Turbulence-Diffusion - TOUCANS A: SOMs and TOMs

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TOUCANS

TOUCANS

T - Third

O - Order moments (TOMs)

U - Unified

C - Condensation

A - Accounting and

N - N-dependent

S - Solver (for turbulence and diffusion)

Reynolds-averaged basic equations:

$$\frac{D\overline{u}}{\partial t} = S_u \boxed{-\frac{\partial \overline{u'w'}}{\partial z}}$$

$$\frac{D\overline{v}}{\partial t} = S_v \boxed{-\frac{\partial \overline{v'w'}}{\partial z}}$$

$$\frac{D\overline{\theta}}{\partial t} = S_\theta \boxed{-\frac{\partial \overline{\theta'w'}}{\partial z}}$$

$$\frac{D\overline{q}}{\partial t} = S_q \boxed{-\frac{\partial \overline{q'w'}}{\partial z}}$$

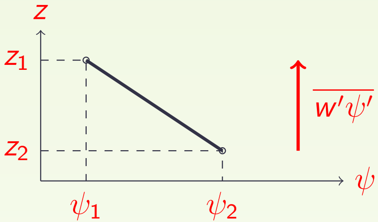
(u, v, w -wind components, θ - potential temperature, q - specific humidity, $S_{u/v/\theta/q}$ - external source terms, $\frac{D()}{\partial t} = \frac{\partial()}{\partial t} + \overline{u}\frac{\partial()}{\partial x} + \overline{v}\frac{\partial()}{\partial y}$, $\overline{()}$ - average, $()'$ - fluctuation)

Modeling of moments

- modeling of Second Order Moments (SOMs) $\overline{\partial u' w'}$, $\overline{\partial u' w'}$, $\overline{\partial \theta' w'}$, $\overline{\partial q' w'}$ require 15 prognostic equation (11 in dry case) with appearance of TOMs
- Third Order Moments (TOMs) additional 26 prognostic equation (16 in dry case) with appearance of FOMs
- Fourth Order Moments (FOMs) additional 9 prognostic equation (Cheng, Canuto, Howard (2005), with parameterisation of some FOMs as a function of SOMs)

Local turbulent diffusion

- reduction of the system to 0 prognostic equation for SOMs
- analogy with molecular diffusion
- depends only on local gradients
  - down-gradient transport
- $\overline{w'\psi'} = -K_\psi \frac{\partial \bar{\psi}}{\partial z}$



(K_ψ - coefficient of turbulent diffusion)

Turbulent diffusion - local transport

$$\frac{\partial\theta}{\partial t} = g\frac{\partial\rho\overline{w'\theta'}}{\partial p}$$

$$\frac{\theta^{*}-\theta^{-}}{\delta t} = \frac{\partial\left(-g\rho K_h\left(\frac{\partial\overline{\theta^{*}}}{\partial z}\right)\right)}{\partial p}$$

$$\text{at surface: } \overline{w'\theta'} = -C_h|V_L|\left(\theta_L^{*}-\theta_s\right)$$

θ- potential temperature, θ^* is θ at next time step after local diffusion, V -horizontal wind speed, w -vertical wind component , $_L$-lowest model level, $_s$ -surface, δt - time step, K_h - exchange coefficient for θ, C_h - drag coefficient for θ, ρ - density, p - pressure, z -height, g - acceleration of gravity

Exchange coefficients $K_{m/h}$ and drag coefficients $C_{m/h}$ in Louis scheme (LPTKE=.FALSE., LCOEFKTKE=.FALSE.):

$$K_{m/h} = l_{m/h} l_m \left[\left(\frac{\partial \bar{u}}{\partial z} \right)^2 + \left(\frac{\partial \bar{v}}{\partial z} \right)^2 \right]^{\frac{1}{2}} F_{m/h}(Ri)$$

$$C_{m/h} = C^N_{m/h}(z, z_0, \kappa).F_{m/h}(Ri)$$

$l_{m/h}$ - Prandtl mixing length for momentum and pot. temperature
$F_{m/h}(Ri)$ - stability functions, Ri - Richardson gradient number
u, v , w - wind components , z - height, z_0 - roughness
$C^N_{m/h}$ - drag coefficient at neutrality($Ri = 0$)
κ - von Karman constant

Louis stability functions F_m and F_h:

stable case:

$$F_m(Ri) = \frac{1}{1+\frac{2bRi'_m}{\sqrt{1+\frac{d}{k}Ri'_m}}}, \quad Ri'_m = \frac{Ri}{1+\frac{Ri}{Ri_{lim}}}$$

$$F_h(Ri) = \frac{1}{1+3bRi'_h\sqrt{1+d\,k\,Ri'_h}}, \quad Ri'_h = \frac{Ri}{\left(1+\alpha\frac{Ri}{Ri_{lim}}\right)^{\frac{1}{\alpha}}}$$

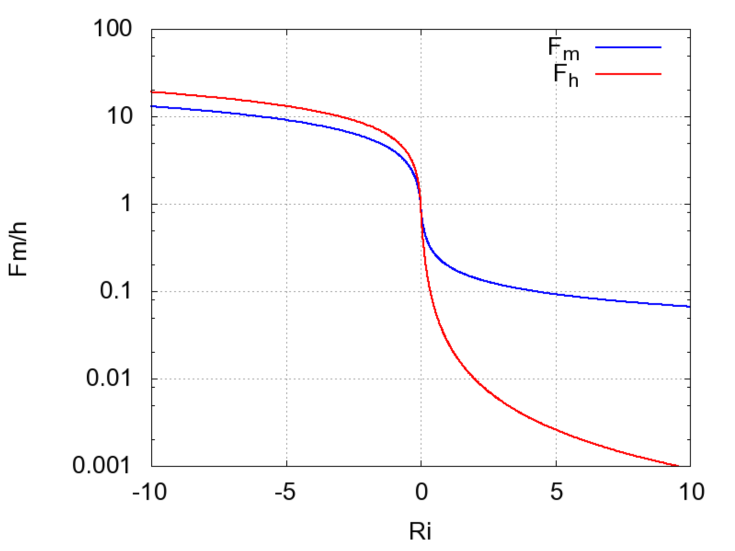
unstable case:

$$F_m(Ri) = 1 - \frac{2bRi}{1+3bc\sqrt{\frac{|Ri|}{27}}\left(\frac{l_m}{z+z_0}\right)^2}$$

$$F_h(Ri) = 1 - \frac{3bRi}{1+3bc\sqrt{\frac{|Ri|}{27}}\left(\frac{l_h}{z+z_{0h}}\right)\left(\frac{l_m}{z+z_0}\right)}$$

b, c, d, k - constants, z_0, z_{0h} - roughness, $Ri_{lim}(z)$-limiting Ri, $\alpha(z, Ri)$- coefficient

Louis scheme - stability functions



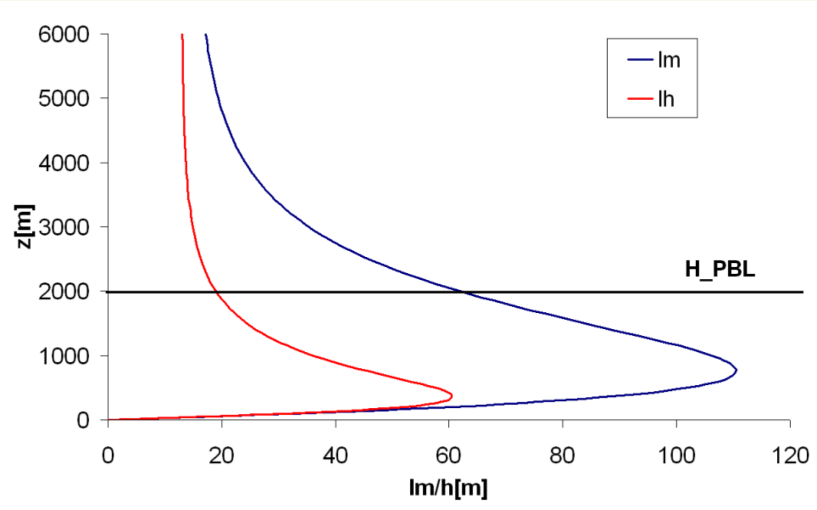
Prandtl-type mixing lengths l_m and l_h

(CGMIXLEN='AY', in ALARO0='CG') :

$$l_{m/h}^{GC} = \frac{\kappa z}{1 + \frac{\kappa z}{\lambda_{m/h}} \left[\frac{1+\exp\left(-a_{m/h}\sqrt{\frac{z}{H_{pbl}}}+b_{m/h}\right)}{\beta_{m/h}+\exp\left(-a_{m/h}\sqrt{\frac{z}{H_{pbl}}}+b_{m/h}\right)} \right]}$$

(κ is Von Kármán constant, z is height, $a_{m/h}$, $b_{m/h}$, $\beta_{m/h}$ and $\lambda_{m/h}$ are tuning constants and H_{pbl} is PBL height)

Prandtl-type mixing lengths:



TOUCANS and pseudo-TKE (LPTKE=.TRUE.)

- addition of 1 prognostic equation for SOMs

$$\frac{\partial e}{\partial t} = Adv(e) + \overbrace{\frac{1}{\rho}\frac{\partial}{\partial z}\left(\rho K_E \frac{\partial e}{\partial z}\right)}^{\text{TKE diffusion}} + \underbrace{K_m\left[\left(\frac{\partial \overline{u}}{\partial z}\right)^2 + \left(\frac{\partial \overline{v}}{\partial z}\right)^2\right]}_{\text{shear}} \underbrace{-\frac{g}{\theta}K_h\frac{\partial\overline{\theta}}{\partial z}}_{\text{buoyancy}} \underbrace{-C_\epsilon\frac{(e)^{\frac{3}{2}}}{L_\epsilon}}_{\text{dissipation}}$$

$$K_m = L_K C_K \sqrt{e}\chi_3(Ri), \qquad K_h = L_K C_K C_3 \sqrt{e}\phi_3(Ri)$$

$e = \frac{1}{2}(\overline{u'\cdot u' + v'\cdot v' + w'\cdot w'})$ =TKE, K_E- auto-diffusion coefficient for TKE, $\chi_3(Ri)$,$\phi_3(Ri)$ - stability functions, C_K,C_ϵ - closure constants, C_3 - inverse Prantl number at neutrality, $L_{K/\epsilon}$ - mixing lengths

Prognostic TKE scheme - code implementation

$$\frac{\partial e}{\partial t} = \underbrace{Adv(e)}_{\text{advection}} + \underbrace{\frac{1}{\rho}\frac{\partial}{\partial z}\left(\rho K_E \frac{\partial e}{\partial z}\right)}_{\text{diffusion with antifibrilation}} + \underbrace{\frac{1}{\tau_\epsilon}(\tilde{e} - e)}_{\text{relaxation}}$$

- numerically stable
- with antifibrilation for TKE diffusion
- enables shallow convection parametrisation with Richardson number's modification

$\tilde{e}$ - TKE at stationary equilibrium, τ_ϵ - dissipation time scale

Prognostic TKE scheme - code implementation

$$\frac{\partial e}{\partial t} = Adv(e) + \frac{1}{\rho}\frac{\partial}{\partial z}\left(\rho K_E \frac{\partial e}{\partial z}\right) + \frac{1}{\tau_\epsilon}(\tilde{e} - e)$$

$$\widetilde{e} = \left(\frac{K^*}{\nu l_m}\right)^2, \qquad K_m = \nu l_m \sqrt{e}\sqrt{F_m},$$

$$\tau_\epsilon = \frac{l_m}{\nu^3\sqrt{e}}\frac{1}{F_\epsilon} = \frac{l_m^2}{\nu^2 K^*}\frac{1}{F_\epsilon}, \qquad \underbrace{K_h = K_m \frac{l_h F_h}{l_m F_m}}_{\text{after TKE solver}},$$

$$K_E = \frac{l_m\sqrt{e}}{\nu}F_\epsilon = \underbrace{\frac{K^*}{\nu^2}F_\epsilon}_{\text{first time step}},$$

$$K^* = \frac{\widetilde{K_m}}{\sqrt{F_m}}, \widetilde{K}_{m/h} = l_{m/h} l_m \left[\left(\frac{\partial \bar{u}}{\partial z}\right)^2 + \left(\frac{\partial \bar{v}}{\partial z}\right)^2\right]^{\frac{1}{2}} F_{m/h}(Ri)$$

$\nu = (C_K C_\epsilon)^{\frac{1}{4}}$, F_ϵ - stability function

Prognostic TKE scheme

-TOUCANS is analytically equivalen with 'full TKE scheme' (LCOEFKTKE=.TRUE.)
-pseudo-TKE uses Louis stability functions (LCOEFKTKE=.FALSE.)

stability function	TOUCANS	pseudo-TKE
F_ϵ	$\frac{f(Ri)^{\frac{3}{4}}}{\chi_3(Ri)^{\frac{3}{2}}}\beta_e$	1.0
F_m	$\chi_3(Ri)\sqrt{\chi_3(Ri)(1-Ri_f)}$	Louis scheme
F_h	$\frac{\phi_3(Ri)}{\chi_3(Ri)}F_m(Ri)$	Louis scheme

$f(Ri) = \chi_3(1 - Ri_f)$, $Ri_f = Ri\frac{K_h}{K_m}$ - flux Richardson number,

β_e - 'dry' antifibrilation coefficient for TKE

Prognostic TKE scheme

Computation of 'static' $\widetilde{K_m}$, $\widetilde{K_h}$

↓

TKE solver:
update of e $\Rightarrow$ update of K_m, K_h

↓

Local turbulent diffusion (with AF scheme)
with K_m, K_h :
computation of $\overline{\theta' w'}$, $\overline{q' w'}$, $\overline{u' w'}$,$\overline{v' w'}$

Vertical stagerring

FULL LEVEL ——— u, v, θ, q, e

HALF LEVEL ——— $\widetilde{e}, K_E, \tau_\epsilon, l_m, Ri, K_{m/h}$

FULL LEVEL ——— u, v, θ, q, e

Non-local transport

- thermals can cause counter- gradient transport
- influence of higher order moments
- Canuto, Cheng, Howard (2005):
  $\overline{w'\theta'} = -K_h \frac{\partial \overline{\theta}}{\partial z} + A_1^\theta \frac{\partial \overline{w'^3}}{\partial z} + A_2^\theta \frac{\partial \overline{w'\theta'^2}}{\partial z} + A_3^\theta \frac{\partial \overline{w'^2\theta}}{\partial z}$
- A_1^θ, A_2^θ, A_3^θ - functions of Ri, $\overline{\theta}$, $\frac{\partial \overline{\theta}}{\partial z}$, e, L,
  and of constants of turbulence scheme

L - mixing length ($L = \left(L_K^3 L_\epsilon\right)^{\frac{1}{4}}$ in TOUCANS)

TOMs after Canuto, Cheng, Howard (2007):

- simplification of the complex set of equations (FOMs included) to:

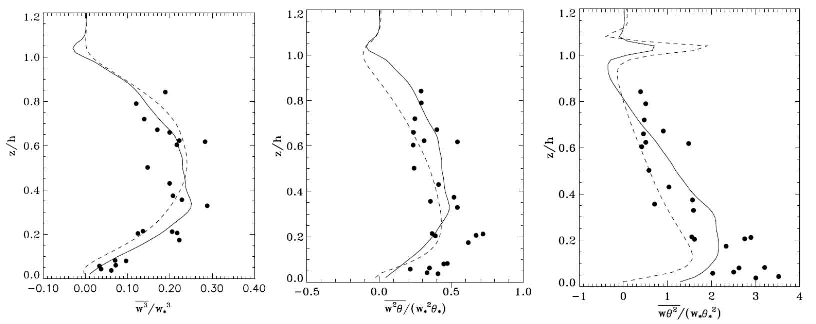
$$\overline{w'^3} = -0.06\frac{g}{\overline{\theta}}\tau^2\overline{w'^2}\frac{\partial\overline{w'\theta'}}{\partial z}$$

$$\overline{w'\theta'^2} = -\tau\overline{w'\theta'}\frac{\partial\overline{w'\theta'}}{\partial z}$$

$$\overline{w'^2\theta'} = -0.3\tau\overline{w'^2}\frac{\partial\overline{w'\theta'}}{\partial z}$$

$\tau = 2\frac{e}{\epsilon}$ -TKE dissipation time scale, $\epsilon = C_\epsilon\frac{e^{\frac{3}{2}}}{L}$ -TKE dissipation rate

TOMs after Canuto, Cheng, Howard (2007):



• aircraft data, – – LES data, – Canuto, Cheng, Howard (2007),

$\frac{z}{h}$ - height normalized by the PBL depth.

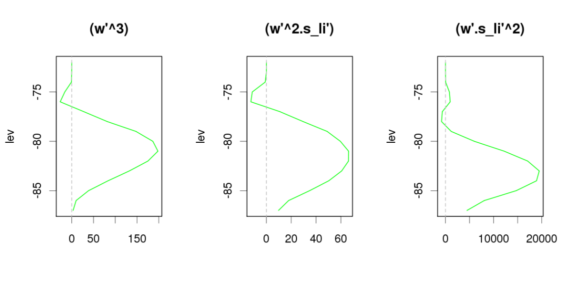
TOUCANS :

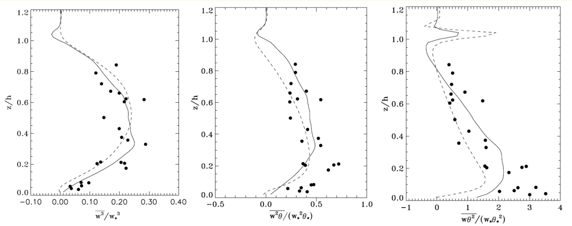
$$\overline{w'^2} = e\frac{2}{3}\left[1 - \frac{(3\lambda_3 - \lambda_2)\left(1 + \frac{4\lambda_4 Ri_f}{(3\lambda_3 - \lambda_2)}\right)}{1 - Ri_f}\right]$$

$$\overline{w'^2} = eF_{\overline{w'^2}}$$

λ_{2-4} - turbulence scheme constants

TOMs in TOUCANS:





Chosen profile from 3D run for TOUCANS with QNSE A setting vs. CCH2007.
($s_{li} = c_p T + gz - L_v q_l - L_s q_i$ - ice-liquid water static energy)

Determination of A_1^θ, A_2^θ, A_3^θ:

- starting from equation for $\overline{\theta'^2}$ without neglecting TOM term:

$$\frac{\partial \overline{w'\theta'^2}}{\partial z} = -2\frac{\partial \overline{\theta}}{\partial z}\overline{w'\theta'} - 2C_3\frac{\overline{\theta'^2}}{\tau} \Rightarrow$$

$$\overline{w'\theta'} = -K_h.\frac{\partial \overline{\theta}}{\partial z} - K_h T_h(Ri)\frac{\tau}{e}\frac{g}{\overline{\theta}}\frac{\partial \overline{w'\theta'^2}}{\partial z} \Rightarrow$$

$$A_2^\theta = -K_h T_h(Ri)\frac{\tau}{e}\frac{g}{\overline{\theta}}$$

C_3 - inverse Prandtl number at neutrality,

$T_h(Ri)$ - depends on Ri and on constants of turbulence scheme

Determination of A_1^θ, A_2^θ, A_3^θ:

- A_1^θ (for $\frac{\partial \overline{w'^3}}{\partial z}$), A_3^θ (for $\frac{\partial \overline{w'^2\theta'}}{\partial z}$) are computed with ratio method from A_2^θ (for $\frac{\partial \overline{w'\theta'^2}}{\partial z}$)
- ratios determined from prognostic equations for $\overline{w'^2}$, $\overline{w'\theta'}$, e, and $\overline{\theta'^2}$:

$$
\begin{aligned}
\overline{w'\theta'} \quad = \quad & (-4\,(1-3\,\lambda_3+\,\lambda_2)\left(2\,\tau\,e+\tau^2\,\frac{\partial e}{\partial t}\right)\frac{\partial \theta}{\partial z}+\\
& 3\,(\lambda+3\,\lambda_3-\lambda_2)\,\tau^2\frac{\partial \theta}{\partial z}\frac{\partial \overline{w'^3}}{\partial z}+6\,\lambda\tau^2\frac{\partial \theta}{\partial z}\frac{\partial \overline{w'^2}}{\partial t}\\
& -6\,\lambda_8\,\frac{g}{\theta}\,\tau^2\left(\frac{\partial \overline{w'\theta'^2}}{\partial z}+\frac{\partial \overline{\theta'^2}}{\partial t}\right)-12\,\tau\left(\frac{\partial \overline{w'^2\theta'}}{\partial z}+\frac{\partial \overline{w'\theta'}}{\partial t}\right))\cdot\\
& \left[\frac{g}{\theta}\frac{\partial \theta}{\partial z}\tau^2\,(12\lambda_8+4\lambda_4+4\,(3\,\lambda_3-\lambda_2))+12\,\lambda_5\right]^{-1}
\end{aligned}
$$

Determination of A_1^θ, A_2^θ, A_3^θ:

- resulting A_1^θ, A_2^θ, A_3^θ:

$$
\begin{aligned}
A_2^\theta &= -K_h T_h(Ri) \frac{\tau}{e} \frac{g}{\overline{\theta}} \\
A_1^\theta &= K_h T_h(Ri) \frac{C_3 \left(3\,\lambda_3 - \lambda_2 + \lambda\right)}{2 O_\lambda} \frac{\tau}{e} \frac{\partial \overline{\theta}}{\partial z} \\
A_3^\theta &= -K_h T_h(Ri) \frac{2 C_3}{O_\lambda} \frac{1}{e}
\end{aligned}
$$

(λ, O_λ - turbulence scheme constants)

Resulting equation for $\overline{w'\theta'}$:

- resulting A_1^θ, A_2^θ, A_3^θ:

$$\overline{w'\theta'} = -K_h \frac{\partial\overline{\theta}}{\partial z} + I^* \frac{\partial\overline{w'\theta'}}{\partial z} + I^{**} \frac{\partial^2\overline{w'\theta'}}{\partial z^2} \Rightarrow$$

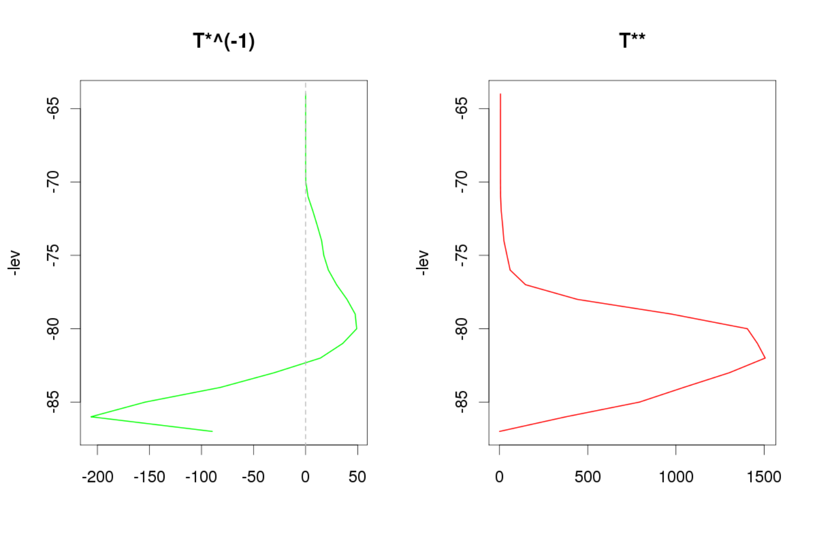
$$J_h = \rho K_h \frac{\partial\overline{\theta}}{\partial z}$$

$$+\rho K_h T_h(Ri) T_*^{-1} \frac{\partial J_h}{\partial p} - \rho g K_h T_h(Ri) T_{**} \frac{\partial\left(\frac{\partial J_h}{\partial p}\right)}{\partial z}$$

- T_*^{-1}, T_{**} - functions of: $\overline{w'\theta'}$, $\overline{w'^2}$, Ri, N^2, $\overline{\theta}$, $\frac{\partial\overline{\theta}}{\partial z}$, e, L

($J_h = -\rho\overline{w'\theta'}$)

Typical vertical profile of T_*^{-1}, T_{**}:



Solver:

- stable and accurate algorithm immune against singularities:
- rewritten in tendencies through $\frac{\partial J_h}{\partial p} = -\frac{1}{g}\frac{\partial \theta}{\partial t}$:

$$\frac{\partial \theta}{\partial t} = \underbrace{\frac{\partial\left(-g\rho K_h\left(\frac{\partial\theta}{\partial z} + T_h(Ri)\,T_{**}\frac{\partial\left(\frac{\partial\theta}{\partial t}\right)}{\partial z}\right)\right)}{\partial p}}_{\text{'diffusive term'}} + \underbrace{\frac{\partial\left(\rho K_h . T_h(Ri)\left(T_*^{-1}\frac{\partial\theta}{\partial t}\right)\right)}{\partial p}}_{\text{'mass-flux type term'}}$$

Solver:

- when we substract equation for local tendencies:

$$\frac{\theta^* - \theta^-}{\delta t} = \frac{\partial\left(-g\rho K_h\left(\overline{\frac{\partial\theta^*}{\partial z}}\right)\right)}{\partial p},$$

we get:

$$\frac{\delta\theta^+}{\delta t} = \frac{\partial\left(-g\rho K_h\left(1 + \frac{T_h(Ri)T_{**}}{\delta t}\right)\frac{\partial(\delta\theta^+)}{\partial z}\right)}{\partial p} + \frac{\partial\left(\rho K_h . T_h(Ri)(T_*^{-1}\widehat{\left(\frac{\delta\theta^+}{\delta t}\right)})\right)}{\partial p}$$
$$+ \frac{\partial\left(-g\rho K_h\left(\frac{T_h(Ri)T_{**}}{\delta t}\frac{\partial(\theta^* - \theta^-)}{\partial z}\right)\right)}{\partial p} + \frac{\partial\left(\rho K_h . T_h(Ri)(T_*^{-1}\widehat{\left(\frac{\theta^* - \theta^-}{\delta t}\right)}\right.}{\partial p}$$

(δt is time step,'$^+$', and '$^-$' mark next and current time step, θ^* is θ at next time step after local diffusion $\delta\theta^+ \equiv \theta^+ - \theta^*$, $\widehat{\ }$ - 'hat' is used for averaging from full levels to half levels, T_{**} is on half level, T_*^{-1} is on full level)

Solver:

- $\overline{w'\theta'}$ for T_{**}, T_{*}^{-1} is taken from local diffusion as 'first guess' : $\left(\overline{w'\theta'}\right)_0 = \overline{w'\theta^{*\prime}}$
- the non-local part of the solver is iterated once in order to update $\left(\overline{w'\theta'}\right)_0$ with resulting $\overline{w'\theta'}$ (including contribution from TOMs)
- TKE is updated (for the next time step) according to the non-local correction of $\overline{w'\theta'}$
- surface fluxes are updated according to TOMs contribution

Stability of the solver:

- T_* needs special treatment adapted from mass flux approach (convection) in order to avoid non-linear instability
- T_{**} term is diffusion term, matrix is diagonal dominant if also $T_{**} > 0$, therefore we secure $T_{**} = max(0.0, T_{**})$

TOUCANS with non-local transport:

Computation of 'static' $\widetilde{K_m}$, $\widetilde{K_h}$

↓

TKE solver:
update of e $\Rightarrow$ update of K_m, K_h

↓

Local turbulent diffusion with K_m, K_h:
computation of $\overline{\theta' w'}$, $\overline{q' w'}$, $\overline{u' w'}$, $\overline{v' w'}$

↓

Computation of T_*^{-1}, T_{**} from e, L, $\overline{\theta' w'}$, $\overline{q' w'}$

↓

Non-local turbulent diffusion with K_h, T_*^{-1}, T_{**}:
computation of $\overline{\theta' w'}_{TOMs}$, $\overline{q' w'}_{TOMs}$

Time scale of non-local transport:

- from prognostic equation for $\overline{w'\theta'}$:

$$
\begin{aligned}
\overline{w'\theta'} + A_t \frac{\partial \overline{w'\theta'}}{\partial t} &= -K_h \frac{\partial \overline{\theta}}{\partial z} + A_1^{\theta} \frac{\partial \overline{w'^3}}{\partial z} + A_2^{\theta} \frac{\partial \overline{w'\theta'^2}}{\partial z} \\
&\quad + A_3^{\theta} \frac{\partial \overline{w'^2\theta}}{\partial z} \\
A_t &= A_h \tau \equiv K^{(A_2,A_3)} \frac{C_K C_\epsilon}{2} T_h(Ri) \phi_3(Ri) \tau
\end{aligned}
$$

Time scale of non-local transport:

- with assuming $\frac{\partial A_h}{\partial p} = 0$, $\theta^- - \theta^{--} \cong \theta^* - \theta^-$,we get:

$$\frac{\delta\theta^+}{\delta t} = \frac{1}{1 + \overbrace{\frac{A_t}{\delta t}}} [\frac{\partial\left(-g\rho K_h\left(1 + \frac{T_h(Ri)T_{**}}{\delta t}\right)\frac{\partial(\delta\theta^+)}{\partial z}\right)}{\partial p} + \frac{\partial\left(\rho K_h . T_h(Ri)(T_*^{-1}\widehat{\left(\frac{\delta\theta^+}{\delta t}\right)})\right)}{\partial p}$$

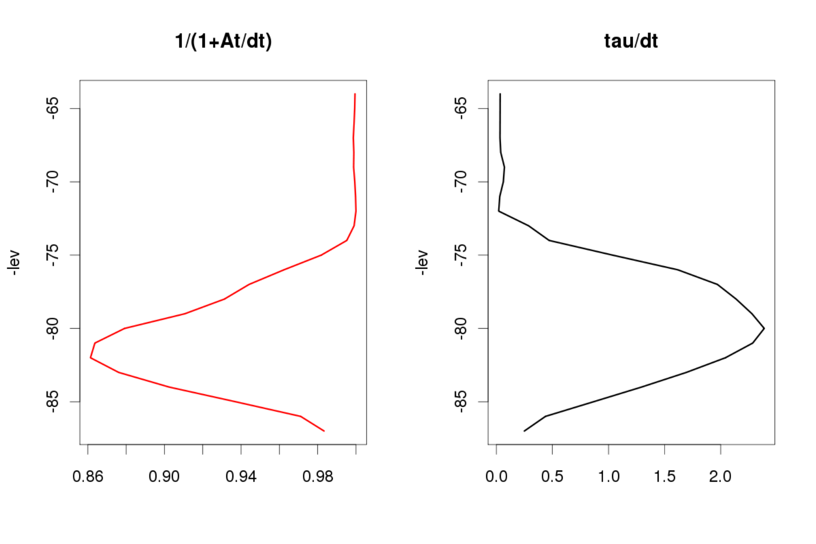
$$+ \frac{\partial\left(-g\rho K_h\left(\frac{T_h(Ri)T_{**}}{\delta t}\frac{\partial(\theta^* - \theta^-)}{\partial z}\right)\right)}{\partial p}$$

$$+ \frac{\partial\left(\rho K_h . T_h(Ri)(T_*^{-1}\widehat{\left(\frac{\theta^* - \theta^-}{\delta t}\right)})\right)}{\partial p}]$$

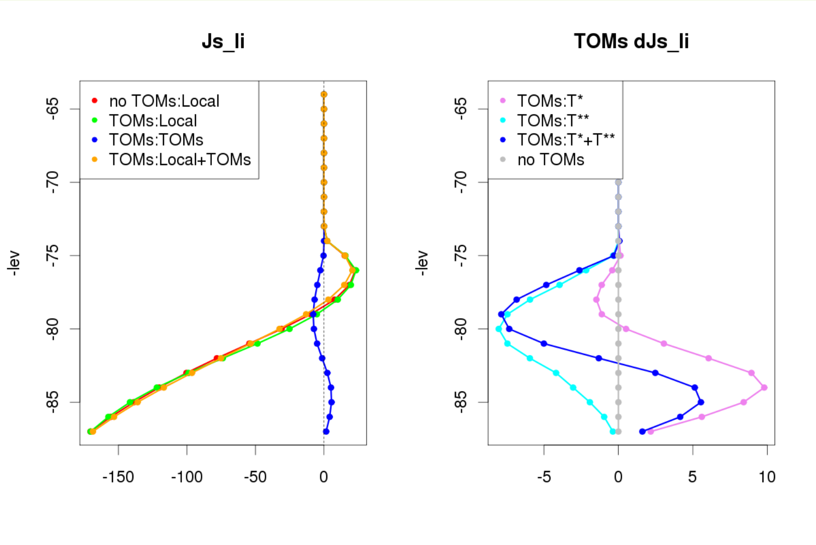
($\overbrace{\quad}$ - 'overbrace' is used for averaging from half levels to full levels,

A_t - is on half level)

Typical vertical profile of time scale and τ:



Contribution from TOMs to heat flux:



Chosen profile from 3D run at 11 a.m. for TOUCANS with QNSE A setting

Moist aspect:

- usage of conservative variables for turbulent transport:

$$
\begin{aligned}
q_t &= q_v + q_l + q_i \\
s_{li} &= c_p T + \phi - L_v q_l - L_s q_i
\end{aligned}
$$

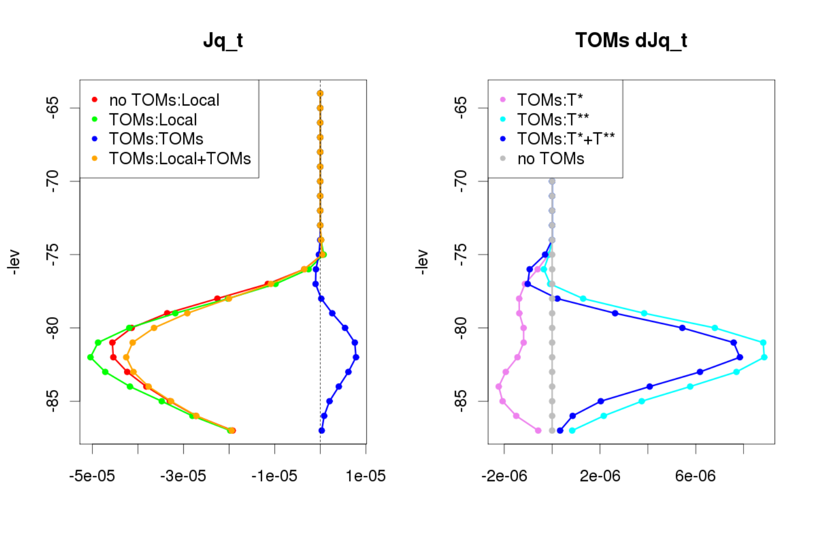
(q_t - total specific humidity,q_v - specific humidity , q_l- specific humidity of liquid water, q_i- specific humidity of ice, ϕ - geopotential, c_p - specific heat capacity, L_v - latent heat of vaporization, L_s - latent heat of sublimation)

Moist aspect:

- Ri is computed from moist entropy potential temperature $\theta_{s1} = \theta_l \exp\left(\Lambda q_t\right)$ after Marquet (2011) and from 'external' C cloudiness - shallow convection parametrisation (affects also local diffusion) $\Rightarrow$
- simple way of extending TOMs parametrisation for moist case without touching TOMs solver
- we assume that both q_t, s_{li} use the same T_{**}, T_*^{-1} (analogy with K_h)
- $\left(\overline{w'\theta'}\right)_0$ is replaced by $\left(\overline{w'\theta'_{s1}}\right)_0$

$(\theta_l = \theta - \frac{\theta}{T}\left(\frac{L_v}{c_p} q_l + \frac{L_s}{c_p} q_i\right))$

Contribution from TOMs to flux of moisture:



Chosen profile from 3D run at 11 a.m. for TOUCANS with QNSE A setting

TKE correction after TOMs:

- 'dry' case: $e_T = e_l + \delta t \, \frac{g}{c_p.T} \left(\overline{s'w'}_T - \overline{s'w'}_l\right)$
- 'moist' case
  - using relation for moist BVF - Marquet, Geleyn 2012 (with buoyancy term rewritten through fluxes):

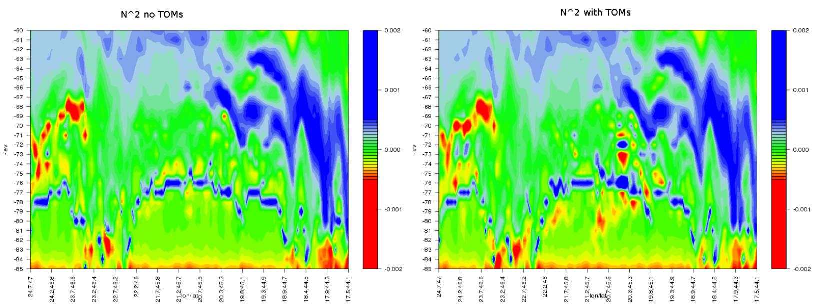
$$
\begin{aligned}
e_T &= e_l + \delta t \, \frac{gM(C)}{c_p T} \left(\left[\overline{w's'_{li}}_T - \overline{w's'_{li}}_l \right] \right. \\
&\quad \left. + C_{term} \, c_p \, T \, \left[\overline{w'q'_t}_T - \overline{w'q'_t}_l \right] \right) \\
C_{term} &\equiv \left[\frac{(1+r_v)\,R_v}{R_d.q_d + R_v.q_v} F(C) - \frac{1}{M(C)\,(1-q_t)} \right]
\end{aligned}
$$

($_l$- after local diffusion, $_T$- after TOMs; $M(C)$, $F(C)$ - functions of C, ..)

Vertical diffusion of $q_{l/i}$

- after Smith 1990: turbulent fluxes $\overline{w'q'_{l/i}}$ can be computed from fluxes of conservative variables : $\overline{w'q'_t}$ and $\overline{w's'_{li}}$ (cloud fraction C (on half levels) needed) { More details in TOUCANS B presentation}
- TOMs parametrisation
  - extends $q_{l/i}$ diffusion (TOMs influence)
  - increases stability of $q_{l/i}$ diffusion (algorithmic analogy with TOMs solver)

Brunt–Väisälä frequency vertical cross section:



Results after 30 h of integration at 12 a.m. TOUCANS with QNSE A setting in ARPEGE/ ALADIN model.

Summary

- local turbulent diffusion:
  - no prognostic SOMs - Louis scheme
  - prognostic TKE - stable solver:
    - pseudo-TKE - Louis stability functions
    - TOUCANS (without TOMs) - analytically equivalent with 'full TKE scheme'
- non local turbulent transport:
  - TOUCANS - parametrisation of TOMs for heat and moisture

Summary

TOUCANS parametrisation of TOMs:

- usage of TOMs expression derived from system with prognostic equations till FOMs (Canuto, Cheng, Howard (2007))
- influence of both $\frac{\partial \overline{w'\theta'}}{\partial z}$ (T_*^{-1}) and $\frac{\partial^2 \overline{w'\theta'}}{\partial z^2}$ (T_{**}) is included
- considering time scale of non-local transport

Summary

TOUCANS parametrisation of TOMs:

- simple and relative cheap extension of local diffusion
- stable solver
- possible extension towards moist cases through Ri modification (thanks to Marquet (2011)) 'orthogonal' to TOMs solver
- extension of $q_{l/i}$ diffusion

Thank you for your attention!