$F_M(Ri) = 1 - \frac{2bRi}{1 + 3bc\sqrt{\frac{|Ri|}{27}}\left(\frac{l_m}{z+z_0}\right)^2}$$

$$F_H(Ri) = 1 - \frac{3bRi}{1 + 3bc\sqrt{\frac{|Ri|}{27}}\left(\frac{l_h}{z+z_{0h}}\right)\left(\frac{l_m}{z+z_0}\right)}$$

b, c, d, k - constants, z_0, z_{0h} - roughness, $Ri_{lim}(z)$-limiting Ri, $\alpha(z, Ri)$- coefficient

Louis scheme - stability functions



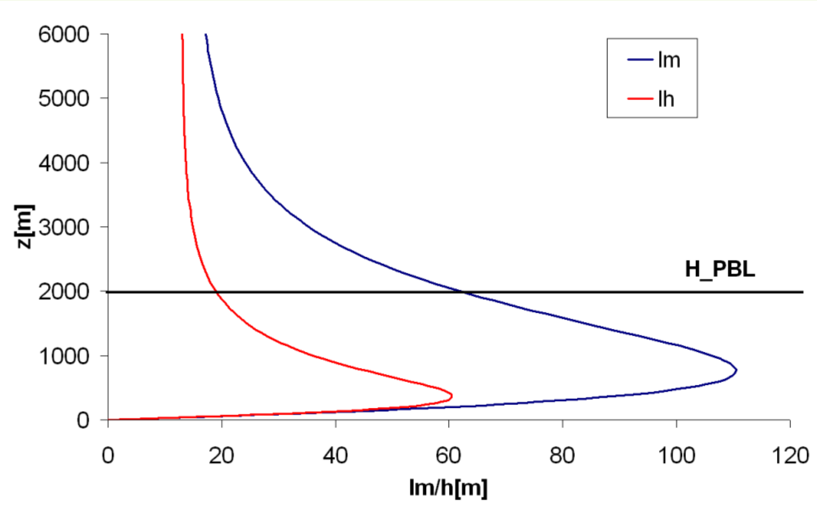
Prandtl-type mixing lengths l_m and l_h

(CGMIXLEN='AY', in ALARO0='CG') :

$$l_{m/h}^{GC} = \frac{\kappa z}{1 + \frac{\kappa z}{\lambda_{m/h}} \left[\frac{1+\exp\left(-a_{m/h}\sqrt{\frac{z}{H_{pbl}}}+b_{m/h}\right)}{\beta_{m/h}+\exp\left(-a_{m/h}\sqrt{\frac{z}{H_{pbl}}}+b_{m/h}\right)} \right]}$$

(κ is Von Kármán constant, z is height, $a_{m/h}$, $b_{m/h}$, $\beta_{m/h}$ and $\lambda_{m/h}$ are tuning constants and H_{pbl} is PBL height)

Prandtl-type mixing lengths:



TOUCANS and pseudo-TKE (LPTKE=.TRUE.)

- addition of 1 prognostic equation for SOMs

$$\frac{\partial e}{\partial t} = Adv(e) + \overbrace{\frac{1}{\rho}\frac{\partial}{\partial z}\left(\rho K_E \frac{\partial e}{\partial z}\right)}^{\text{TKE diffusion}} + \underbrace{K_M\left[\left(\frac{\partial \overline{u}}{\partial z}\right)^2 + \left(\frac{\partial \overline{v}}{\partial z}\right)^2\right]}_{\text{shear}} \underbrace{-\frac{g}{\theta}K_H\frac{\partial\overline{\theta}}{\partial z}}_{\text{buoyancy}} \underbrace{-C_\epsilon\frac{(e)^{\frac{3}{2}}}{L_\epsilon}}_{\text{dissipation}}$$

$e = \frac{1}{2}(\overline{u' \cdot u' + v' \cdot v' + w' \cdot w'})$ =TKE, K_E- auto-diffusion coefficient for TKE,

C_ϵ - free parameter, L_ϵ - length scale

Exchange coefficients

$$K_M = \frac{\nu^4}{C_\epsilon}\chi_3(Ri_f)\sqrt{e}L_K, \quad K_H = C_3\frac{\nu^4}{C_\epsilon}\phi_3(Ri_f)\sqrt{e}L_K$$

free parameters / stab. functions	TKE	length scale
given by turbulence scheme	prognostic meassue of intensity	quasi-independent, may depend on TKE and BVF

$\chi_3(Ri_f)$,$\phi_3(Ri_f)$ - stability functions, ν - free parameter, C_3 - inverse Prantl number at neutrality, L_K - length scale, $Ri_f = Ri\frac{K_H}{K_M}$ - flux Richardson number

Prognostic TKE scheme - code implementation

$$\frac{\partial e}{\partial t} = \underbrace{Adv(e)}_{\text{advection}} + \underbrace{\frac{1}{\rho}\frac{\partial}{\partial z}\left(\rho K_E \frac{\partial e}{\partial z}\right)}_{\text{diffusion with antifibrilation}} + \underbrace{\frac{1}{\tau_\epsilon}(\tilde{e} - e)}_{\text{relaxation}}$$

- numerically stable
- with antifibrilation for TKE diffusion
- enables shallow convection parametrisation with Richardson number's modification

Prognostic TKE scheme - code implementation

$$\frac{\partial e}{\partial t} = Adv(e) + \frac{1}{\rho}\frac{\partial}{\partial z}\left(\rho K_E \frac{\partial e}{\partial z}\right) + \frac{1}{\tau_\epsilon}(\tilde{e} - e)$$

$$\tau_\epsilon = \frac{l_m}{\nu^3\sqrt{e}}\frac{1}{F_\epsilon} = \frac{l_m^2}{\nu^2 K^*}\frac{1}{F_\epsilon}, \qquad \widetilde{e} = \left(\frac{K^*}{\nu l_m}\right)^2,$$

$$K_E = \frac{l_m\sqrt{e}}{\nu}F_\epsilon = \underbrace{\frac{K^*}{\nu^2}F_\epsilon}_{\text{first time step}}, \qquad K_M = \nu l_m \sqrt{e}\sqrt{F_M}$$

$$K^* = \frac{\widetilde{K_M}}{\sqrt{F_M}}, \widetilde{K}_{M/H} = l_{m/h} l_m \left[\left(\frac{\partial \bar{u}}{\partial z}\right)^2 + \left(\frac{\partial \bar{v}}{\partial z}\right)^2\right]^{\frac{1}{2}} F_{m/h}(Ri)$$

$\tilde{e}$ - TKE at stationary equilibrium, τ_ϵ - dissipation time scale, $\nu = (C_K C_\epsilon)^{\frac{1}{4}}$,

Prognostic TKE scheme

-TOUCANS is analytically equivalen with 'full TKE scheme' (LCOEFKTKE=.TRUE.)
-pseudo-TKE uses Louis stability functions (LCOEFKTKE=.FALSE.)

stability function	TOUCANS	pseudo-TKE
F_ϵ	$\frac{f(Ri)^{\frac{3}{4}}}{\chi_3(Ri_f)^{\frac{3}{2}}}\beta_e$	1.0
F_M	$\chi_3(Ri_f)\sqrt{f(Ri)}$	Louis scheme
F_H	$\frac{\phi_3(Ri_f)}{\chi_3(Ri_f)}F_M(Ri_f)$	Louis scheme

$f(Ri) = \chi_3(Ri_f) - Ri\, C_3\, \phi_3(Ri_f)$,

β_e - 'dry' antifibrilation coefficient for TKE

Prognostic TKE scheme

TKE equation inputs:
Computation of source terms (via $\tilde{e}$ and τ_ϵ) and K_E

↓

TKE solver:
update of $e \Rightarrow$ computation of K_M, K_H

↓

Local turbulent diffusion (with AF scheme)
with K_M, K_H :
computation of $\overline{s'_{sL}w'}$, $\overline{q'_t w'}$, $\overline{u'w'}$,$\overline{v'w'}$

Stability dependency functions χ_3, ϕ_3 and free parameters

- derived from Louis stability functions + RMC01 (pTKE)
- based on (Bašták Ďurán, Geleyn, and Váňa, 2014) (TOUCANS)
  - analytical framework given by four free parameters: C_ϵ, ν, C_3, O_λ (+ two possible functional dependencies)
  - enables emulation of various schemes: EFB, QNSE, RMC01,..

Moisture influence

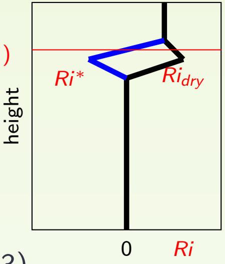
- expansion and latent heat release influences BVF - $N^2(\rightarrow Ri,\ Ri_f,\ ..)$ (Marquet and Geleyn, 2013):
  $N^2 = E_{s_{sL}}\,(SCC)\,\frac{\partial s_{sL}}{\partial z} + E_{q_t,s_{sL}}\,(SCC)\,\frac{\partial q_t}{\partial z}$
  - Shallow Convection Cloudiness (SCC) needed
- currently parametrised by modification of Ri (Geleyn, 1987):

$$Ri^* = \underbrace{\frac{g}{c_p T}\Bigg(\frac{\frac{\partial(c_p T+gz)}{\partial z}}{\left[\frac{\partial u}{\partial z}\right]^2+\left[\frac{\partial v}{\partial z}\right]^2}}_{Ri} + \frac{L.min\left[0,\frac{\partial(q_t-q_{sat})}{\partial z}\right].\delta_h}{\left[\frac{\partial u}{\partial z}\right]^2+\left[\frac{\partial v}{\partial z}\right]^2}\Bigg)$$

δ_h - switches ON/OFF the parametrisation

-requires AntiFibrillation scheme

-Ri^* enables (backwards) computation of SCC via (Marquet and Geleyn, 2013)



$q_{l/i}$ diffusion

- turbulent diffusion of q_l and q_i (Smith, 1990)
- transports condensates in 'clouded' region
- cloud fraction (SCC) required on half levels

TKE based length scales L

- Bougeault a Lacarrère (1989) :

$$L_{BL}(E) = \left(\frac{L_{up}^{-\frac{4}{5}} + L_{down}^{-\frac{4}{5}}}{2} \right)^{-\frac{5}{4}}$$

$L_{up}(E)$ ($L_{down}(E)$) - L upward (downward)

- $L_N = \sqrt{\frac{2.E}{N^2}}$ for stable straification
- with possibility to use moist BVF (computed from Ri^*)
- prognostic treatment of L (under development)

Conversion between L and l_m

- following RMC01:

$$L_K = \frac{C_\epsilon}{\nu^3} l_m \frac{f(Ri)^{\frac{1}{4}}}{\chi_3^{\frac{1}{2}}}, \quad L_\epsilon = \frac{C_\epsilon}{\nu^3} l_m \frac{\chi_3^{\frac{3}{2}}}{f(Ri)^{\frac{3}{4}}}$$

- assuming: $L \equiv \left(L_K^3 L_\epsilon\right)^{\frac{1}{4}}$ we get:

$$L = \frac{C_\epsilon}{\nu^3} l_m$$

TOUCANS

Lengt scale computation: $L(e^-)$

Stab. functions: $Ri^* \rightarrow \chi_3, \phi_3$

TKE equation inputs:
Computation of source terms (via $\tilde{e}$ and τ_ϵ) and K_E

TKE solver: update of e

$$K_M = \frac{\nu^4}{C_\epsilon}\chi_3(Ri_f)\sqrt{e}L_K, \quad K_H = C_3\frac{\nu^4}{C_\epsilon}\phi_3(Ri_f)\sqrt{e}L_K$$

Local turbulent diffusion
computation of $\overline{s'_{sL}w'}$, $\overline{q'_t w'}$, $\overline{u'w'}$, $\overline{v'w'}$

Third Order Moments (TOMs)

- distant turbulent transport caused by presence of semi-organised large eddies
- parametrisation for heat and moisture
- following (Canuto, Cheng, and Howard, 2007):

$$\overline{w'\theta'} = -K_H \frac{\partial\overline{\theta}}{\partial z} + A_1^\theta \frac{\partial\overline{w'^3}}{\partial z} + A_2^\theta \frac{\partial\overline{w'\theta'^2}}{\partial z} + A_3^\theta \frac{\partial\overline{w'^2\theta}}{\partial z}$$

$$\overline{w'^3} = -0.06\frac{g}{\overline{\theta}}\tau^2\overline{w'^2}\frac{\partial\overline{w'\theta'}}{\partial z}, \quad \overline{w'\theta'^2} = -\tau\overline{w'\theta'}\frac{\partial\overline{w'\theta'}}{\partial z}, \quad \overline{w'^2\theta'} = -0.3\tau\overline{w'^2}\frac{\partial\overline{w'\theta'}}{\partial z}$$

θ - potential temperature, A_1^θ, A_2^θ, A_3^θ - coefficients, τ - dissipation time scale

- two step solver: local + nonlocal correction

Prognostic Total Turbulent Energy (TTE)

- parametrisation of counter-gradient heat transport maintained by velocity shear following (Zilitinkevich et al., 2013)
- pair of prognostic turbulent energies - TKE and TTE
- equilibrium assumption links energy ratio share $\Pi = \frac{TTE - TKE}{TKE}$ to stability parameters Ri_f, Ri
- Π used as new stability parameter
- usage of TKE solver also for TTE

Summary

- exchange coefficients K_M and K_H given by:
  - stability functions and free parameters
  - lenght scale
  - prognostic TKE
  - moisture influence via Ri - SCC required
  - prognostic TTE via Π
- heat and moisture flux with TOMs parametrisation
- $q_{l/i}$ diffusion - SCC required

Thank you for your attention!