TOUCANS -special issues

Ivan Bašták Ďurán, Jean-François Geleyn and Filip Váňa

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Motivation

Relations between mixing lengths l_m (Prandtl type) and L_K, L_ϵ (TKE)

- enables usage of TKE mixing lengths (conversion from $L \equiv \sqrt{L_K . L_\epsilon}$ to l_m)
- required for derivation of stability functions $F_{m/h}$ in eTKE scheme

Motivation

Derivation of stability functions $F_{m/h}$:
condition of equivalence with full TKE scheme:

$$\tilde{E}\left(L_K\right) = \frac{E}{\epsilon\left(L_\epsilon\right)}\left[I\left(L_K\right) + II\left(L_K\right)\right]$$

definition of $F_{m/h}$:

$$F_{m/h} = \frac{\widetilde{K}_{m/h}}{l_m l_{m/h}\sqrt{\left[\left(\frac{\partial \overline{u}}{\partial z}\right)^2 + \left(\frac{\partial \overline{v}}{\partial z}\right)^2\right]}}$$

E - TKE (Turbulence Kinetic Energy), $\tilde{E}$ - TKE at stationary equilibrium
I - shear term, II - buoyancy term, ϵ - dissipation
$K_{m/h}$ - exchange coefficients

Derivation

Idea from RMC01 to compare two formalisms:

similarity laws:

$$\widetilde{E} = \alpha \kappa^2 z^2 \left[\left(\frac{\partial \bar{u}}{\partial z} \right)^2 + \left(\frac{\partial \bar{v}}{\partial z} \right)^2 \right] \phi_E \left(\frac{z}{L_{MO}} \right)$$

$$\overline{u'w'}^2 + \overline{v'w'}^2 = \kappa^4 z^4 \left[\left(\frac{\partial \bar{u}}{\partial z} \right)^2 + \left(\frac{\partial \bar{v}}{\partial z} \right)^2 \right]^2 \phi_m^{-4} \left(\frac{z}{L_{MO}} \right)$$

κ - von Karman constant, α -constant

$\phi_E \left(\frac{z}{L_{MO}} \right)$,$\phi_m \left(\frac{z}{L_{MO}} \right)$ - stability functions

L_{MO} - Monin Obukhov mixing length

Derivation

with TKE schemes:

$$\widetilde{E} = \frac{C_K}{C_\epsilon} L_K L_\epsilon \left[\left(\frac{\partial \overline{u}}{\partial z} \right)^2 + \left(\frac{\partial \overline{v}}{\partial z} \right)^2 \right] f(Ri)$$

$$\overline{u'w'}^2 + \overline{v'w'}^2 = {\chi_3}^2 \frac{C_K^3}{C_\epsilon} L_K^3 L_\epsilon \left[\left(\frac{\partial \overline{u}}{\partial z} \right)^2 + \left(\frac{\partial \overline{v}}{\partial z} \right)^2 \right]^2 f(Ri)$$

$$f(Ri) = \chi_3(Ri) - Ri C_3 \phi_3(Ri)$$

C_K, C_ϵ - closure constants

$\chi_3(Ri)$,$\phi_3(Ri)$ - stability functions, Ri - gradient Richardson number

Derivation

Result:

$$L_K C_K \chi_3 = \frac{\kappa z}{\sqrt{\alpha}} \frac{1}{\phi_m^2 \sqrt{\phi_E}}$$

$$\frac{L_\epsilon}{C_\epsilon} = \kappa z \alpha^{\frac{3}{2}} \frac{\phi_m^2 \phi_E^{\frac{3}{2}} \chi_3}{f(Ri)}$$

Conversion to *Ri*-form

Conditions:

$$L_K = L_\epsilon \text{ for } Ri = 0 \Rightarrow \frac{1}{\alpha^2} = C_K C_\epsilon \equiv \nu^4$$

$$\text{from CCH02} : \phi_m = \frac{1}{\chi_3 (Ri)^{\frac{1}{2}} f(Ri)^{\frac{1}{4}}}$$

Assumption:

$$\phi_E \phi_m^2 = 1$$

Prolongation:

$$\kappa z \longrightarrow l_m$$

Conversion to *Ri*-form

Result:

$$
\begin{aligned}
L_K C_K &= \nu l_m \frac{f(Ri)^{\frac{1}{4}}}{\chi_3^{\frac{1}{2}}} \\
\frac{L_\epsilon}{C_\epsilon} &= \frac{l_m}{\nu^3} \frac{\chi_3^{\frac{3}{2}}}{f(Ri)^{\frac{3}{4}}}
\end{aligned}
$$

pTKE scheme

pTKE scheme - TKE equation

$$\underbrace{\frac{\partial E}{\partial t} + ADV(E)}_{\text{advection}} = \underbrace{-\frac{\partial}{\partial z}\left(\overline{-K_E \frac{\partial E}{\partial z}}\right)}_{\text{diffusion with AF sch.}} + \underbrace{\frac{1}{\tau_\epsilon}\left(\tilde{E} - E\right)}_{\text{relaxation}}$$

$\tau_\epsilon = \frac{E}{\epsilon}$ - dissipation time scale

$K_E = -\frac{\overline{E'w'} + \frac{\overline{p'w'}}{\rho}}{\frac{\partial E}{\partial z}}$ - auto-diffusion vertical coefficient for the TKE

pTKE scheme

FULL LEVEL ———— E_l

HALF LEVEL ———— $\widetilde{E}, K_E, \tau_\epsilon, l_m, \beta_E$

FULL LEVEL ———— E_{l+1}

$\beta_E = sqrt\beta$ - decentering factor for TKE

pTKE scheme

pTKE scheme:

$$\widetilde{E} = \left(\frac{\widetilde{K}_*}{\nu l_m}\right)^2$$

$$\tau_\epsilon = \frac{\nu^3\sqrt{E}}{l_m} = \frac{l_m^2}{\nu^2 K^*}$$

$$K_E = \frac{l_m\sqrt{E}}{\nu} = \underbrace{\frac{K^*}{\nu^2}}_{\text{first time step}}$$

$$\nu = (C_K C_\epsilon)^{\frac{1}{4}}, K^* = \sqrt{K_m K_N}$$

K_N - K_m for neutral stratification ($Ri = 0$)

pTKE scheme

TKE solver:

$$\widetilde{K}_m, \widetilde{K}_N \rightarrow \widetilde{K}_* = \sqrt{\widetilde{K}_m, \widetilde{K}_N}$$

$$\widetilde{K}_*, E^0, l_m, \nu \rightarrow \widetilde{E}, K_E, \tau_\epsilon$$

$$\left.\widetilde{E}, K_E, \tau_\epsilon, E^0, \beta_E \rightarrow E^+ \right\} \text{ TKE equation - diffusion}$$

$$E^+, l_m, \nu \rightarrow K_* = \nu l_m \sqrt{E^+}$$

$$K_*, \widetilde{K}_m, \widetilde{K}_* \rightarrow K_m = K_* \widetilde{K}_m / \widetilde{K}_*$$

$$K_m, \widetilde{K}_m, \widetilde{K}_h \rightarrow K_h = K_m \widetilde{K}_h / \widetilde{K}_m$$

eTKE scheme

Differences between eTKE and pTKE:

Stability functions:

$$
\begin{aligned}
F_m(Ri) &= \chi_3(Ri)\sqrt{f(Ri)} \\
F_h(Ri) &= \frac{\phi_3(Ri)}{\chi_3(Ri)} F_m(Ri)
\end{aligned}
$$

Expression for K_m:

$$K_m = L_K C_K \chi_3 \sqrt{E}$$

Relation for ϕ_m (influences $L_{K/\epsilon}(l_m)$ conversion):

$$
\begin{aligned}
\mathrm{pTKE}: \phi_m &= \frac{1}{f(Ri)} \\
\mathrm{eTKE}: \phi_m &= \frac{1}{\chi_3(Ri)^{\frac{1}{2}} f(Ri)^{\frac{1}{4}}}
\end{aligned}
$$

eTKE scheme

Modification of $\widetilde{E}$, τ_ϵ and K_E in eTKE:
From TKE scheme:

$$\begin{aligned}\frac{1}{\tau_\epsilon} &= \frac{C_\epsilon}{L_\epsilon}\sqrt{E} \\ K_m &= L_K C_K \chi_3 \sqrt{E}\end{aligned}$$

with $L_{K/\epsilon}(l_m)$ conversion:

$$\begin{aligned}\frac{1}{\tau_\epsilon} &= \frac{\nu^3}{l_m}\frac{f(Ri)^{\frac{3}{4}}}{\chi_3(Ri)^{\frac{3}{2}}}\sqrt{E} \\ K_m &= \nu l_m f(Ri)^{\frac{1}{4}}\chi_3(Ri)^{\frac{1}{2}}\sqrt{E}\end{aligned}$$

eTKE scheme

using K_*:

$$K_* = \sqrt{K_m.K_N} = K_m.f(Ri)^{\frac{1}{4}}\chi_3(Ri)^{\frac{1}{2}}$$

we get:

$$\frac{1}{\tau_\epsilon} = \frac{\nu^3}{l_m}\frac{f(Ri)^{\frac{3}{4}}}{\chi_3(Ri)^{\frac{3}{2}}}\sqrt{E} \quad \text{different from pTKE}$$

$$K_* = \nu l_m \sqrt{E} \Rightarrow \widetilde{E} = \left(\frac{\widetilde{K}_*}{\nu l_m}\right)^2 \quad \text{identical with pTKE}$$

eTKE scheme

Relation for K_E modified according to change in τ_ϵ in order to keep ratio $\frac{\frac{1}{\tau_\epsilon}}{K_E}$ the same as in pTKE - ensures that matrix of the solver is diagonally dominant:

$$K_E = \frac{l_m\sqrt{E}}{\nu}\frac{f(Ri)^{\frac{3}{4}}}{\chi_3(Ri)^{\frac{3}{2}}}$$

eTKE scheme

TKE solver:

$\widetilde{K}_m, \widetilde{K}_N \rightarrow \widetilde{K}_* = \sqrt{\widetilde{K}_m, \widetilde{K}_N}$

↓

$\widetilde{K}_*, E^0, l_m, \nu, \chi_3, \phi_3 \rightarrow \widetilde{E}, K_E, \tau_\epsilon$

↓

$\widetilde{E}, K_E, \tau_\epsilon, E^0, \beta_E \rightarrow E^+$ } TKE equation - diffusion

↓

$E^+, l_m, \nu \rightarrow K_* = \nu l_m \sqrt{E^+}$

↓

$K_*, \widetilde{K}_m, \widetilde{K}_* \rightarrow K_m = K_* \widetilde{K}_m / \widetilde{K}_*$

↓

$K_m, \widetilde{K}_m, \widetilde{K}_h \rightarrow K_h = K_m \widetilde{K}_h / \widetilde{K}_m$

Prandtl number

Turbulent Prandtl number:

$$Prt = \frac{K_m}{K_h}$$

in turbulent schemes:

Louis scheme:

$$Prt = \frac{l_m}{l_h}\frac{F_m(Ri)}{F_h(Ri)}$$

$$\Rightarrow Prt(Ri=0) \equiv Prt_0 = \frac{l_m}{l_h}$$

TKE scheme:

$$Prt = \frac{1}{C_3}\frac{\chi_3(Ri)}{\phi_3(Ri)}$$

$$\Rightarrow Prt_0 = \frac{1}{C_3}$$

Vertical aspect of Prandtl number

Louis scheme :
vertical profile of *Prt* given by:

$$l_{m/h}^{AY} = \frac{\kappa z}{1 + \frac{\kappa z}{\lambda_{m/h}}\left[\frac{1+\exp\left(-a_{m/h}\sqrt{\frac{z}{H_{PBL}}}+b_{m/h}\right)}{\beta_{m/h}+\exp\left(-a_{m/h}\sqrt{\frac{z}{H_{PBL}}}+b_{m/h}\right)}\right]}$$

$$\text{at surface:}\quad l_m = l_h \Rightarrow Prt_0 = 1.0$$

H_{PBL} - PBL height, $a_{m/h}$, $b_{m/h}, \lambda_{m/h}$ - tuning constants

TKE scheme:
C_3 given for isotropic turbulence: free atmosphere

Match of *Prt*

eTKE uses combination of

Louis formalism and TKE formalism

Prt must match for every stratification:

$$\frac{F_m(Ri)}{F_h(Ri)} = \frac{\chi_3(Ri)}{\phi_3(Ri)} \qquad \text{always valid}$$

and in free atmosphere ($z \longrightarrow \infty$):

$$\frac{l_m}{l_h} = \frac{1}{C_3} \qquad \text{requires modification of } l_{m/h}$$

Match of Prt

Conditions:

$$\text{free atmosphere:} \quad Prt_0 = \frac{l_m}{l_h} = \frac{1}{C_3}$$

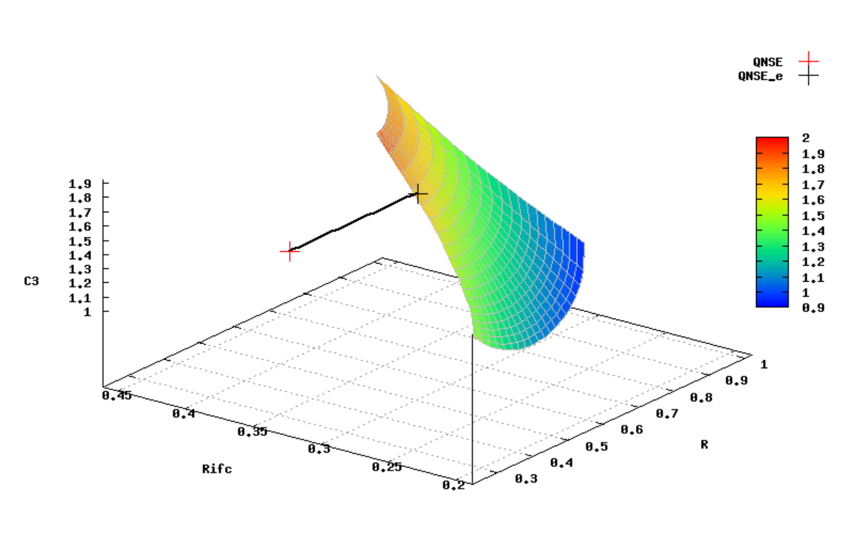
$$\text{surface:} \quad Prt_0 = \frac{l_m}{l_h} = 1$$

Solution with use of $l_{m/h}^{AY}$:

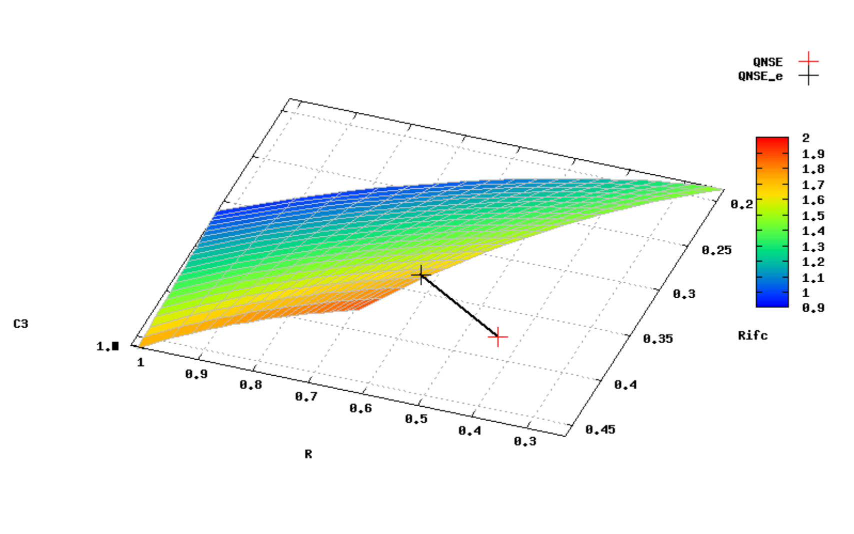
$$\frac{\lambda_m}{\lambda_h} = \frac{1}{C_3}$$

$$\frac{\beta_m}{\beta_h} = 1$$

3D space of degrees of freedom



3D space of degrees of freedom



3D space of degrees of freedom