(An)Isotropy of background error structure functions

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Introduction

In ALADIN 3d-var, background error structure functions are defined as horizontally homogeneous and isotropic in physical space. This means, that horizontal covariances are supposed to be invariant to the horizontal translation and direction, they depend thus only on the horizontal separation distance (Berre, 2000). However, due to the spectral representation of background errors, some anisotropy is implemented into the structure functions, that we illustrate on fig.1. The picture shows a simple graphical check of the isotropy, namely the resulting analysis increment field (i.e. observation minus background field) of a single observation experiment. The increment isolines are not fully spherical, especially at a larger distance from the observation's location, which obviously indicates anisotropy. Our short writing is dealing with one possible reason for the deviation of structure functions from the full isotropy and describes the experiment we have performed in order to get rid of the problem.

Investigating anisotropy

(a) Discretization of the isotropy assumption

First of all, one should think over, how the homogeneity and especially the isotropy assumption is applied to the spectral background error covariances in ALADIN. The covariance computations between the (m, n) spectral coefficients of the background errors are greatly simplified because of the homogeneity assumption. For example, for a given variable x the covariance is computed as:

$$cov(x_{m,n}, x_{m',n'}) = \delta_m^{m'} \delta_n^{n'} cov(x_{m,n}, x_{m,n}) = \sigma^2(x_{m,n})$$

where $\delta_m^{m'}$ is the Kronecker delta. It means, that only the covariances between the same (m, n) pairs are considered (Berre, 2000). The isotropy assumption is applied then by an averaging of the $\sigma^2(x_{m,n})$ spectral variances over the (m, n) pairs corresponding to the same $k^*_{m,n}$ total wavenumber, that is:

$$k^*_{m,n} = N\sqrt{\left(\frac{m}{M}\right)^2 + \left(\frac{n}{N}\right)^2} \qquad (1)$$

where M and N are the maximum wavenumbers in x and y directions. As a consequence, the covariances are not any more dependent on m and n separately, which ensures the invariance to the horizontal direction in physical space. The problem here is that the different (m, n) pairs never correspond exactly to the same real $k^*_{m,n}$ value, so within the averaging a discretization of the real $k^*_{m,n}$ total wavenumber vector and the real $\sigma^2(x_{m,n})$ variance spectrum is introduced as follows next. Let's denote by k^*_i the nearest integer to the real $k^*_{m,n}$ total wavenumber ($k^*_i = 0, 1, ..., N$). The average mentioned above determines the isotropic spectral variance spectrum in the function of the k_i^* integer total wavenumber vector:

$$\sigma^2(x_{k_i^*}) = \frac{1}{J} \sum_{m=0}^{M} \sum_{n=0}^{N} \sigma^2(x_{m,n}) \tag{2}$$

for which,

$$k_{m,n}^* = k_i^* \pm \epsilon. \tag{3}$$

In (2) J is the number of (m, n) pairs for which (3) is true and ϵ is a constant real number specifying the half interval for which the isotropic average above is done. As a consequence the B matrix finally consists of a set of isotropic spectral variances representing horizontal scales corresponding to the k_i^* integer total wavenumber vector.

(b) The normalization

On the other hand, in the J_b part of the 3d-var code, the change of variable

$$\chi = B^{-1/2}\delta x$$

(Fischer, 2002) is done for the usual spectral coefficients corresponding to (m, n) wave pairs and not to the integer total wavenumbers k_i^*. It is clear then, that an inevitable estimation of the real $\sigma^2(x_{m,n})$ variances should be done from the available $\sigma^2(x_{k_i^*})$ isotropic variances, in order to do the

$$\chi_{m,n} = \delta x_{m,n}/\hat{\sigma}(x_{m,n}) \tag{4}$$

normalization, where $\hat{\sigma}(x_{m,n})$ is the square-root of $\hat{\sigma}^2(x_{m,n})$, which is the estimation of the real $\sigma^2(x_{m,n})$. The presently used estimation (all cycles up to now), is very simple, namely

$$\hat{\sigma}^2(x_{m,n}) = \sigma^2(x_{k_i^*}) \tag{5}$$

where k_i^* is the nearest integer to the given $k_{m,n}^*$.

(c) Rectangular domains

An other thing to be considered is how the normalization above acts in case of a rectangular domain. If the domain is rectangular, that is $M \neq N$, then

$$k_{m,n}^* \neq k_{n,m}^*$$

with

$$m \neq n \qquad m = 0, ..., M \qquad n = 0, ..., N.$$

Substituting into (1), one can easily see that $k_{0,1}^* = 1$ and $k_{1,0}^* = N/M$ for example, which means that if we have only one wave in x or y directions, we have got total wavenumber that differ by the N/M ratio. On the other hand, using the estimation (5) in (4), we will normalize both $\delta x_{m,n}$ and $\delta x_{n,m}$ with the same $\sigma^2(x_{k_i^*})$ variance, because $k^*_{m,n}$ and $k^*_{n,m}$ are corresponding to the same k_i^* value. Our statement was that this is an approximation of the isotropy assumption, because we assume x to have the same variance on both of the horizontal scales corresponding to (m, n) and (n, m), however the exact isotropy would rather mean that only two wave pairs that correspond exactly to the same $k^*_{m,n}$ have exactly the same variance.

The experiment

Our proposal was to try to make the approximation of the isotropy assumption more realistic by applying the following estimation of $\sigma^2(x_{m,n})$ instead of (5):

$$\hat{\sigma}^2(x_{m,n}) = \sigma^2(x_{k_i^*})\frac{k^*_{i+1} - k^*_{m,n}}{k^*_{i+1} - k^*_i} + \sigma^2(x_{k^*_{i+1}})\frac{k^*_{m,n} - k^*_i}{k^*_{i+1} - k^*_i} \tag{6}$$

which is a linear interpolation between the two neighbouring $\sigma^2(x_{k_i^*})$ variances of the given $\sigma^2(x_{m,n})$. Note, that taking into account that $(k^*_{i+1} - k^*_i = 1)$, the interpolation above can be written in a simpler form:

$$\hat{\sigma}^2(x_{m,n}) = \sigma^2(x_{k_i^*})(k^*_{i+1} - k^*_{m,n}) + \sigma^2(x_{k^*_{i+1}})(k^*_{m,n} - k^*_i)$$

On fig.2 one can see the result of the same single observation experiment as shown on fig.1, but using the interpolation above instead of the original nearest integer estimation. It is noticeable, that the increment isolines are closer to be spherical at a larger distance from the observation, in comparison with fig.1, however the shape of the increments is not visually changed near the observation point. The same features were observed making comparison of vertical cross-sections.

In order to be able to make a more reliable comparison of the isotropic properties of the analysis increments, we have prepared a simple tool that measures quantitatively the degree of anisotropy. We have defined the measure of the anisotropy by the absolute value of the ratio

$$\frac{\delta x_{dx}}{\delta x_{dy}},$$

where δx_{dx} and δx_{dy} are the analysis increment values, at the same d horizontal distance from the observation point, in x and y directions (fig.3). The result of this diagnostic, comparing the (5) and (6) estimations is shown on fig.4. The two most obvious things that one can see on the plot, are that the degree of anisotropy is increasing with the distance for both experiments and that for a given distance the degree of anisotropy is smaller for the experiment using the linear interpolation of the $\sigma^2(x_{k_i^*})$ variances than for the one using the nearest integer estimation.

As a summary we can say, that the more realistic approximation of the isotropy assumption could decrease the unwanted anisotropy but in a quite moderate extent.

Figures

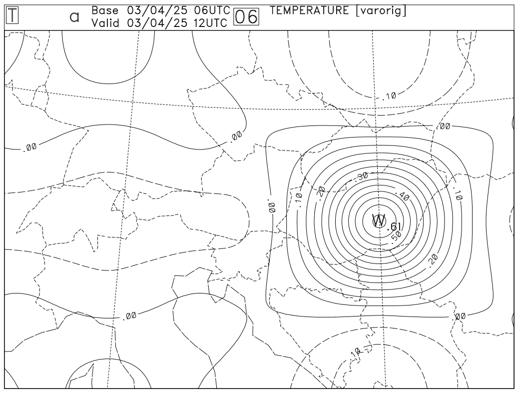

Figure 1: 3d-var analysis increment field on model level 16, due to a temperature single observation at 500 hPa. The domain is the formal Hungarian operational domain with 200 points in x and 144 points in y direction including the extension zone.

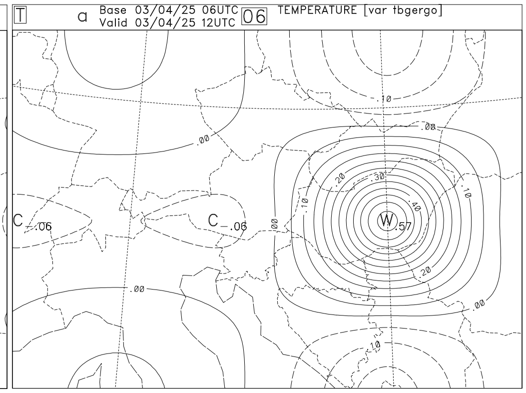

Figure 2: Similar analysis increment as shown on fig.1 but using the (6) interpolation.

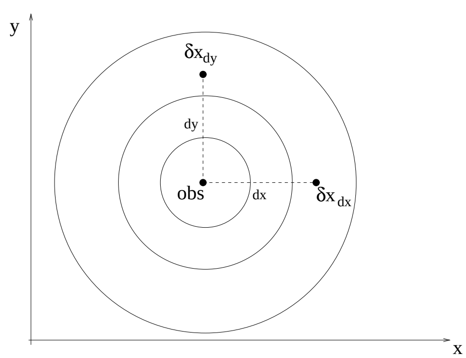

Figure 3: measuring anisotropy

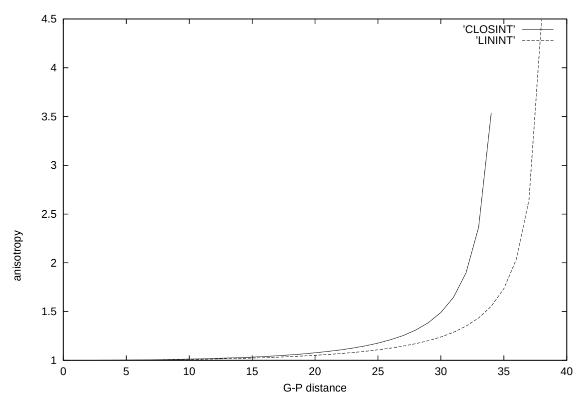

Figure 4: Degree of (an)isotropy. CLOSINT: nearest integer estimation (5), LININT: linear interpolation (6)

References

Berre, L., 2000 : Estimation of Synoptic and Mesoscale Forecast Error Covariances in a Limited-Area Model. *Mon. Wea. Rev.*, 128, 644-667

Fischer, C., 2002 : The variational computation inside ARPEGE/ALADIN cycle CY25T1. *ALADIN Technical documentation*, 59 pp.