‘TOUCANS’
Complements on:
- the QNSE fitting procedure;
- the details of the ‘moist’ part.

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QNSE fitting

with some complements on the
stationarity-based ‘filter’

The '*f*' function (RMC01) and its computation

- A bridge is needed between the shear- and buoyancy- terms of the TKE prognostic equation.

- The 'CBR' approach obtains it in a case where the only stability dependency is the one linked with the parameterisation of the TKE ⇔ TPE term, but this result can be shown to be absolutely general.

$$f = \frac{c_\varepsilon}{c_K} \frac{E}{L^2\left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right]}$$

- There are two ways to compute '***f***' in practice:
  - Either explicitly while solving the TKE equation;
  - Or by solving a characteristic equation that expresses the stationnary solution **shear term + buoyancy term + dissipation = 0** . ***This delivers a second order equation for f(R_i) that admits a solution for R_i going from -∞ to +∞*** .

The '*f*' function (RMC01) and its computation

- We follow here the second path, since:
  - We wish a solution without restriction of the range of possible Richardson-numbers;
  - We obtain this feature in a way very similar to the argument of Zilitinkevitch et al.: '***f***' acts as a 'filter' imposing that '***stationarity of the TKE equation + diagnostic TPE equation*** $\Leftrightarrow$ ***conservation of TTE***'.
- Under these conditions it can be shown that the characteristic equation leading to '***f***' factorises as

$$f(R_i) = \chi_3(R_i)(1 - R_{if})$$

with $\boldsymbol{R_{if}}$ the flux-Richardson-number. With this, $\boldsymbol{\chi_3(R_i)}$ has the same range of validity as '***f***', i.e. from $-\infty$ to $+\infty$. Idem for $\boldsymbol{\phi_3(R_i)}$.
.

A key relationship

- We do not have yet the conditions for a full analytical solution of the problem.
- But, adding one constraint (too complex to be explicited here), that anyhow takes a different shape depending on which problem one wants to solve, one can obtain a unique equation linking the two stability dependency functions:

$$C_3 R_i \phi_3^{\;2} - \phi_3 (\chi_3 + C_3 R_i / R_{ifc}) + \chi_3 = 0$$

with $\boldsymbol{C_3}$ the inverse Prandtl number at neutrality and $\boldsymbol{R_{ifc}}$ the critical flux-Richarson-number, i.e. two of the three 'physical' quantities relevant to our proposal (note indeed that '***R***' does not appear in this equation).

Choice of the method (stable range)

- We recall our 'universal' equation for RANS models.

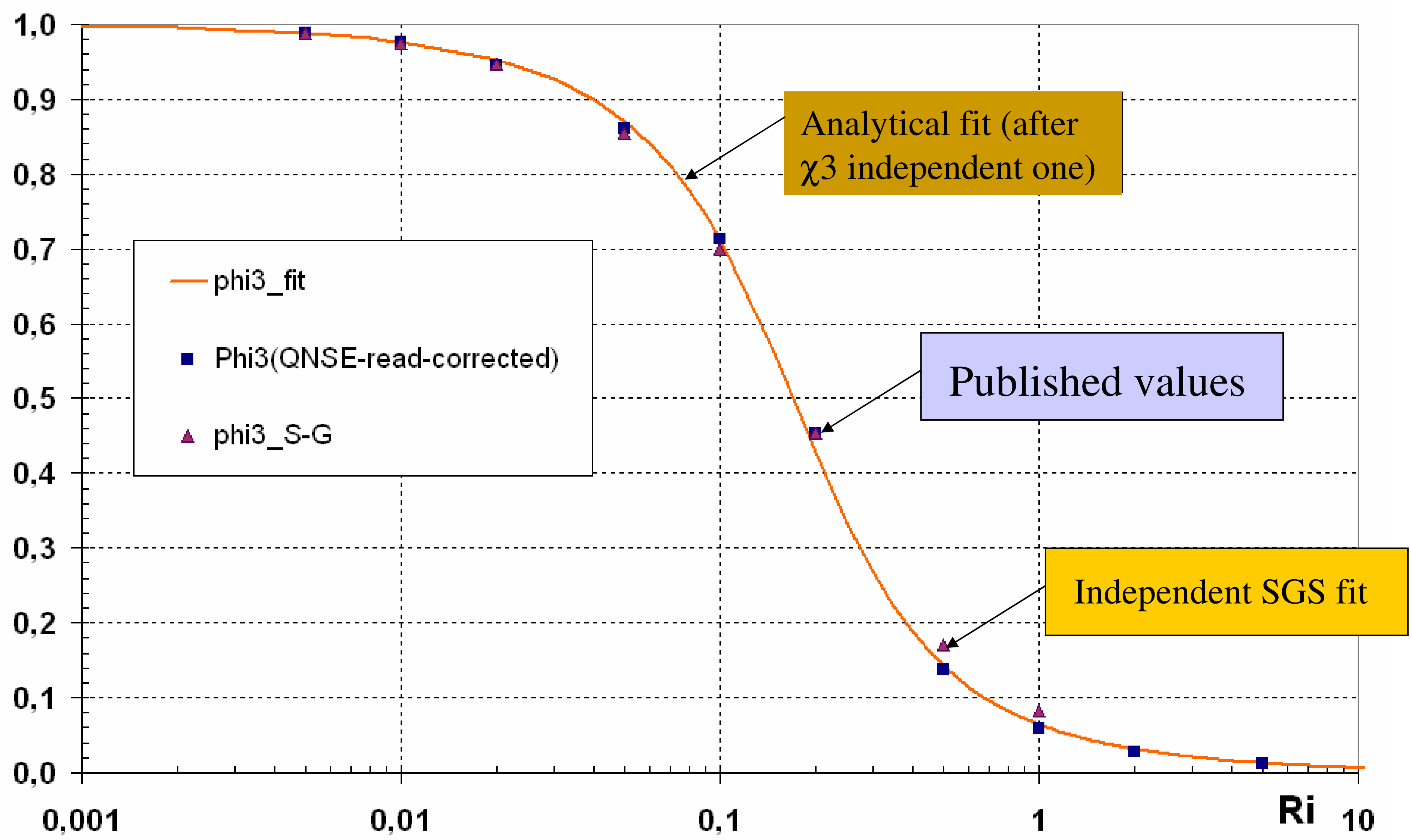
$$C_3 R_i \phi_3^{\,2} - \phi_3(\chi_3 + C_3 R_i / R_{ifc}) + \chi_3 = 0$$

- At first sight, using it in order to simplify the QNSE fitting procedure would mean fitting independently $\boldsymbol{\phi_3(R_i)}$ and using a first order equation to obtain $\boldsymbol{\chi_3(R_i)}$.
- But, for high $\boldsymbol{R_i}$ values, the numerical QNSE procedure is less secure for the $\boldsymbol{\phi_3}$ values than for the $\boldsymbol{\chi_3}$ ones. So we shall solve a second order equation for $\boldsymbol{\phi_3}$ after the first fit of $\boldsymbol{\chi_3}$.
- For all this the available information (in the stable range) is:
  - the derivatives at neutrality: ***-2.48*** for $\boldsymbol{\chi_3}$ and $\boldsymbol{D=-2.3}$ for $\boldsymbol{\phi_3}$
  - the asymptotic $\boldsymbol{\chi_3}$ value at infinity: ***~ 0.232***
  - $\boldsymbol{C_3=1.39}$ and $\boldsymbol{R_{ifc}=C_3/(C_3-D)}$
- One then obtain a one-parameter Pade fitting procedure with:

$$\chi_3 = (1 + 0.75\, R_i\, (1 + X\, R_i)) / (1 + (0.75 + 2.48)\, R_i\, (1 + X\, R_i)) \qquad \boldsymbol{(X=13)}$$

[0.75 ≈ 2.48 x 0.232/(1. - 0.232)]

Verification of the function $\phi_3(R_i)$ for QNSE, after fitting $\chi_3(R_i)$ and solving the linking second-order equation



A remaining degree of freedom (‘*R*’)

- On top of c_K, c_ε (Reynolds case only) and C_3, R_{ifc} (general case), a dependency analysis shows that we still have a degree of freedom to consider in our new system of equations.
- Let us define, for the time being as a function of stability (and by ‘eliminating’ the ‘f’ function),

$$R(R_i) = R_{if}\,/(1 - f(R_i))$$

- R can be seen as a measure of the anisotropy. For an isotropic flow one shall have $R \equiv 1$ (CBR case for instance); lower and lower R values will indicate more and more anisotropy.
- The interesting feature here is that the asymptotic value of $\chi_3(R_i)$ for R_i going to minus infinity is $1/R$. So we may simply postulate that the QNSE unstable extension has a constant asymptote for the very unstable case and maximum continuity at neutrality.

Choice of the method (unstable range)

- Recall of the available information:
  - the derivatives at neutrality: *-2.48* for χ_3 and $D=-2.3$ for ϕ_3
  - $C_3=1.39$ and $R_{ifc}=C_3/(C_3-D)$
- One then obtain a homographic fitting procedure with:

$$\chi_3 = (1 - Y\, R_i)) / (1 - (Y - 2.48)\, R_i)$$

(Y=4.16 is obtained from the little information available on QNSE functions in the slightly unstable case)

- ϕ_3 is again obtained by solving:

$$C_3 R_i \phi_3^{\,2} - \phi_3(\chi_3 + C_3 R_i / R_{ifc}) + \chi_3 = 0$$

- A last verification can be made in the unstable range. Using

$$F_h(Ri) = \phi_3(Ri)\sqrt{\chi_3(Ri)(1 - R_{if})}$$

and comparing it with the Louis formulation allows computing the free convection constant C* of the latter (with 5.3 'observed' value) as:

$$C^*_h = \sqrt{\frac{(3R_{ifc})^3}{C_3}} \Big/ \kappa^2 = 6.37$$

What about the handling of anisotropy?

- After doing the analytical fit of $\chi_3(R_i)$ one may look at what are the implicit values of $\boldsymbol{R}$ associated with the resulting function (fitted exclusively from published values)
  - For $R_i \rightarrow -\infty$, we get $R=0.404$ *(through extrapolation)*
  - For $R_i = 0$, we get $R=0.359$
  - For $R_i \rightarrow +\infty$, we get $R=0.440$
- After the quality of the ‘double fit’, the relative homogeneity of these three values is an indirect proof of the ‘solidity’ of our 3 parameter / 3 equation system.
- The other constants corresponding to the QNSE fit are $C_3=1.39$ (given by the authors) and $R_{ifc}=0.377$ (vs. $\boldsymbol{0.4}$ suggested by the authors).

Remaining (and indeed pending) ‘moist’ issues

Classification

- The general description of the link between turbulence and diffusion may have given the impression that all ‘moist’ aspects are under control, once the SCC is supposed to be known.
- This not exaxctly true. Three issues (at least) still deserve special attention:
  - The influence of moisture on buoyancy via density effects;
  - How to do the SCC vs. 1-SCC averaging?
  - The way to compute the TOM’s terms for q_t in case of non-zero SCC.

Influence of moisture on buoyancy via density effects

- In the case when one assumes zero phase changes, the solution has been known for a long time. θ should be replaced by θ_{vl}, obtained via:

$$\theta_{vl} = \theta (1 + \frac{R_v}{R_d} q_v - q_t)$$

- This converts into a modification of the 'dry' ***N*** value.
- We have to derive an equivalent for the 'fully moist' ***N_m*** value (this time of course with phase changes on the menu).

How to do the SCC vs. 1-SCC averaging?

- The immediate temptation is to do it on $\boldsymbol{N^2}$ (or on $\boldsymbol{R_i}$, which is equivalent, since we shall consider the shear $\boldsymbol{S}$ as homogeneous across the whole mesh).
- However, owing to the many non-linearities present in our problem (one of which having been recalled in the previous viewgraph), we shall have to do a complete thermodynamic analysis before confirming this choice.
- The guideline shall here also be that the 'conversion' term can best be written as the Reynolds-type flux of density.

Computing the TOM's terms for q_t in case of non-zero phase changes

- The problematic is roughly the same as that of two viewgraphs before.
- In the calculation of the heat flux correction,

$$(\mathrm{T}_*^{-1})_\theta = \frac{C_1^g c_\varepsilon}{4c_\theta}\left[\frac{g}{\theta}\overline{\tau}\right]\frac{g^2}{e}$$

may be computed with the gradients corresponding to the effective buoyancy flux.
- But we also need an equivalent

$$(\mathrm{T}_*^{-1})_q = \frac{C_1^g c_\varepsilon}{4c_\theta}\left[g\overline{\tau}\left(\frac{R_v q_v}{R_d q_t}-1\right)\right]\frac{g^2}{e}$$

given here in the shape obtained without influence of the phase changes. That state of affairs shall have to be consistently adapted to the most general case.

Conclusion

Help from people interested in applied themodynamics would now be most welcome!