A two-energies turbulence scheme

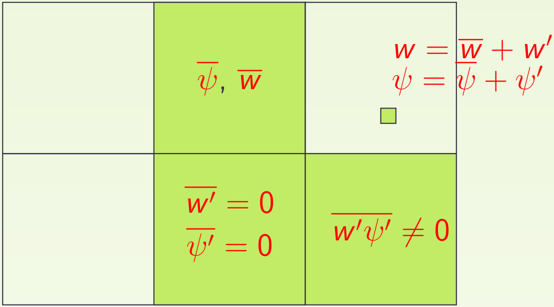
Ivan Bašták Ďurán[1], Jean-François Geleyn[2], Filip Váňa[3], Juerg Schmidli[1], Radmila Brožková[4]

Goethe University Frankfurt[1]
Prev. aff.: CNRM Météo-France, Toulouse, France [2]
ECMWF, Reading, United Kingdom[3]
CHMI, Praha, Czech Republic[4]

Parametrization in numerical model

- Representation of unresolved and too complex processes in the model:
    - Turbulence
    - Convection
    - Radiation
    - Cloud processes (microphysics)
    - Surface processes
    - ...
- Based on physical understanding and/or heuristic knowledge

Reynolds-averaging on grid



ψ - prognostic variable

Reynolds-averaged basic equations for conservative variables:

$$\frac{D\overline{u}}{\partial t} = S_u \boxed{-\frac{\partial\overline{u'w'}}{\partial z}}, \quad \frac{D\overline{v}}{\partial t} = S_v \boxed{-\frac{\partial\overline{v'w'}}{\partial z}},$$

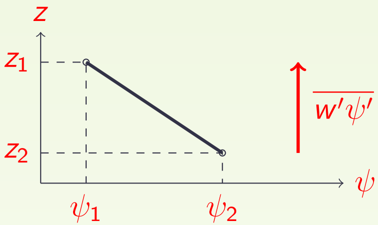
$$\frac{D\overline{\theta_l}}{\partial t} = S_{\theta_l} \boxed{-\frac{\partial\overline{\theta_l'w'}}{\partial z}}, \quad \frac{D\overline{q_t}}{\partial t} = S_{q_t} \boxed{-\frac{\partial\overline{q_t'w'}}{\partial z}}$$

u, v, w -wind components, θ_l - liquid water potential temperature, q_t total specific water content, S_ψ - remaining source terms

Local down-gradient turbulent diffusion

$$\overline{u'w'} = -K_M \frac{\partial u}{\partial z}, \quad \overline{v'w'} = -K_M \frac{\partial v}{\partial z},$$
$$\overline{\theta_l' w'} = -K_H \frac{\partial \theta_l}{\partial z}, \quad \overline{q_t' w'} = -K_H \frac{\partial q_t}{\partial z},$$

K_M and K_H - turbulent diffusion coefficients for momentum and heat/moisture



Turbulent diffusion - discretisation

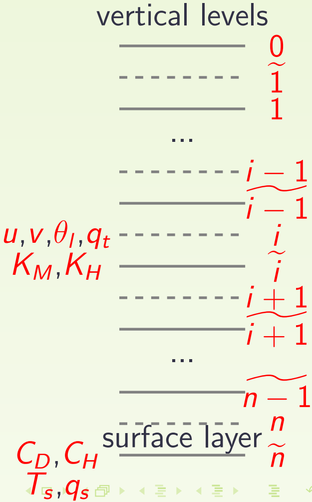
$$\frac{\partial \psi}{\partial t} = -\frac{\partial \overline{\psi' w'}}{\partial z}$$

$$\text{upper air}: \frac{\partial \psi}{\partial t} = \frac{\partial K_\psi \frac{\partial \psi}{\partial z}}{\partial z},$$

$$\text{surface}: \overline{w' \psi'} = -C_\psi \sqrt{u^2 + v^2}\,(\psi_n - \psi_s)$$

$$\psi_i^+ - \psi_i^- = \frac{\Delta t}{\delta z_i}[\widetilde{K_{\psi,i}}\,(\psi_i^+ - \psi_{i+1}^+) - \widetilde{K_{\psi,i-1}}\,(\psi_{i-1}^+ - \psi_i^+)]$$

leads to inversion of 3-diagonal matrix



RANS for Second Order Moments

$$
\begin{aligned}
\frac{\overline{s_L'^2}}{\tau_k \frac{C_4}{2\,C_3}} &= -\overline{u_i' s_L'}\frac{\partial \overline{s_L}}{\partial x_i}, \\
\frac{\overline{q_t'^2}}{\tau_k \frac{C_4}{2\,C_3}} &= -\overline{u_i' q_t'}\frac{\partial \overline{q_t}}{\partial x_i}, \\
\frac{2\overline{q_t' s_L'}}{\tau_k \frac{C_4}{2\,C_3}} &= -\frac{\partial s_L}{\partial z}\overline{w' q_t'} - \frac{\partial q_t}{\partial z}\overline{w' s_L'}, \\
A_{ij}\overline{u_j' s_L'} &= -\tau_k \overline{u_i' u_j'}\frac{\partial \overline{s_L}}{\partial x_j} \\
&\quad + \frac{2\,O_\lambda}{C_4}\tau_k\left(\beta_{s_L,i}\overline{s_L'^2} + \beta_{q_t,i}\overline{s_L' q_t'}\right), \\
A_{ij}\overline{u_j' q_t'} &= -\tau_k \overline{u_i' u_j'}\frac{\partial \overline{q_t}}{\partial x_j} \\
&\quad + \frac{2\,O_\lambda}{C_4}\tau_k\left(\beta_{s_L,i}\overline{s_L' q_t'} + \beta_{q_t,i}\overline{q_t'^2}\right), \\
A_{ij} &= \lambda_5\delta_{ij} + \lambda_6\tau_k S_{ij} + \lambda_7\tau_k R_{ij},
\end{aligned}
$$

$$
\begin{aligned}
S_{ij} &= \frac{1}{2}\left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i}\right), \\
R_{ij} &= \frac{1}{2}\left(\frac{\partial \bar{u}_i}{\partial x_j} - \frac{\partial \bar{u}_j}{\partial x_i}\right), \\
\beta_{s_L,i} &\equiv (0,0,E_{s_L}), \quad \beta_{q_t,i} \equiv (0,0,E_{q_t}), \\
b_{ij} &\equiv \overline{u_i' u_j'} - \frac{2}{3}e_k\delta_{ij}, \\
b_{ij} &= -\lambda_1 e_k\tau_k S_{ij} - \lambda_2\tau_k\Sigma_{ij} - \lambda_3\tau_k Z_{ij} \\
&\quad + \lambda_4 B_{ij}, \\
\Sigma_{ij} &= b_{ik}S_{kj} + S_{ik}b_{kj} - \frac{2}{3}\delta_{ij}b_{km}S_{mk}, \\
Z_{ij} &= R_{ik}b_{kj} - b_{ik}R_{kj}, \\
B_{ij} &= \beta_{s_L,i}\overline{u_j' s_L'} + \beta_{q_t,i}\overline{u_j' q_t'} \\
&\quad + \beta_{s_L,j}\overline{u_i' s_L'} + \beta_{q_t,j}\overline{u_i' q_t'} \\
&\quad - \frac{2}{3}\delta_{ij}\left(\beta_{s_L,k}\overline{u_k' s_L'} + \beta_{q_t,k}\overline{u_k' q_t'}\right),
\end{aligned}
$$

Turbulent diffusion coefficients in TKE scheme

$$K_M = \frac{\nu^4}{C_\epsilon}\chi_3(Ri_f)\sqrt{e_k}L, \quad K_H = C_3\frac{\nu^4}{C_\epsilon}\phi_3(Ri_f)\sqrt{e_k}L$$

- closure constants
- stability functions - influence of stratification
- stability parameter - influence of stratification
- length scale - scale of the problem
- TKE - measure of turb. intensity

$Ri_f \equiv (\frac{g}{\theta_v}\overline{\theta_v' w'})/(\overline{u' w'}\frac{\partial u}{\partial z} + \overline{v' w'}\frac{\partial v}{\partial z}) = Ri\frac{K_H}{K_M}$ - flux Richardson number,

$Ri \equiv (\frac{g}{\theta_v}\frac{\partial \theta_v}{\partial z})/(\sqrt{\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2})$ - gradient Richardson num.

Framework of stability functions (Bastak et al. 2014):

$$\chi_3(Ri) = \frac{1 - \frac{Ri_f}{R}}{1 - Ri_f} ,$$

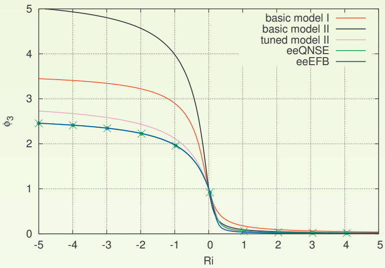
$$\phi_3(Ri) = \frac{1 - \frac{Ri_f}{P}}{1 - Ri_f} ,$$

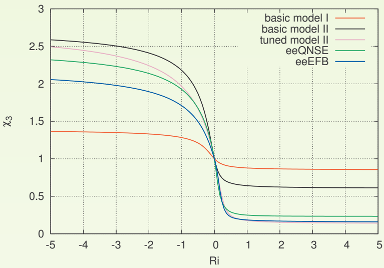
$$\frac{Ri}{Ri_f} = \frac{P(R - Ri_f)}{C_3\, R\,(P - Ri_f)}$$

$$0 < \lim_{Ri\to\infty} P = Ri_{fc} < 1,\ \ Ri_{fc} < \lim_{Ri\to\infty} R \equiv R_\infty \le 1.$$

(R, P - constants or functional dependencies, $Ri_{fc} = \lim_{Ri\to\infty} Ri_f$ - critical flux Richardson number)

Framework of stability functions (Bastak et al. 2014):





Length scales L

- Prandtl-type mixing length (Cedilnik, 2005):

$$L_1 = \frac{(C_K C_\epsilon)^{\frac{1}{4}}}{C_K} \frac{\kappa z}{1+\frac{\kappa z}{\lambda_m}\left[\frac{1+\exp\left(-a_m\sqrt{\frac{z}{H_{pbl}}}+b_m\right)}{\beta_{m/h}+\exp\left(-a_m\sqrt{\frac{z}{H_{pbl}}}+b_m\right)}\right]}$$

- Bougeault a Lacarrère (1989) :

$$L_2 = \left(\frac{L_{up}^{-\frac{4}{5}}+L_{down}^{-\frac{4}{5}}}{2}\right)^{-\frac{5}{4}}$$

- combination : $L = \min(L_1, L_2)$

a_m, b_m, λ_m - tuning constants, H_{pbl} - PBL height, $L_{up/down}(e_k)$ - upward/downward free path

Prognostic TKE equation

$$\frac{de_k}{dt} = \frac{\partial}{\partial z}\left(K_{e_k}\frac{\partial e_k}{\partial z}\right) + SHEA + BUOY - \epsilon_k,$$

$$e_k \equiv \frac{\overline{u'u'} + \overline{v'v'} + \overline{w'w'}}{2} \quad \text{-Turbulence Kinetic Energy},$$

$$SHEA \equiv -\overline{u'w'}\frac{\partial u}{\partial z} - \overline{v'w'}\frac{\partial v}{\partial z} \quad \text{-Shear term},$$

$$BUOY \equiv \frac{g}{\theta_v}\overline{\theta_v' w'} = E_{q_t}\overline{q_t' w'} + E_{\theta_l}\overline{\theta_l' w'} \quad \text{-Buoyancy term}$$

$$\epsilon_k \equiv \frac{2\,e_k}{\tau_k} \quad \text{-Dissipation term}$$

K_{e_k} - turb. exchange coefficients for e_k; τ_k and τ_s - are dissipation time scales; E_{q_t} and E_{θ_l} are cloud-dependent buoyancy flux coefficients.

Buoyancy flux coefficients (Marquet and Geleyn, 2013)

$$
\begin{aligned}
E_{s_{sL}} &= \frac{g\,M(C)}{\overline{c_p}\,\overline{T}}, \\
E_{q_t} &= g\,M(C)\Big\{\left(\frac{R_v - R_d}{R_d.\overline{q_d} + R_v.\overline{q_v}} - \frac{c_{pv} - c_{pd}}{\overline{c_p}}\right) \\
&\quad + C^* \left[\frac{L_{vs}(\overline{T})(R_d.\overline{q_d} + R_v.\overline{q_v})}{\overline{c_p}\,\overline{T} R_v} - 1\right] \cdot \\
&\quad \left[\frac{R_v - R_d}{R_d.\overline{q_d} + R_v.\overline{q_v}} + \frac{1}{(1 - q_t)\,(1 + D_C)}\right]\Big\}
\end{aligned}
$$

($M(C) = \frac{1+D_C}{1+D_C\left(1+C\left[\frac{L_{vs}(\overline{T})(R_d.\overline{q_d}+R_v.\overline{q_v})}{\overline{c_p}\,\overline{T}R_v}-1\right]\right)}, D_C = \frac{L_{vs}(\overline{T})\,\overline{r_s^{li}}}{R_d\,\overline{T}} = \frac{\overline{T}}{\overline{p}-e_{sat}(\overline{T})}\frac{\partial e_{sat}(\overline{T})}{\partial \overline{T}}$)

(C^* - turbulence cloud fraction influenced by skewness, C -cloud fraction in the

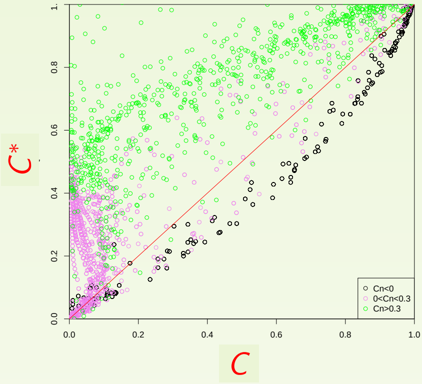
Turbulence cloud fraction influenced by skewness

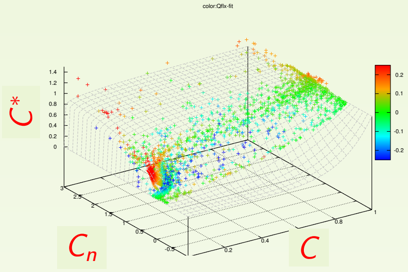
$$C^* = C^{F(C_n)}, \quad F(C_n) = 0.5\left[\sqrt{(6.25\,C_n)^2 + 4} - 6.25\,C_n\right]$$

$$C_n = \frac{-\frac{\overline{w' s'_{sL}}}{\widehat{c_p\,T}} - \left(\frac{R_v - R_d}{R_d.\widehat{q_d} + R_v.\widehat{q_v}} - \frac{c_{pv} - c_{pd}}{\widehat{c_p}}\right)\overline{w' q'_t}}{\left[\frac{L_{vs}(\widehat{T})(R_d.\widehat{q_d} + R_v.\widehat{q_v})}{\widehat{c_p}\,T R_v} - 1\right]\left[\frac{R_v - R_d}{R_d.\widehat{q_d} + R_v.\widehat{q_v}} + \frac{1}{(1-\widehat{q_t})(1+D_C)}\right]\overline{w' q'_t}}$$

(C_n - skewness parameter)

Fitting of $C^*(C, C_n)$ on LES data (courtesy of D. Lewellen)





Limitations of TKE scheme

- down-gradient parameterization of turbulent fluxes
- stability parameter approximated via local gradients:
  $$Ri_f \equiv -\frac{BOUY}{SHEAR} \approx Ri\frac{K_H}{K_M}$$
  $$Ri \equiv \left(\frac{g}{\theta_v}\frac{\partial\theta_v}{\partial z}\right)/\left(\sqrt{\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2}\right)$$
  $\Rightarrow$ may require shallow conv. parametr. to mix across and in to the locally stable layers
- feedback between mixing and stability parameter may cause oscillations

Two-energies scheme

- based on Zilitinkevich et al. (2013)
  - dry case and stable stratification
- addition of second prognostic energy to TKE scheme
- stability parameter depends on prognostic turbulence energies
  $\Rightarrow$ (attains) prognostic and non-local properties
- (still) down-gradient parameterization of turbulent fluxes

Two prognostic turbulence energies (Bastak et al., submitted to JAS)

$$
\begin{aligned}
\frac{d\, e_k}{dt} &= \frac{\partial}{\partial z}\left(K_{e_k}\frac{\partial e_k}{\partial z}\right) + \boxed{SHEA + BUOY - \frac{2\, e_k}{\tau_k}}, \\
\frac{d\, e_s}{dt} &= \frac{\partial}{\partial z}\left(K_{e_s}\frac{\partial e_s}{\partial z}\right) + \boxed{SHEA - \frac{2\, e_s}{\tau_s}}, \\
e_s &\equiv e_k + \frac{E_{q_t}\overline{q_t'^2}}{2\frac{\partial q_t}{\partial z}} + \frac{E_{\theta_l}\overline{\theta_l'^2}}{2\frac{\partial \theta_l}{\partial z}}, \\
Ri_f^{TE} &\equiv \boxed{-\frac{BOUY}{SHEA} = \frac{e_s - e_k}{e_s + e_k\left(\frac{C_4}{2\, C_3} - 1\right)}}
\end{aligned}
$$

K_{e_s} - turb. exchange coefficients for e_s; τ_s - dissipation time scale

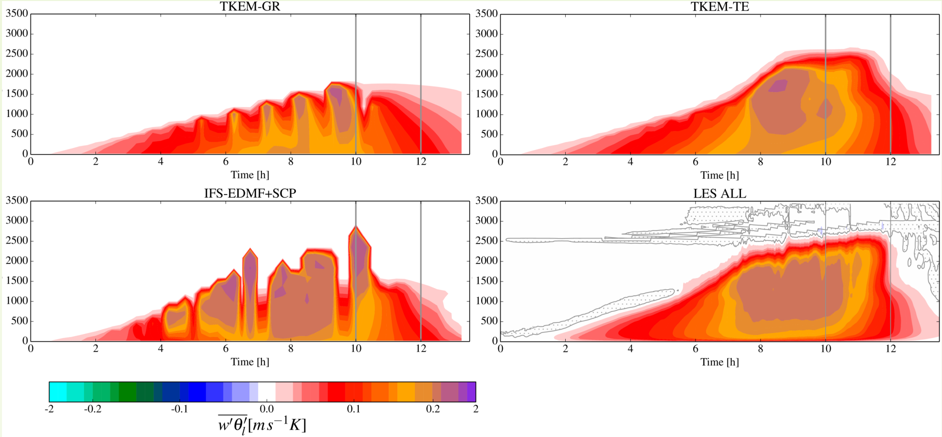
Single Column Model (SCM)

- OpenIFS model
- idealized environment with only one model column
- external forcings (tendencies, fluxes, boundary conditions) can be prescribed
- study of individual parametrizations / specific properties
- Large Eddy Simulation (LES) - microHH - used as reference

SCM experiments

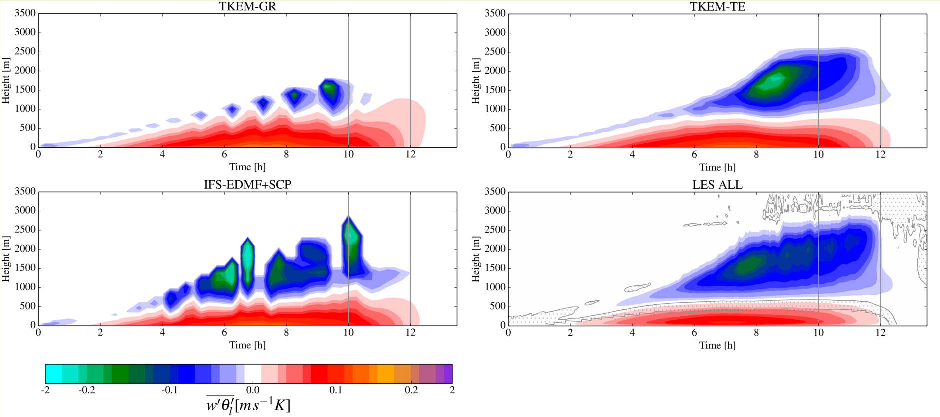
- ARM: Continental shallow cumulus
  - diurnal cycle
- BOMEX: Non-precipitating trade cumulus
  - quasi steady state
- GABLS(1): Stable stratification
- configuration: time step=900 seconds, 91 atmospheric vertical levels, 17 levels in the lowest 2 km

ARM case - $\overline{w'q'_t}$



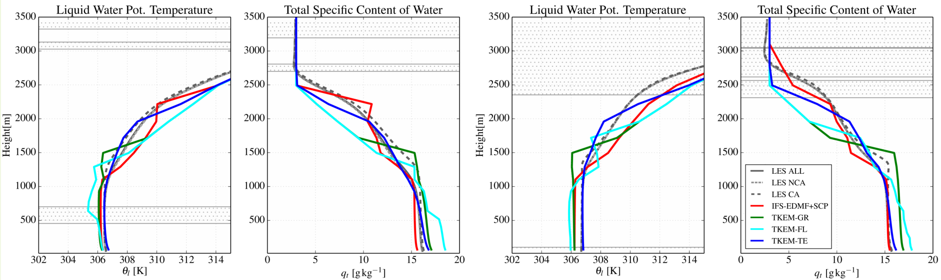
TKEM-GR - TKE scheme with Ri_f computed from gradients, TKEM-TE - 2 energy scheme, IFS-EDMF+SCP - IFS turbulence scheme with EDMF and parametrization of shallow convection, LES-ALL - microHH model. Dotted areas indicate counter-gradient regions.

ARM case - $\overline{w'\theta'_l}$



ARM case - vertical profiles of θ_l and q_t

after 10 hours of integration | after 12 hours of integration

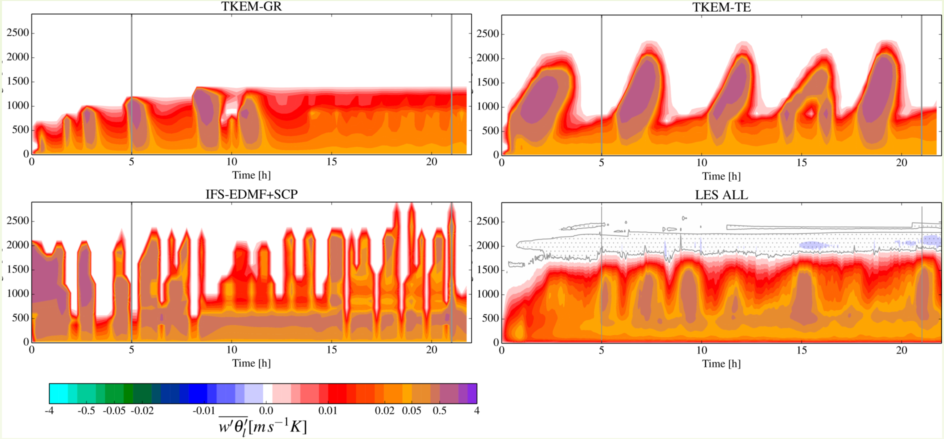


TKEM-FL - TKE scheme with Ri_f computed from fluxes

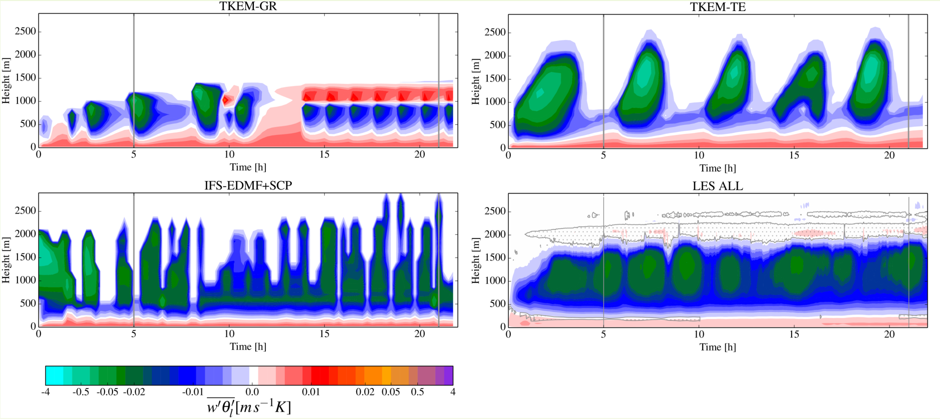
Counter-gradient fluxes

- two-energy scheme is unable to parametrize counter-gradient fluxes
- counter-gradient regions in ARM case:
  - above the PBL
    - small magnitude of fluxes
  - in PBL layers with near constant θ_l
    - two-energy scheme has near-constant θ_l profile, but enforces opposite signs of the vertical gradient and the turbulent flux
- down-gradient approach is sufficiently accurate for ARM case

BOMEX case - $\overline{w'q'_t}$



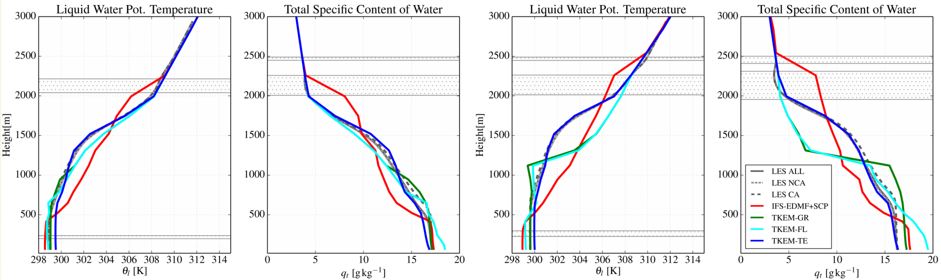
BOMEX case - $\overline{w'\theta_l'}$



BOMEX case - vertical profiles of θ_l and q_t

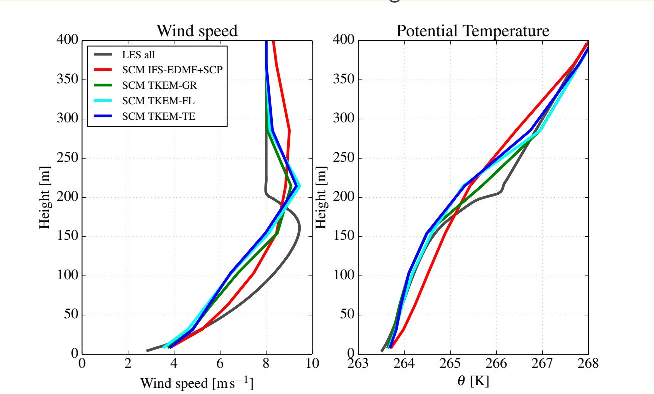
after 5 hours of integration

after 21 hours of integration



GABLS(1) case - vertical profiles of θ_l and q_t

after 7 hours and 30 minutes of integration



Long-term three-dimensional global simulations

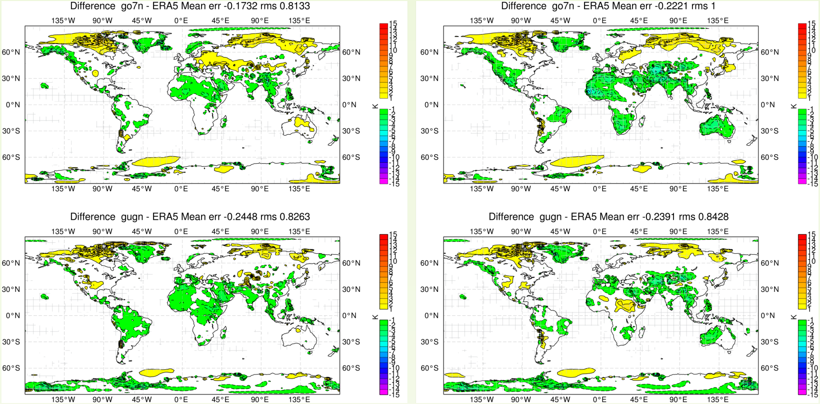
- 4-member ensemble simulation of the uncoupled global IFS model for the 2000-2001
- two-energy scheme replaces turbulence scheme and shallow convection parametrization is turned off
- comparison with full IFS physics (CY43R3)
- reanalyses and observations used as reference

2m-temperature and 2m-dew point

2m-temperature | 2m-dew point

IFS physics

2TE scheme in IFS



ERA5 data as reference | ERA5 data as reference

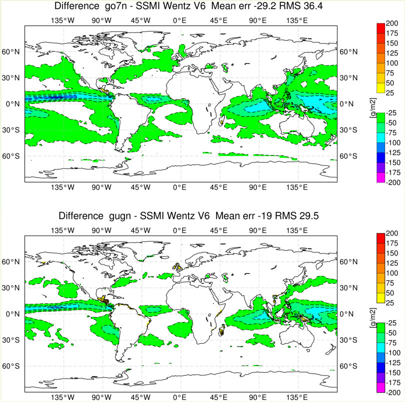
go7n - IFS physics, gugn - two-energies scheme in IFS physics

Liquid water path and total cloud cover

IFS physics

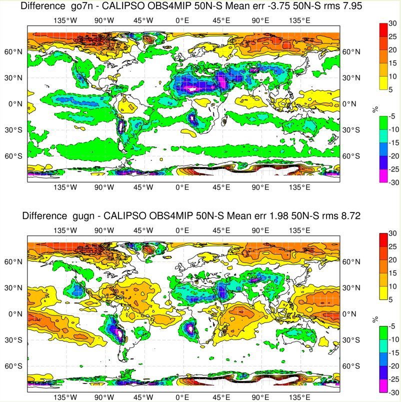
2TE scheme in IFS

Liquid water path



SSM/I V6 dataset as reference

Total cloud cover



CALIPSO data as reference

Summary (1)

- two-energy scheme is extension of TKE scheme with additional prognostic turbulence energy
- stability parameter depends on prognostic turbulence energies
  $\Rightarrow$ (attains) prognostic and non-local properties
- turbulent fluxes are parametrized in down-gradient way

Summary (2)

- parametrization of both turbulence and clouds in the PBL - no shallow convection parametrization is used
- the turbulent fluxes of two-energy scheme are more continuous in time and space than in TKE scheme and EDMF
- two-energy scheme enables transport across locally stable layers and deeper mixing
- thermals in the two-energies scheme are too intense and less frequent than in the LES for BOMEX case

Summary (3)

- 2TE scheme behaves reasonable well in a full atmospheric model in the long-term 3D simulation
- 2m-temperature and 2m-dew point scores are comparable between IFS and 2TE, 2TE has better Arctic region
- 2TE scheme in IFS overestimates cloud cover on low levels
- further calibration possible/required especially for cloud cover

What next?

- more testing
  - Stratocumulus cases, transition cases,..
- extension with the Assumed PDF method (Golaz et al. 2002)
- revision of length scale formulation
- introduction of scale-awareness:
  - dynamics of the model
  - stochastic parametrizations (Bengtsson et al., 2013; Sakradzija et al., 2016)
  - 3D turbulence