pTKE

Pseudo-prognostic TKE scheme

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Vertical diffusion scheme in ALADIN

- Louis type scheme (K-closure)
  - explicitly resolves the boundary layer
  - in analogy with molecular diffusion assumes the fluxes proportional to gradients:

$$\overline{w'X'} = -K_{M/H}\frac{\partial X}{\partial z}, \quad X = u, v, s, q$$

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SC: replace Ri by Ri^*:

$$Ri^* = \frac{g}{C_p T} \frac{\frac{\partial s}{\partial z} + \boxed{L \min\left(0, \frac{\partial (q - q_s)}{\partial z}\right)}}{\left| \frac{\partial \vec{u}}{\partial z} \right|^2}$$

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$$\left.\frac{\partial X}{\partial t}\right|_{\text{v\_diff}} = \frac{\partial}{\partial z}\left(K_{M/H}\frac{\partial X}{\partial z}\right)$$

discretized into time-shifted formulation:

$$\frac{X^{+} - X^{0}}{\Delta t} = \left[(1-\beta)(K_{M/H}X_z^0) + \beta(K_{M/H}X_z^{+})\right]_z$$

To avoid fibrillations $\beta \geq 1$.

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- replace full TKE equation by a pseudo one converging toward the Louis scheme
- modify $K_{M/H}$ according the TKE to obtain space-consistent variation around the static solution

Full TKE equation

Prognostic equation for TKE ($E = \frac{1}{2}(\overline{u'^2} + \overline{v'^2} + \overline{w'^2})$:

$$\frac{\partial E}{\partial t} + \underbrace{u\frac{\partial E}{\partial x} + v\frac{\partial E}{\partial y} + w\frac{\partial E}{\partial z}}_{\text{advection}} = \underbrace{-\overline{u'w'}\frac{\partial u}{\partial z} - \overline{v'w'}\frac{\partial v}{\partial z}}_{\text{I}}$$

$$\underbrace{-\frac{g}{\varrho_0}\overline{w'\varrho'}}_{\text{II}} \quad \underbrace{-\frac{\partial}{\partial z}\left(\overline{E'w'} + \frac{\overline{p'w'}}{\varrho}\right)}_{\text{III}} \quad \underbrace{-\varepsilon}_{\text{IV}}$$

I = mechanical production/destruction of E by wind shear

II = production/consumption of E by buoyancy

III = transport or diffusion terms

IV = dissipation

pTKE equation

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What about K_E, $\tau_\varepsilon = E/\varepsilon$ and $\tilde{E}$?

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- $K_m = K_*(\tilde{\tilde{K_m}}/\tilde{\tilde{K_*}})$, $K_h = K_*(\tilde{\tilde{K_h}}/\tilde{\tilde{K_*}})$,

Stability choices

Oscillatory tests $-X(t-dt)+2X(t)-X(t+dt)$

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- SLHD or QM for sL advection - SLHD together with QM slightly less stable.

pTKE - Algorithmics

- two tuning parameters: $\tilde{K}_* = R_l \tilde{K}_n^{1-\gamma} \tilde{K}_m^{\gamma}$ (`GAMTKE=0.5`), ν (`NUPTKE=0.52`)

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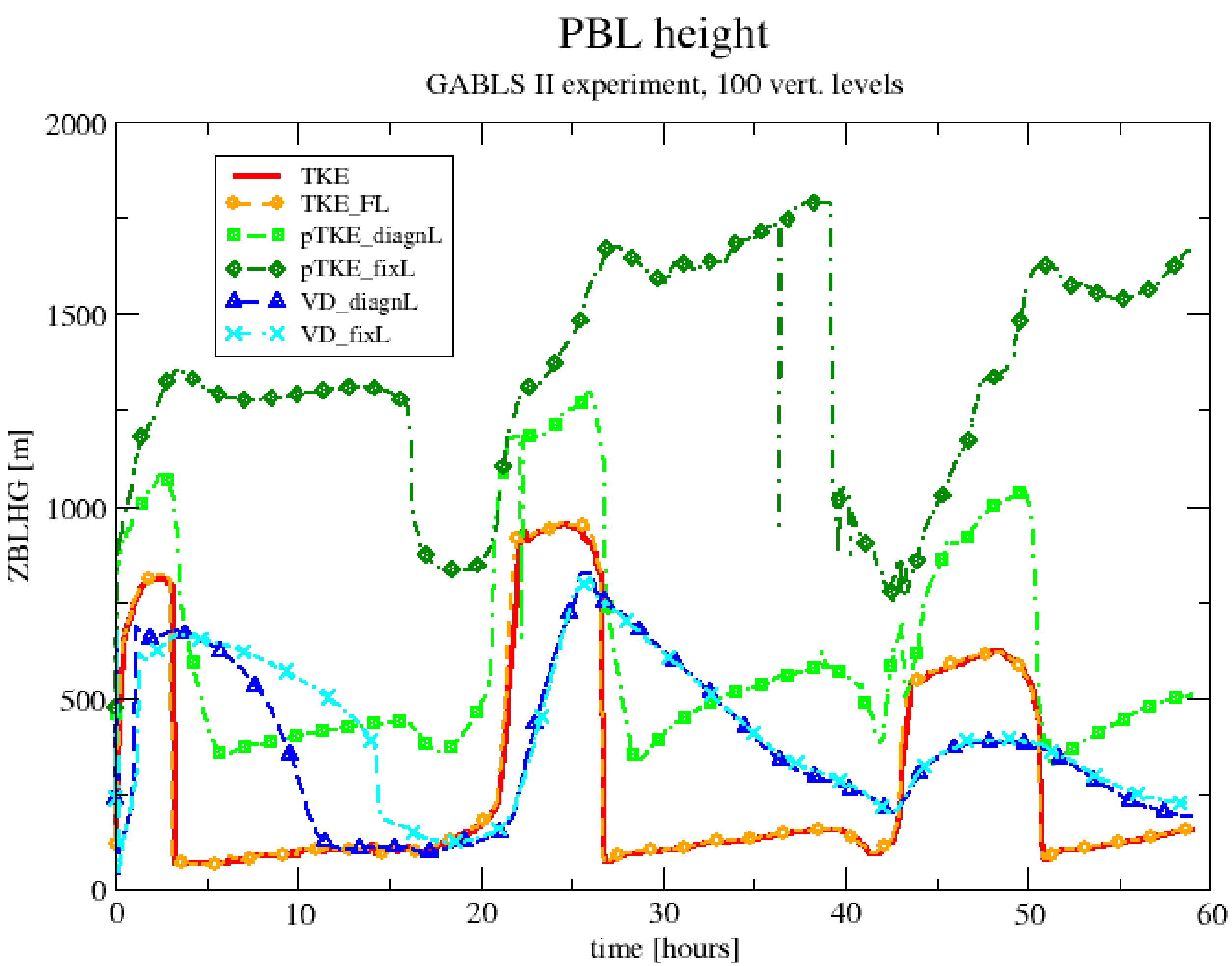
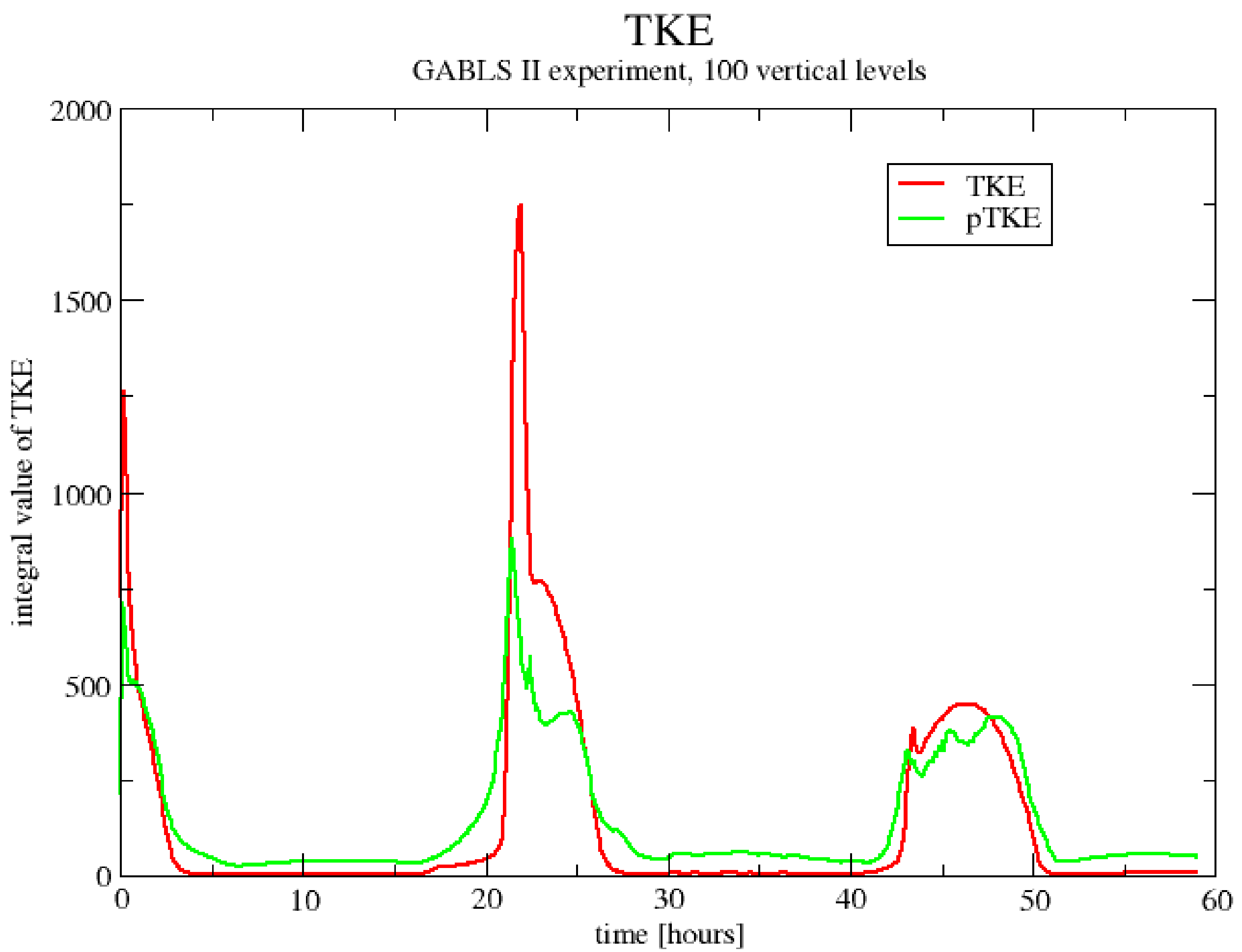
- diffusion should dominate the relaxation $\Rightarrow$ positive off-diagonal terms are not allowed (through R_l)
- anti-fibrillation scheme works for TKE diffusion

Validation



PBL height
GABLS II experiment, 100 vert. levels
TKE
TKE_FL
pTKE_diagnL
pTKE_fixL
VD_diagnL
VD_fixL
ZBLHG [m]
time [hours]
0
500
1000
1500
2000
0
10
20
30
40
50
60

Validation


TKE
GABLS II experiment, 100 vertical levels
integral value of TKE
time [hours]
TKE
pTKE
0
500
1000
1500
2000
0
10
20
30
40
50
60

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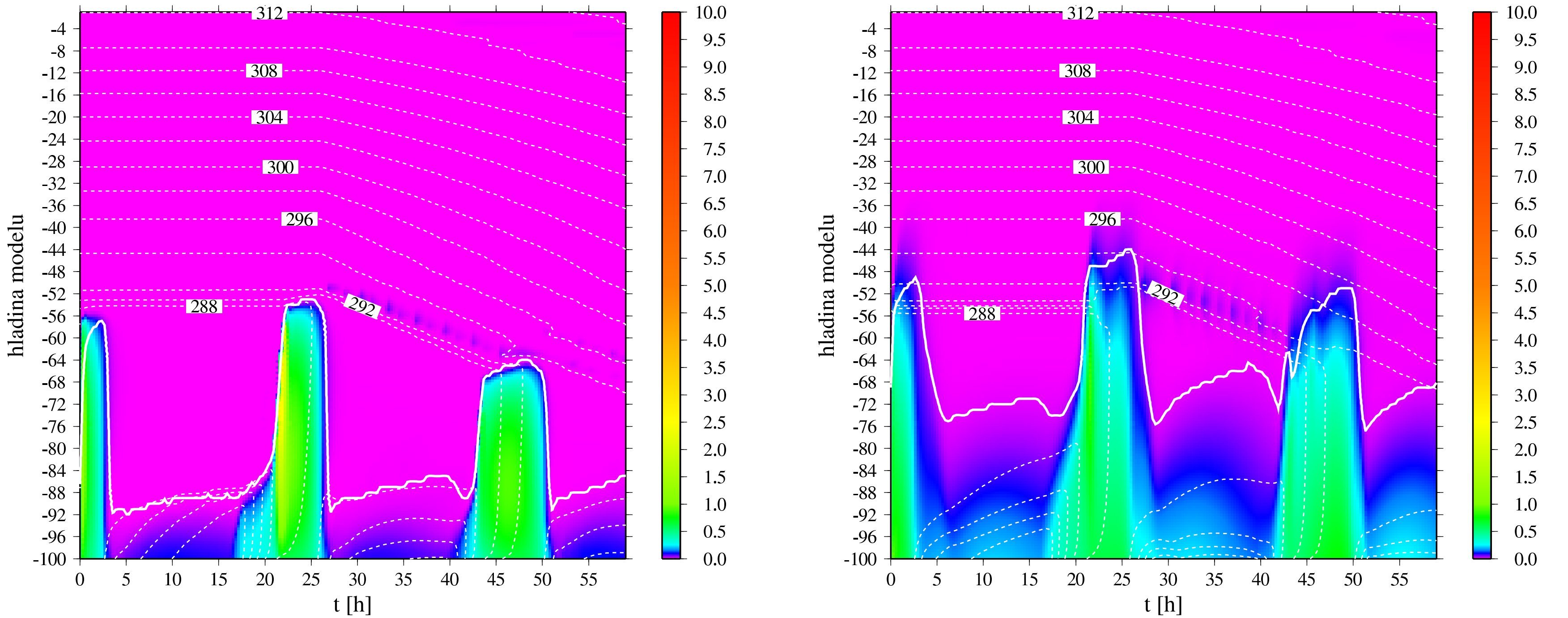
1D model simulation with GABLS II experiment



Full TKE scheme vs. pTKE (GC mxl. length)

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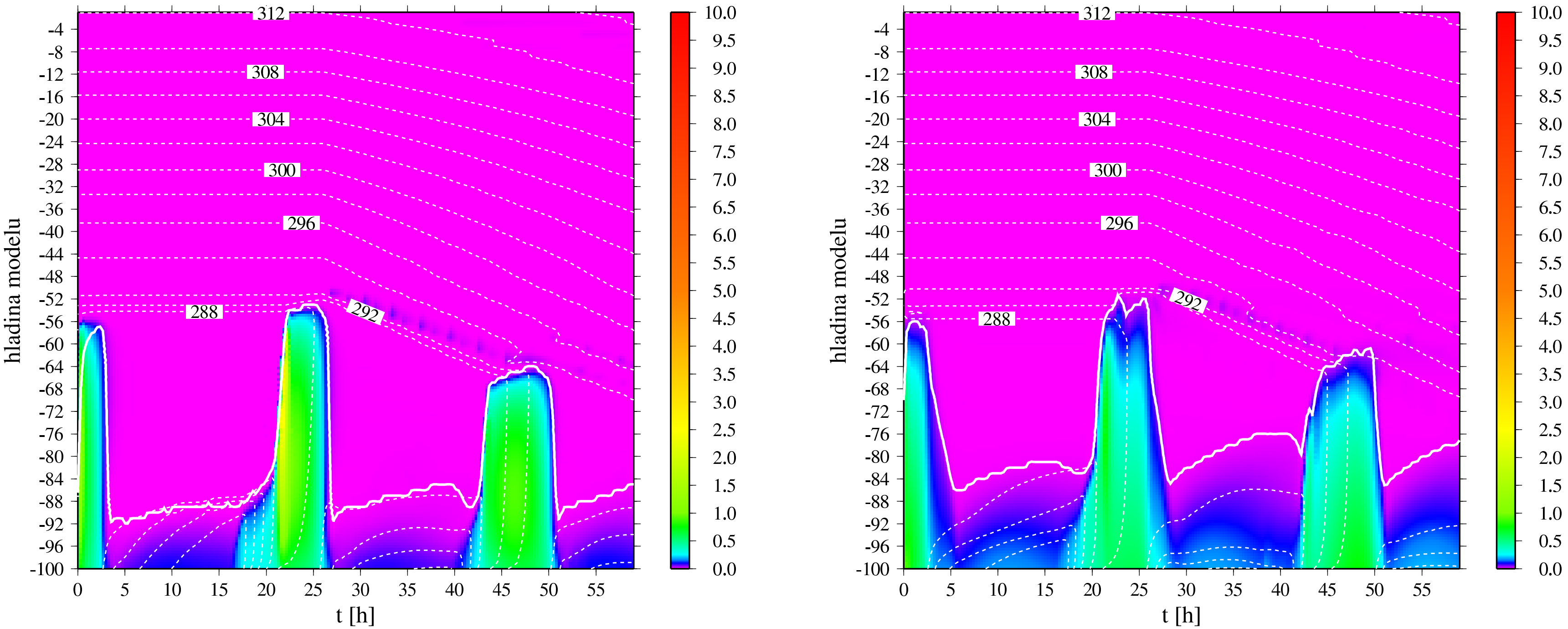
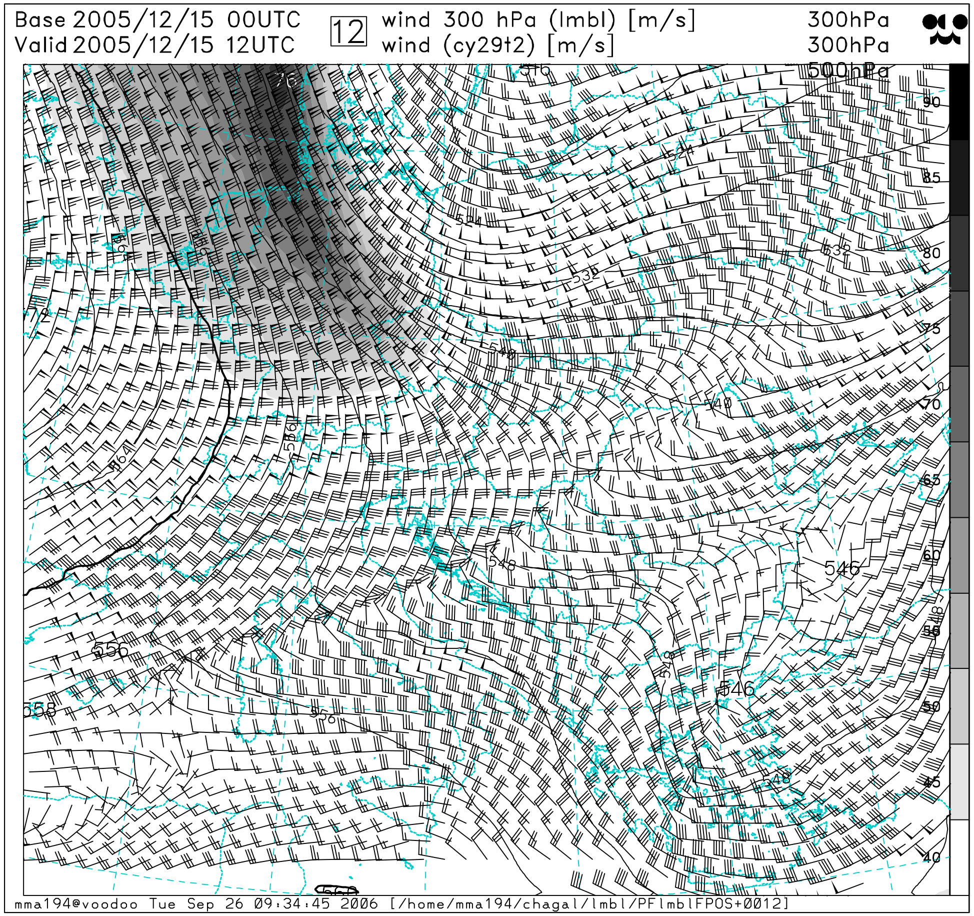


Full TKE scheme vs. pTKE (with mod GC mxl. length)

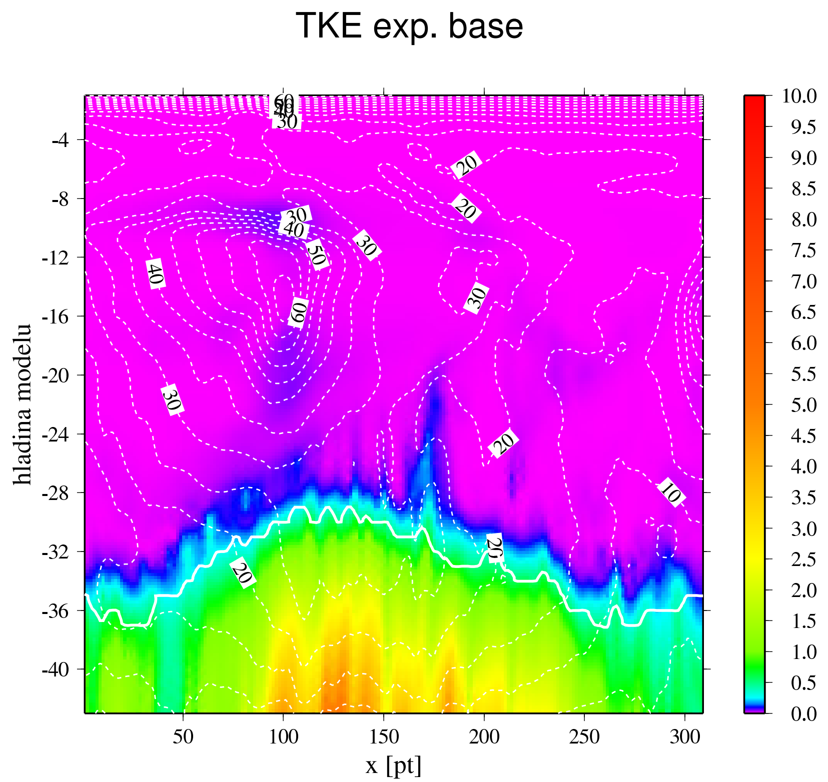
Full model results

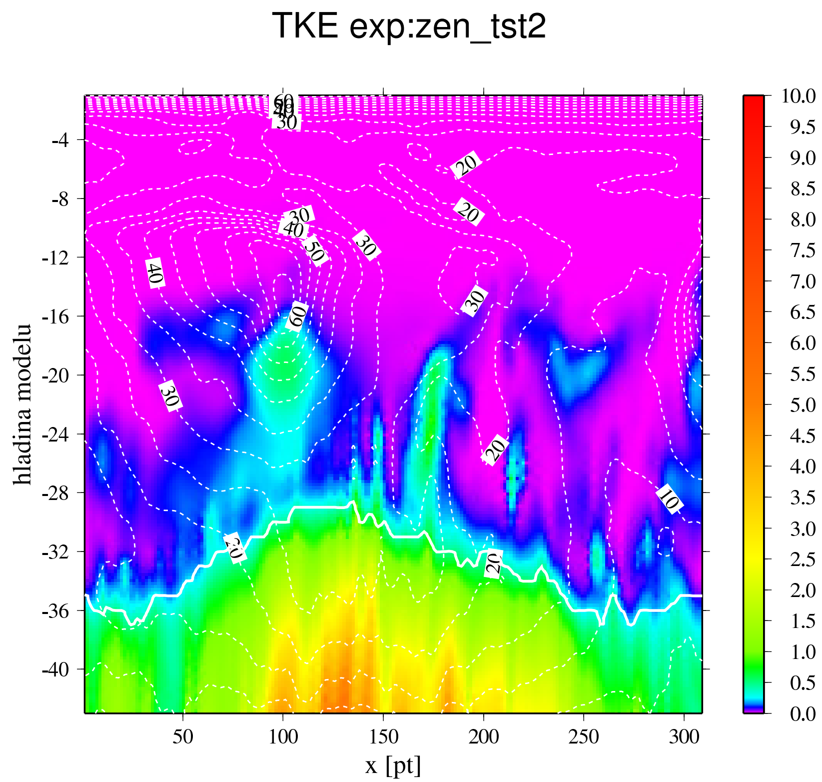


500 hPa geopotential and 300 hPa wind

Full model results

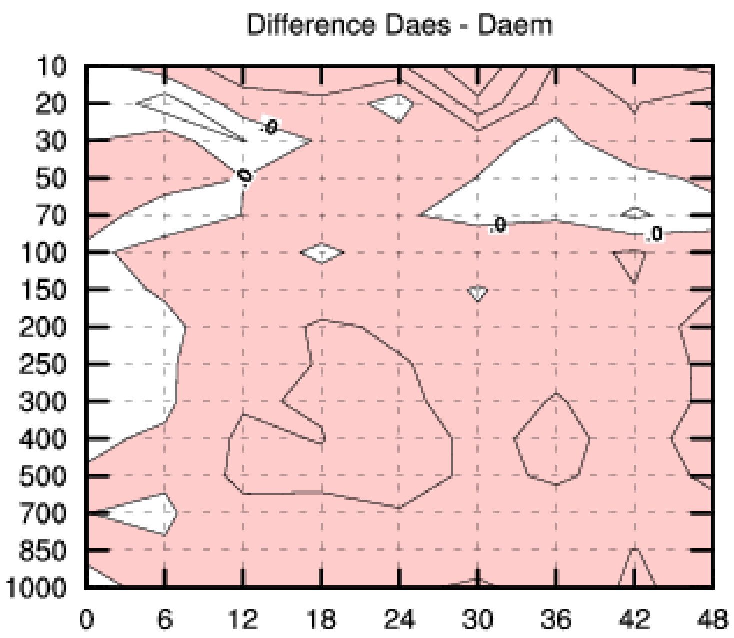
Strong jet over northern Germany

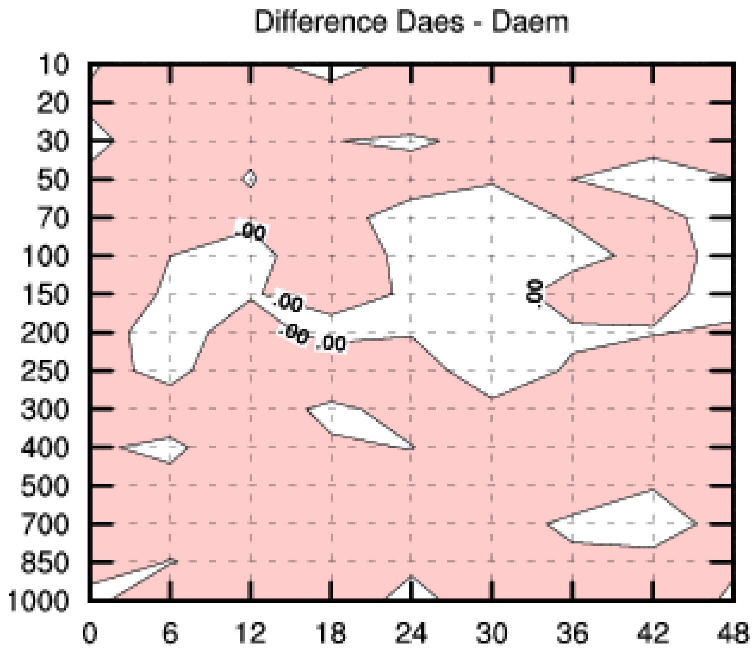




TKE reference vs. modified GC mxl. length

Parallel test





RMSE difference evolution of geopotential and temperature

Near future plans

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- shallow convection + $q_{l/i}$ problem

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- stable vertical staggering while full level TKE values
- special effort to stabilize the solver $\longrightarrow$ no negative TKE values (from physics)
- offers elegant way to sequential convergence towards a full TKE scheme (mixing length, $\tilde{E}$)
- suitable for TL/AD code