# Turbulence-Diffusion - TOUCANS C: Pre-operational choices

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# TOUCANS

- T - Third
- O - Order moments (TOMs)
- U - Unified
- C - Condensation
- A - Accounting and
- N - N-dependent
- S - Solver (for turbulence and diffusion)

# TOUCANS 'colors':

- compact and flexible turbulence parametrisation - enables usage of different aproaches:
  - emulation of different turbulent schemes: CCH02, QNSE, EFB by choice of stability functions $\chi_3$, $\phi_3$ (or rather degrees of freedom $C_3$, $Ri_{fc}$, $R$)
  - usage of different mixing lengths: Prandtl-type, TKE-type
  - four types of shallow convection parametrisation through $Ri$ (linked also to $q_{li}$ diffusion)
- choices in these three categories are orthogonal
- algorithmic unification whenever possible

# Turbulent diffusion:

$$\frac{D\overline{u}}{\partial t} = S_u \boxed{-\frac{\partial \overline{u'w'}}{\partial z}}$$

$$\frac{D\overline{v}}{\partial t} = S_v \boxed{-\frac{\partial \overline{v'w'}}{\partial z}}$$

$$\frac{D\overline{s_{li}}}{\partial t} = S_{s_{li}} \boxed{-\frac{\partial \overline{s'_{li}w'}}{\partial z}}$$

$$\frac{D\overline{q_t}}{\partial t} = S_{q_t} \boxed{-\frac{\partial \overline{q'_t w'}}{\partial z}}$$

($u$, $v$, $w$ -wind components, $s_{li} = c_p T + \phi - L_v q_l - L_s q_i$, $\theta$ - potential temperature, $q_t$ - total specific humidity, $q_v$ - specific humidity , $q_l$- specific humidity of liquid water, $q_i$- specific humidity of ice, $\phi$ - geopotential, $c_p$ - specific heat capacity, $L_v$ - latent heat of vaporization, $L_s$ - latent heat of sublimation, $S_{u/v/s_{li}/q_t}$ - external source terms, $\frac{D()}{\partial t} = \frac{\partial()}{\partial t} + \overline{u}\frac{\partial()}{\partial x} + \overline{v}\frac{\partial()}{\partial y}$, $\overline{()}$ - average, $()'$ - fluctuation )

# Turbulent fluxes

TOMs

$$\overline{w'u'} = -K_m \frac{\partial \bar{u}}{\partial z}$$

$$\overline{w'v'} = -K_m \frac{\partial \bar{v}}{\partial z}$$

$$\overline{w's'_{li}} = -K_h \frac{\partial \overline{s_{li}}}{\partial z} \overbrace{\boxed{-K_h T_h(Ri) T_*^{-1} \frac{\partial J_{s_{li}}}{\partial p} + g K_h T_h(Ri) T_{**} \frac{\partial \left(\frac{\partial J_{s_{li}}}{\partial p}\right)}{\partial z}}}^{\text{TOMs}}$$

$$\overline{w'q'_t} = -K_h \frac{\partial \overline{q_t}}{\partial z} \boxed{-K_h T_h(Ri) T_*^{-1} \frac{\partial J_{qt}}{\partial p} + g K_h T_h(Ri) T_{**} \frac{\partial \left(\frac{\partial J_{q_t}}{\partial p}\right)}{\partial z}}$$

($K_{m/h}(e, \tau, Ri, C_3, Ri_{fc}, R)$ - exchange coefficients for momentum/heat, $e = \frac{1}{2}(\overline{u' \cdot u' + v' \cdot v' + w' \cdot w'})$ =TKE, $\tau$- TKE dissipation time scale, $Ri$- gradient Richardson number, $C_3/Ri_{fc}/R$ - degrees of freedom, $T_h(Ri)$ -stability function, $J_{q_t/s_{li}} = -\rho \overline{w' q_t/s'_{li}}$, $p$-pressure, $\rho$- density, $z$ -height, $g$ - acceleration of gravity, $T_*^{-1}/T_{**}$ functions of $e, \tau, Ri, \overline{w's'_{li}}, \overline{w'q'_t}, C_3, Ri_{fc}, R$)

# Prognostic TKE

$$\frac{\partial e}{\partial t} = Adv(e) + \overbrace{\frac{1}{\rho}\frac{\partial}{\partial z}\left(\rho K_E \frac{\partial e}{\partial z}\right)}^{\text{TKE diffusion}}$$
$$\underbrace{+ K_m\left[\left(\frac{\partial \overline{u}}{\partial z}\right)^2 + \left(\frac{\partial \overline{v}}{\partial z}\right)^2\right]}_{\text{shear}} \underbrace{- \frac{g}{\theta} K_h \frac{\partial \overline{\theta}}{\partial z}}_{\text{buoyancy}} \underbrace{- C_\epsilon \frac{(e)^{\frac{3}{2}}}{L_\epsilon}}_{\text{dissipation}}$$

$$K_m = L_K C_K \sqrt{e}\chi_3(Ri), \qquad K_h = L_K C_K C_3 \sqrt{e}\phi_3(Ri)$$

$K_E$- auto-diffusion coefficient for TKE, $\chi_3(Ri)$,$\phi_3(Ri)$ - stability functions, $C_K$,$C_\epsilon$ - closure constants, $C_3$ - inverse Prantl number at neutrality,

$L_{K/\epsilon}$ - mixing lengths

# Prognostic TKE scheme - code implementation

$$\frac{\partial e}{\partial t} = Adv(e) + \frac{1}{\rho}\frac{\partial}{\partial z}\left(\rho K_E \frac{\partial e}{\partial z}\right) + \frac{1}{\tau_\epsilon}(\tilde{e} - e)$$

$$\tilde{e} = \left(\frac{K^*}{\nu l_m}\right)^2, \qquad K_m = \nu l_m \sqrt{e}\sqrt{F_m},$$

$$\tau_\epsilon = \frac{l_m}{\nu^3\sqrt{e}}\frac{1}{F_\epsilon} = \frac{l_m^2}{\nu^2 K^*}\frac{1}{F_\epsilon}, \qquad \underbrace{K_h = K_m \frac{l_h F_h}{l_m F_m}}_{\text{after TKE solver}},$$

$$K_E = \frac{l_m\sqrt{e}}{\nu}F_\epsilon = \underbrace{\frac{K^*}{\nu^2}F_\epsilon}_{\text{first time step}},$$

$$K^* = \frac{\widetilde{K_m}}{\sqrt{F_m}}, \widetilde{K}_{m/h} = l_{m/h} l_m \left[\left(\frac{\partial \bar{u}}{\partial z}\right)^2 + \left(\frac{\partial \bar{v}}{\partial z}\right)^2\right]^{\frac{1}{2}} F_{m/h}(Ri)$$

$\tilde{e}$ - TKE at stationary equilibrium, $\nu = (C_K C_\epsilon)^{\frac{1}{4}}$, $F_{m/h/\epsilon}$ - stability functions

# Prognostic TKE scheme - code implementation

$$\frac{\partial e}{\partial t} = Adv(e) + \frac{1}{\rho}\frac{\partial}{\partial z}\left(\rho K_E \frac{\partial e}{\partial z}\right) \dots$$

code impl.: $+ \frac{1}{\tau_\epsilon}(\tilde{e} - e)$

versus

full scheme: $+ \underbrace{K_m\left[\left(\frac{\partial \overline{u}}{\partial z}\right)^2 + \left(\frac{\partial \overline{v}}{\partial z}\right)^2\right]}_{I} \underbrace{- \frac{g}{\theta} K_h \frac{\partial \overline{\theta}}{\partial z}}_{II} \underbrace{- C_\epsilon \frac{(e)^{\frac{3}{2}}}{L_\epsilon}}_{-\epsilon}$

equivalence: $\tilde{e} = \frac{e}{\epsilon}(I + II)$

# TOUCANS - stability functions:

(stationary TKE/TTE equation)

$$\tilde{e} = \frac{e}{\epsilon}(I + II) \quad \Leftrightarrow \quad I + II = \epsilon \quad \Leftrightarrow \quad I\frac{f(Ri)}{\chi_3} = \epsilon$$

$$f(Ri) = \chi_3(1 - Ri_f) \quad \text{-'filter'}$$

$$I\frac{f(Ri)}{\chi_3} = \epsilon \quad \text{and} \quad L_{K/\epsilon}(l_m) \Rightarrow$$

$$F_m = \chi_3(Ri)\sqrt{f(Ri)}, \qquad F_h = \frac{\phi_3(Ri)}{\chi_3(Ri)} F_m(Ri)$$

$$F_\epsilon = \frac{f(Ri)^{\frac{3}{4}}}{\chi_3(Ri)^{\frac{3}{2}}}\beta_e$$

$Ri_f = Ri\frac{K_h}{K_m}$ - flux Richardson number, $\beta_e$ - 'dry' antifibrilation coefficient for TKE,
TTE- Total Turbulence Energy

## Stability functions $\chi_3$,$\phi_3$:

- derived from stationary TKE/TTE equation - 'filtering'
- no existence of critical $Ri$
- anisotropy of turbulence $\frac{\partial \chi_3}{\partial Ri} \neq 0$
- valid for whole range of $Ri$
- choice from 3 turbulent schemes: CCH02, QNSE, EFB

# Modified CCH02 scheme:

- CCH02 scheme - Reynolds Stress Modeling scheme
- Modified CCH02 scheme (no critical $Ri$):

$$\chi_3(Ri) = \frac{1 - \frac{Ri_f}{R}}{1 - Ri_f} ,$$

$$\phi_3(Ri) = \frac{1 - \frac{Ri_f}{Ri_{fc}}}{1 - Ri_f} ,$$

$$\frac{Ri}{Ri_f} = \frac{Ri_{fc}(R - Ri_f)}{C_3\, R\, (Ri_{fc} - Ri_f)}$$

# Degrees of freedom

- 3 degrees of freedom for shape of stability functions
  - $Ri_{fc} = \lim_{Ri \to \infty} Ri_f$ - characterising asymptotic behaviour
  - $C_3$ - inverse Prandtl number at neutrality
  - $R$ -parameter characterising the flow's anisotropy
- 1 for 'overall' intensity of turbulence - $\nu \equiv (C_K C_\epsilon)^{\frac{1}{4}}$ , but dependent on $R$ and $C_3$: $\nu(R, C_3)$
- 1 for TKE dissipation - $C_\epsilon$ , but directly dependent on $\nu$ (SS 1989): $C_\epsilon = \pi \nu^2$
- togheter 3 degrees of freedom

## Choice of degrees of freedom

- $C_3 = \frac{1}{P_{rt}(Ri=0)}$, $Ri_{fc} = \lim_{Ri \to \infty} \frac{Ri}{P_{rt}}$, - 'naturally' related to Prandtl number $P_{rt} = \frac{K_m}{K_h}$ (suplied from any turbulent scheme)
- $R$ - counterpart to $Ri_{fc}$ in stability functions $\chi_3$, $\phi_3$ (can be computed from $Ri_f$ and $\chi_3$)
- remaining constants in modified CCH02 scheme ($\lambda$, $F$, $O_\lambda$,..) can be determined from these 3

## A and B system :

Modifications of CCH02 system in order to avoid existence of critical $Ri$ (change in pressure correlation terms):

- A system : dissipation rate for heat flux is dependent on stability
- B system : modification of influence of heat flux on momentum flux

A and B system :

- both have the same shape of stability functions (dependence on 3 degrees of freedom)
- linking relations between $R$, $Ri_{fc}$ and $C_3$ are different
- overall intensity of turbulence $\nu(R, C_3) \equiv (C_K C_\epsilon)^{\frac{1}{4}}$ is different

# QNSE scheme:

- QNSE=Quasi Normal Scale Elimination
- spectral analyses of the flow
- valid mainly for stable stratification ($Ri > 0$)
- no analytical form of stability functions - data points
- no critical $Ri$

# Fitted QNSE scheme:

- fit of $\chi_3(Ri)$ (undirectly fit of $R$ ):

$$\text{for } Ri \geq 0 \quad \chi_3(Ri) = \frac{1 + 0.75Ri(1 + a.Ri)}{1 + 3.23Ri(1 + a.Ri)} \quad ,$$

$$\text{for } Ri < 0 \quad \chi_3(Ri) = \frac{1 - b.Ri}{1 - (b - 2.48).Ri} \quad ,$$

$a = 13.0$, $b = 4.16$ - tuning constants

- $\phi_3(Ri)$ computed from linking equation derived in modified CCH02 (no $R$ dependence):

$$C_3 Ri \phi_3(Ri)^2 - \left[\chi_3(Ri) + \frac{C_3 Ri}{Ri_{fc}}\right] \phi_3(Ri) + \chi_3(Ri) = 0$$

# EFB scheme (not coded):

- EFB=Energy- and Flux-Budget
- Zilitinkevich et al. 2012
- based on budget equations for turbulence energy (kinetic and potential) and fluxes
- prognostic equation for time scale (resp. length scale)
- valid for stable stratification ($Ri > 0$)
- no critical $Ri$

# Fitted EFB scheme (only prognostic TKE):

- fit of $\chi_3(Ri)$ (undirectly fit of $R$ ):

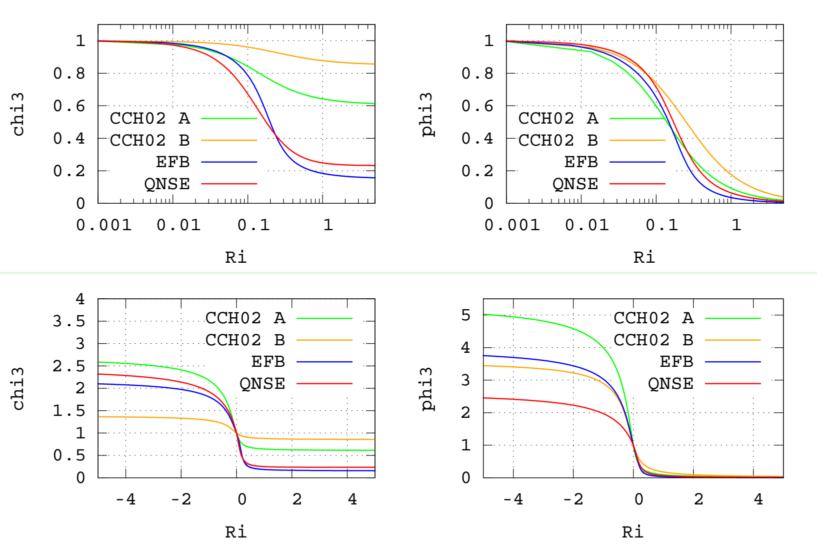
$$\text{for } Ri \geq 0 \quad \chi_3 = \frac{1-1.66\,Ri\,(1-3.15\,Ri\,(2.89\,Ri+1))}{1-0.16\,Ri\,(1-38.96\,Ri\,(16\,Ri+1))},$$

$$\text{for } Ri < 0 \quad \chi_3(Ri) = \frac{1-\frac{Ri_f}{R^{EFB}}}{1-Ri_f}, \; R^{EFB} = 0.455$$

- $\phi_3(Ri)$ computed from linking equation derived in modified CCH02 (no $R$ dependence):

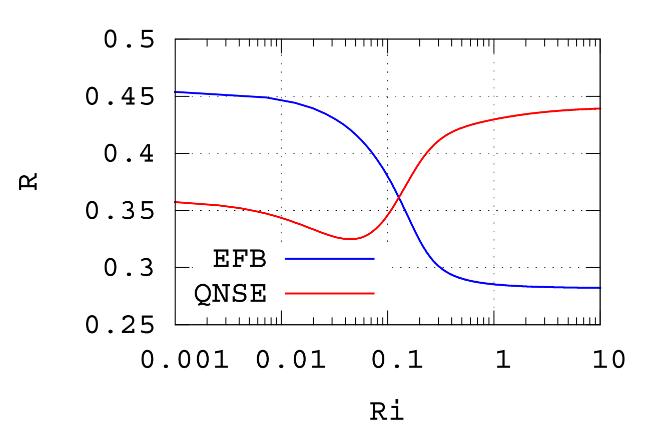
$$C_3 Ri \phi_3(Ri)^2 - \left[\chi_3(Ri) + \frac{C_3 Ri}{Ri_{fc}}\right]\phi_3(Ri) + \chi_3(Ri) = 0$$

# Stability functions comparison:

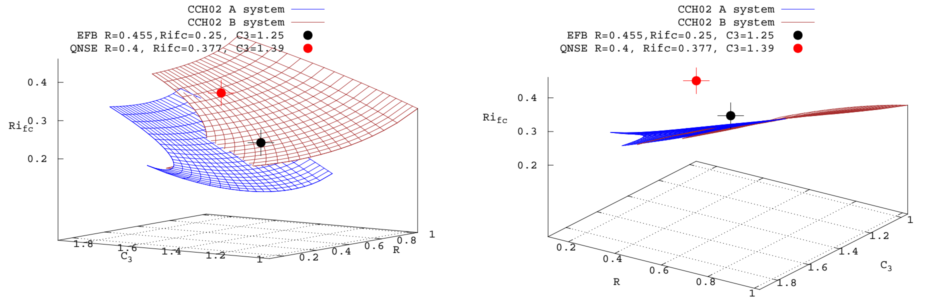


# QNSE and EFB

- QNSE fit and EFB fit have non constant $R = \frac{Ri_f}{\chi_3(Ri_f - 1 + 1)} \cdot \frac{\partial R}{\partial Ri} \neq 0$



## $R, Ri_{fc}, C_3$ - space



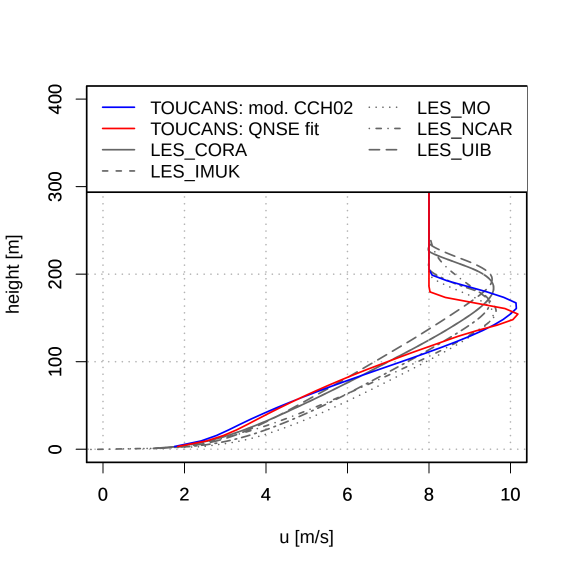
## Degrees of freedom

| Parameter | CCH02 A | CCH02 B | QNSE A/B | EFB A/B |
|---|---|---|---|---|
| $C_3$ | 1.183 | 1.183 | 1.39 | 1.25 |
| $Ri_{fc}$ | 0.1865 | 0.277 | 0.377 | 0.25 |
| $R$ | 0.367 | 0.72 | $\approx 0.4$ | 0.455 |
| $\nu(R, C_3)$ | 0.5265 | 0.477 | 0.504/0.4643 | 0.531/0.483 |
| $C_\epsilon = \pi \nu^2$ | 0.8709 | 0.7148 | 0.798/0.6772 | 0.885/0.732 |

# TOMs in TOUCANS:

- derived in modified CCH02 scheme
- TOMs inputs:
  - $K_h(e, \tau, Ri, C_3, Ri_{fc}, R)$
  - $T_*^{-1}(e, \tau, Ri, \overline{w's'_{li}}, \overline{w'q'_t}, C_3, Ri_{fc}, R)$
  - $T_{**}(e, \tau, Ri, \overline{w's'_{li}}, \overline{w'q'_t}, C_3, Ri_{fc}, R)$
  - $T_h(Ri, C_3, Ri_{fc}, R)$
  - $A_h(Ri, C_3, Ri_{fc}, R)$
  - $\tau(e, L, Ri, C_3, Ri_{fc}, R)$
  - $M(C)$, $C_{term}(C)$ - TKE correction
- in QNSE and EFB $R$ is computed point-wise (for each $Ri$) from $\chi_3$ and $Ri_f$

# GABLS experiment - stable stratification



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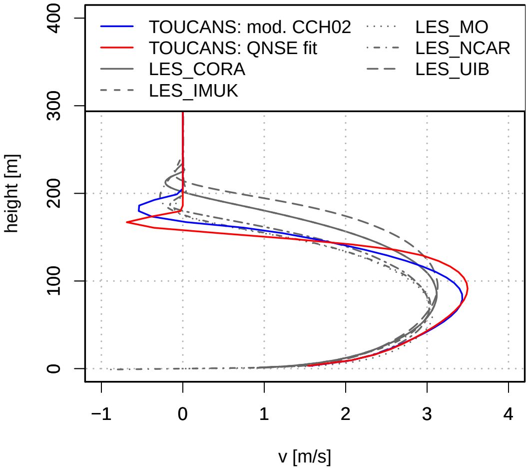
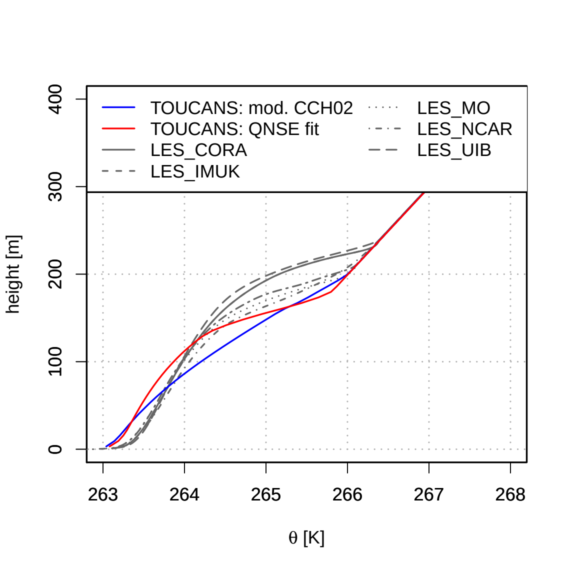


# GABLS experiment - stable stratification



# Mixing lengths

- computed independently before stability functions (with exception for $Ri^{*/**}$ due moist AF scheme)
- choice between Prandtl-type and TKE-type mixing lengths
- TKE-type mixing lengths dependent on $e$ and $Ri$ (influence of shallow convection parametrisation)
- vertical profile connected to Prandtl number (possible change in Unifying perspectives ...)
- possibility of prognostic extension (more in Unifying perspectives ...)

# Mixing length conversion: $L_{K/\epsilon}$ - $l_m$:

- comparison of TKE prognostic scheme with similarity laws (RMC 2001) $\Rightarrow$

$$L_K C_K = \nu l_m \frac{f(Ri)^{\frac{1}{4}}}{\chi_3^{\frac{1}{2}}} \quad ,$$
$$\frac{L_\epsilon}{C_\epsilon} = \frac{l_m}{\nu^3} \frac{\chi_3^{\frac{3}{2}}}{f(Ri)^{\frac{3}{4}}}$$

- choice of one $L$:

$$L \equiv \left(L_K^3 L_\epsilon\right)^{\frac{1}{4}} = \frac{\nu}{C_K} l_m$$

- conversion $L(l_m)$ enables usage of both mixing length types

## Prandtl-type mixing lengths $l_m$ and $l_h$

(CGMIXLEN='AY', in ALARO0='CG') :

$$l_{m/h}^{GC} = \frac{\kappa z}{1 + \frac{\kappa z}{\lambda_{m/h}} \left[ \frac{1+\exp\left(-a_{m/h}\sqrt{\frac{z}{H_{pbl}}}+b_{m/h}\right)}{\beta_{m/h}+\exp\left(-a_{m/h}\sqrt{\frac{z}{H_{pbl}}}+b_{m/h}\right)} \right]}$$

($\kappa$ is Von Kármán constant, $z$ is height, $a_{m/h}$, $b_{m/h}$, $\beta_{m/h}$ and $\lambda_{m/h}$ are tuning constants and $H_{pbl}$ is PBL height)

# TKE mixing lengths:

- modified Bougeault and Lacarrère (1989) approach:

$$L_{BL}(e, N^2) = \left( \frac{L_{up}^{-\frac{4}{5}} + L_{down}^{-\frac{4}{5}}}{2} \right)^{-\frac{5}{4}}$$

$L_{up}(e)$ ($L_{down}(e)$) - upward(downward) mixing distances, $N$ is Brunt-Väisälä Frequency

- mixing length for stable regimes:

$$L_N(e, N^2) = \sqrt{\frac{2.e}{N^2}}$$

# TKE mixing lengths:

- EFB mixing length (not coded)

$$L_\gamma(e, Ri) = \frac{\kappa z}{1 + C_\Omega \frac{\Omega z}{\sqrt{e}}} \left( \frac{e}{\sqrt{\overline{w'u'}^2 + \overline{w'v'}^2}} \right)^{\frac{3}{2}} \phi_3(Ri)$$

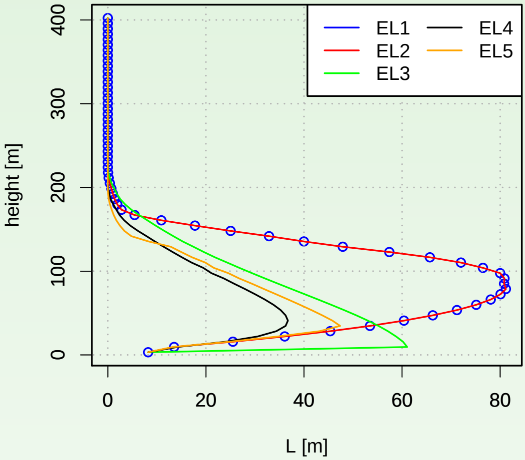
$C_\Omega$ - constant, $\Omega$ angular velocity of Earth's rotation

## Mixing lengths

6 mixing lengths in the code:

| Parameter CGMIXELEN | $Ri > 0$ | $Ri \leq 0$ |
|---|---|---|
| AY | $L_{GC} = \frac{\nu}{C_K} l_m^{GC}$ | $L_{GC}$ |
| EL1 | $L_{BL}$ | $L_{BL}$ |
| EL2 | $L_{BL}$ | $\min\left(\sqrt{L_{BL}\, L_{GC}}, L_{BL}\right)$ |
| EL3 | $\min\left(L_N, L_{max}\right)$ | $L_{GC}$ |
| EL4 | $\frac{L_{GC}\, L_N}{\sqrt{L_{GC}^2 + L_N^2}}$ | $L_{GC}$ |
| EL5 | $\min\left(L_{BL}, L_N\right)$ | $L_{BL}$ |

# Mixing lengths



# Vertical profile of Prandtl number $P_{rt}$

- TKE scheme - $P_{rt}(Ri = 0) = \frac{1}{C_3}$ valid for isotropic turbulence: free atmosphere
- Louis scheme - $P_{rt}$ link to mixing lengths: $P_{rt} = \frac{l_m}{l_h} \frac{F_m(Ri)}{F_h(Ri)} \Rightarrow P_{rt}(Ri = 0) = \frac{l_m}{l_h}$
- TOUCANS - combination of both formalism: $C_3 = \frac{l_h^{GC}}{l_m^{GC}}$:
  Conditions:

$$\text{free atmosphere:} \quad Prt_0 = \frac{l_m}{l_h} = \frac{1}{C_3}$$

$$\text{surface:} \quad Prt_0 = \frac{l_m}{l_h} = 1$$

Solution: $\frac{\lambda_m}{\lambda_h} = \frac{1}{C_3}, \quad \frac{\beta_m}{\beta_h} = 1$

# Shallow convection parametrisation

1. $Ri^*$ after Geleyn 1987 :

$$Ri^* = \underbrace{\frac{g}{c_p T}\left(\frac{\frac{\partial(c_p T + gz)}{\partial z}}{\left[\frac{\partial u}{\partial z}\right]^2 + \left[\frac{\partial v}{\partial z}\right]^2}\right.}_{Ri} + \left.\frac{L.min\left[0, \frac{\partial(q_t - q_{sat})}{\partial z}\right].\delta_h}{\left[\frac{\partial u}{\partial z}\right]^2 + \left[\frac{\partial v}{\partial z}\right]^2}\right)$$

$\delta_h$ -cloudiness switch $\Rightarrow$ (requires moist AF)

2. $Ri^{**}$ - modification of $Ri^*$ with usage of moist entropy potential temperature $\theta_{s_1}$ (Marquet 2010) (requires moist AF)

3. $Ri_m$ - computed from moist BVF (Marquet and Geleyn 2012) - dependent on external cloudiness

4. combination of $Ri_m$ and $Ri_{s_1} = g\left(\frac{\partial \ln(\theta_{s_1})}{\partial z}\right)\frac{1}{\left[\frac{\partial u}{\partial z}\right]^2 + \left[\frac{\partial v}{\partial z}\right]^2}$

# Shallow convection cloudiness - SCC

- required on half levels as input for $q_{li}$ diffusion for separation of $q_t$ flux
- influence on TKE correction after TOMs solver
- in TOUCANS related to $Ri$(on half levels) in shallow convection parametrisation:
    - in $Ri_m$ case directly equal to external cloudiness
    - in remaining cases SCC computed by inversion of $Ri_m(SCC)$ relation from $Ri$ - nonlinear dependence
    - for all $Ri$'s in shallow convection parametrisations consistent computation of SCC

# Summary

- TOUCANS - compact and flexible turbulence parametrisation
  - emulation of 3 turbulent schemes: CCH02, QNSE, EFB from perspective of one theory - 3 degrees of freedom
  - usage of different mixing lengths: Prandtl-type, TKE-type (enabled by $L(l_m)$)
  - four types of shallow convection parametrisation with consistent computation of SCC
- choices in these three categories are orthogonal
- algorithmic unification whenever possible
- scheme uses 6 switches(1 for TOMs) and 7 parameters (2 for TOMs) + some optional tuning

Thank you for your attention!

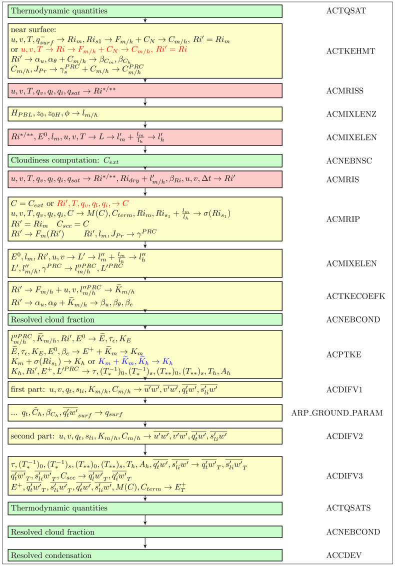



Figure 1: Draft of turbulent scheme (in subroutine APLPAR). Yellow are subroutines of turbulent/diffusion scheme . Red are parts for LCOEFK_RIM=.FALSE. Green are parts connected with turbulent/diffusion scheme.