

*Regional Cooperation for
Limited Area Modeling in Central Europe*



Dynamics in LACE

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thanks to Alexandra Craciun, Jozef Vivoda and other colleagues



ARSO METEO
Slovenia



Outline

- 1. Dynamic definition of the time scheme**
- 2. Finite elements used in vertical in NH**
- 3. The trajectory search in the SL advection scheme**

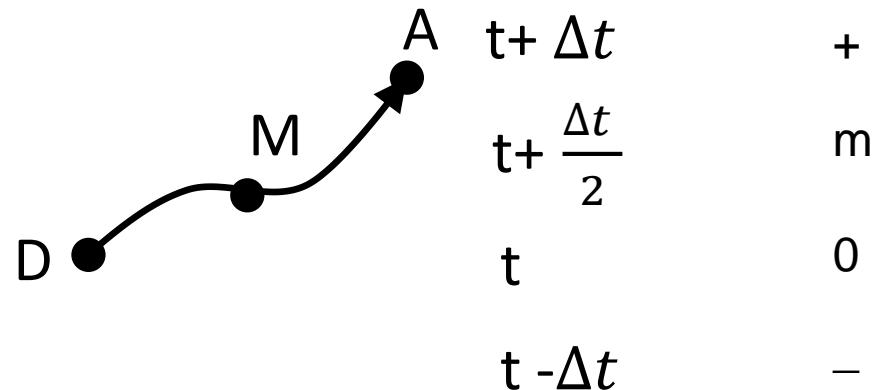
Dynamic definition of the time scheme

work of Jozef Vivoda

Advection equation $\frac{df(t, x)}{dt} = N(t, x)$

Time centered explicit
semi-Lagrangian approach

$$\frac{f_A^+ - f_D^0}{\Delta t} = N_M^m$$

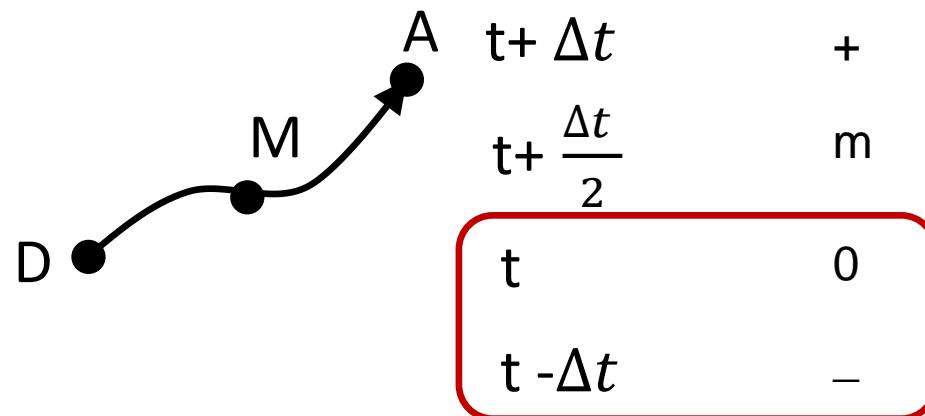


Dynamic definition of the time scheme

Advection equation $\frac{df(t, x)}{dt} = N(t, x)$

Time centered explicit
semi-Lagrangian approach

$$\frac{f_A^+ - f_D^0}{\Delta t} = N_M^m$$



available information

Dynamic definition of the time scheme

First order treatment **NESC**: $N_M^m = \frac{1}{2}(N_A^0 + N_D^0) + \mathcal{O}(\Delta t)$

The time centered scheme **using available information**

$$N_M^m = a_1 N_A^0 + a_2 N_A^- + a_3 N_D^0 + a_4 N_D^-$$

For any α :

$$N_M^m = (-\alpha + \frac{3}{4})N_A^0 + (\alpha - \frac{1}{4})N_A^- + (\alpha + \frac{3}{4})N_D^0 + (-\alpha - \frac{1}{4})N_D^- + \mathcal{O}(\Delta t^2)$$

For $\alpha = \frac{1}{4}$ we get **SETTLS** (Hortal, 2002):

$$N_M^m = \frac{1}{2}(N_A^0 + N_D^0) + \frac{1}{2}(N_D^0 - N_D^-) + \mathcal{O}(\Delta t^2)$$

Dynamic definition of the time scheme

NESC
$$N_M^m = \frac{1}{2}(N_A^0 + N_D^0)$$

SETTLS
$$N_M^m = \frac{1}{2}(N_A^0 + N_D^0) + \frac{1}{2}(N_D^0 - N_D^-)$$

COMBINED SCHEME
$$N_M^m = \frac{1}{2}(N_A^0 + N_D^0) + \frac{\beta}{2}(N_D^0 - N_D^-)$$

We may change β arbitrarily from 0 to 1.

We consider solution in the shape $N(t, x) = (\lambda + i\omega)f(t, x)$
and analyse single Fourier component advected with a constant
wind U

$$f(n\Delta t, j\Delta x) = A^n e^{ijUk\Delta t}$$

Stability reached when $|A| \leq 1$.

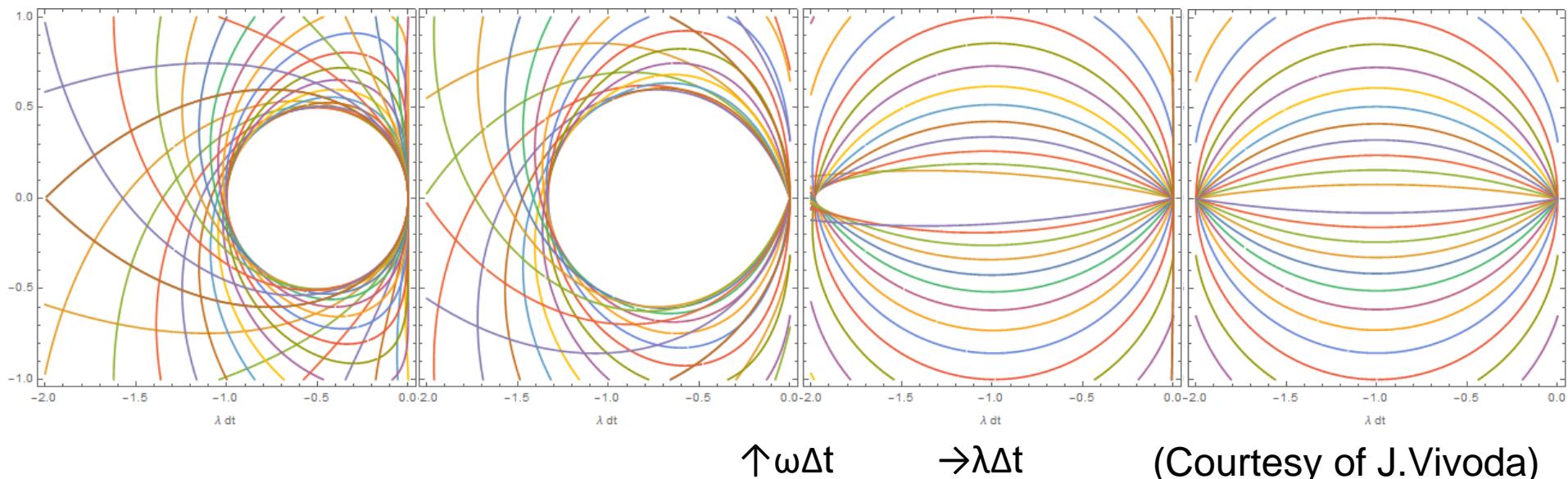
Stability analysis

$\beta = 1$

$\beta = 1/2$

$\beta = 1/100$

$\beta = 0$



SETTLS: λ, ω constraints

NESC: $\omega=0$

Dynamic definition of the time scheme

Introduce implicitness:

$$\frac{df(t, x)}{dt} = N(t, x) + L(t, x)$$

$$\frac{f_A^+ - f_D^0}{\Delta t} = N_M^m + \frac{1}{2}(L_A^+ + L_D^0)$$

Stability analysis for solution

$$L(t, x) = \boxed{\delta i\omega} f(t, x)$$

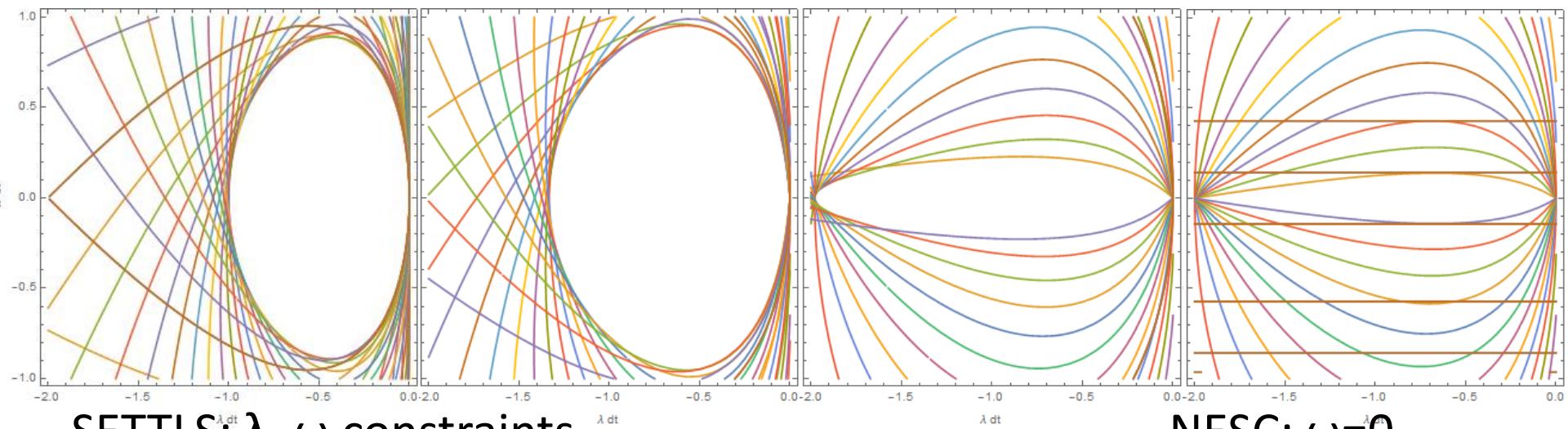
$$N(t, x) = (\lambda + (1 - \delta)i\omega)f(t, x)$$

$\delta=3/4$ $\beta = 1$

$\beta = 1/2$

$\beta = 1/100$

$\beta = 0$



SETTLS: λ, ω constraints

NESC: $\omega=0$

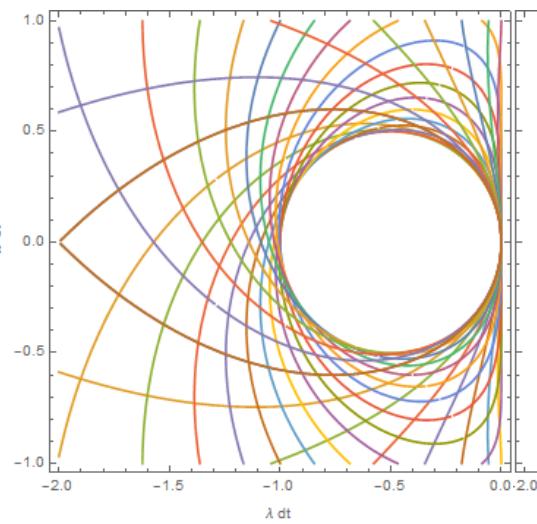
Dynamic definition of the time scheme

Introduce explicit guess:

$$\frac{f_A^+ - f_D^0}{\Delta t} = \left(\frac{1}{4} + \alpha\right)N_A^0 + \left(\frac{1}{4} - \alpha\right)\tilde{N}_A^+ + \left(\frac{3}{4} + \alpha\right)N_D^0 + \left(-\frac{1}{4} - \alpha\right)N_D^- + \frac{1}{2}(L_A^+ + L_D^0)$$

Stability analysis shows better stability in both, ω and λ direction.

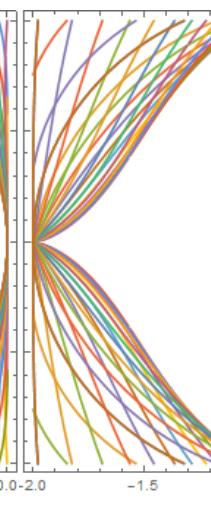
$\delta=0, \alpha = 1/4$



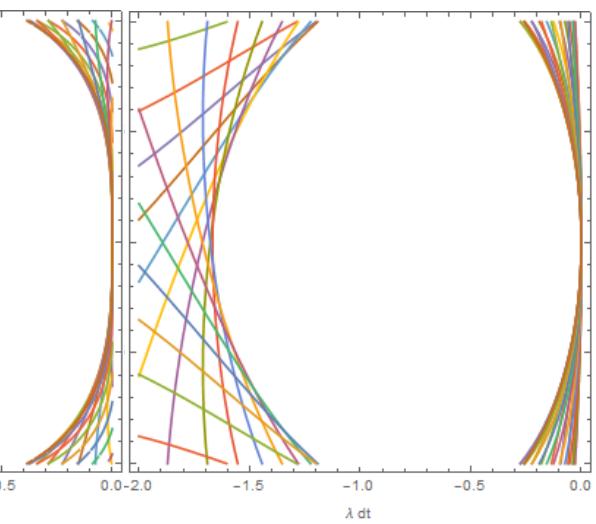
$\delta=0, \alpha = 0$



$\delta=0, \alpha = -1/4$



$\delta=3/4, \alpha = 1/10$



Dynamic definition of the time scheme

Iterative centered implicit scheme

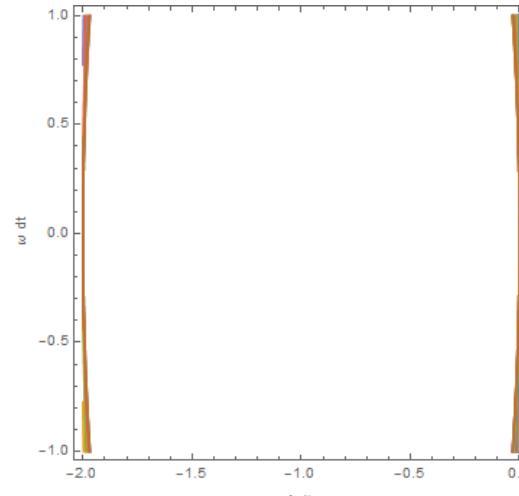
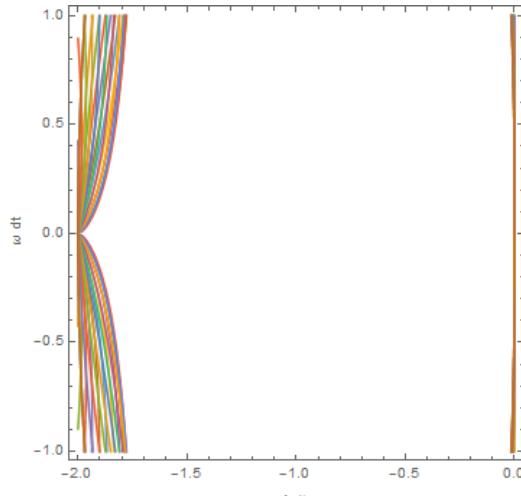
Predictor step using combined scheme:

$$\frac{f_A^{+(0)} - f_D^0}{\Delta t} = \frac{1}{2}(N_A^0 + N_D^0) + \frac{\beta}{2}(N_D^0 - N_D^-) + \frac{1}{2}(L_A^{+(0)} + L_D^0)$$

Corrector step using NESC :

$$\frac{f_A^{+(n)} - f_D^0}{\Delta t} = \frac{1}{2}(N_A^{+(n-1)} + N_D^0) + \frac{1}{2}(L_A^{+(n)} + L_D^0)$$

$\beta = 1$
SETTLS



$\beta = 0$
NESC

Dynamic definition of the time scheme

The advection treatment used

- 1) in SL trajectory search
- 2) for non-linear residuum in the SI scheme

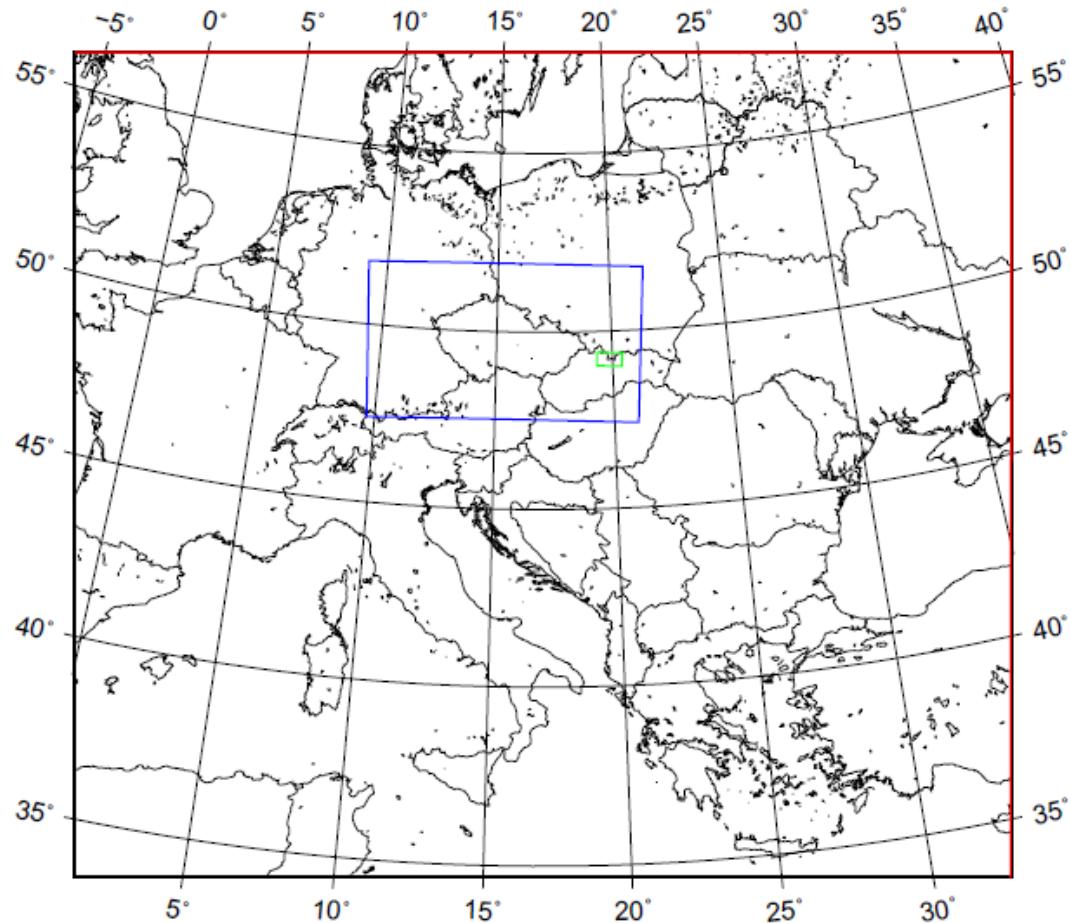
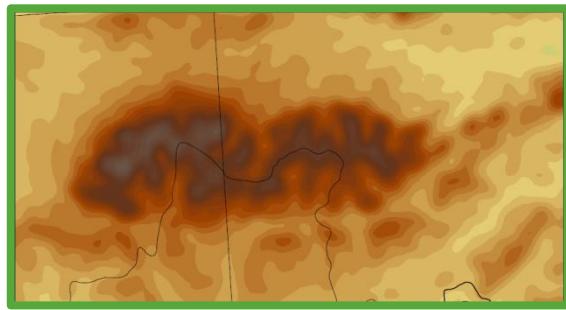
From experience:

- 1) SETTLS beneficial for trajectory search
- 2) Instability may occur when SETTLS applied on non-linear residuum, while NESC is stable (and only first order accurate)

The idea: To use SETTLS whenever possible, and switch to NESC if needed. If it is not “very often”, we keep sufficient accuracy and restore stability.

Dynamic definition of the time scheme

Experiments
on small domain



Experiments on small domain

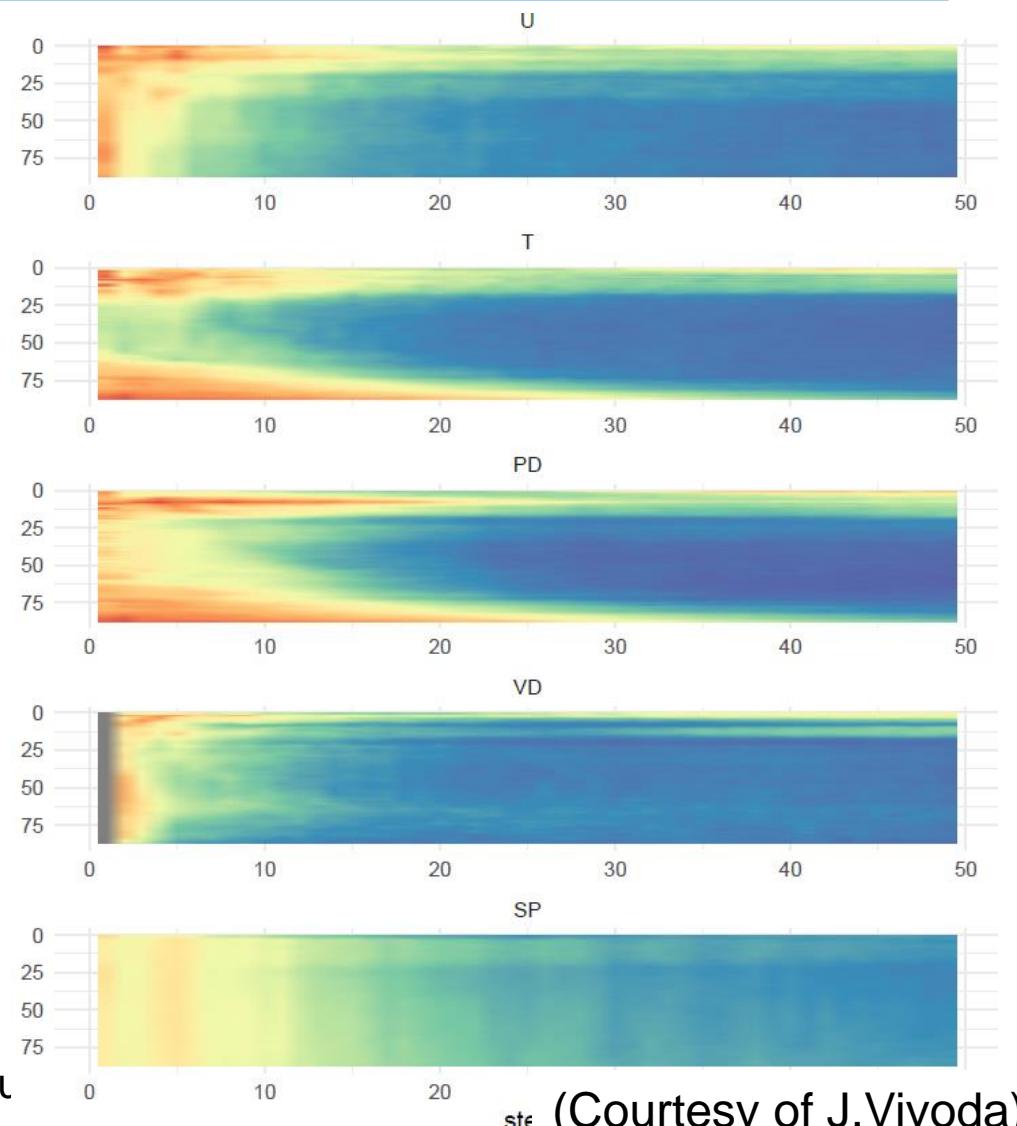
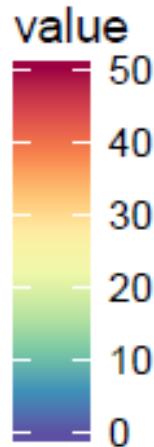
PC with one iteration:

$$\mu = 1 - \frac{|N^0 - N^-|}{|N^0| + |N^-|}$$

for pressure departure.

When $\mu \approx 1$ use SETTLS,
otherwise use NESCI
in each grid point in
predictor.

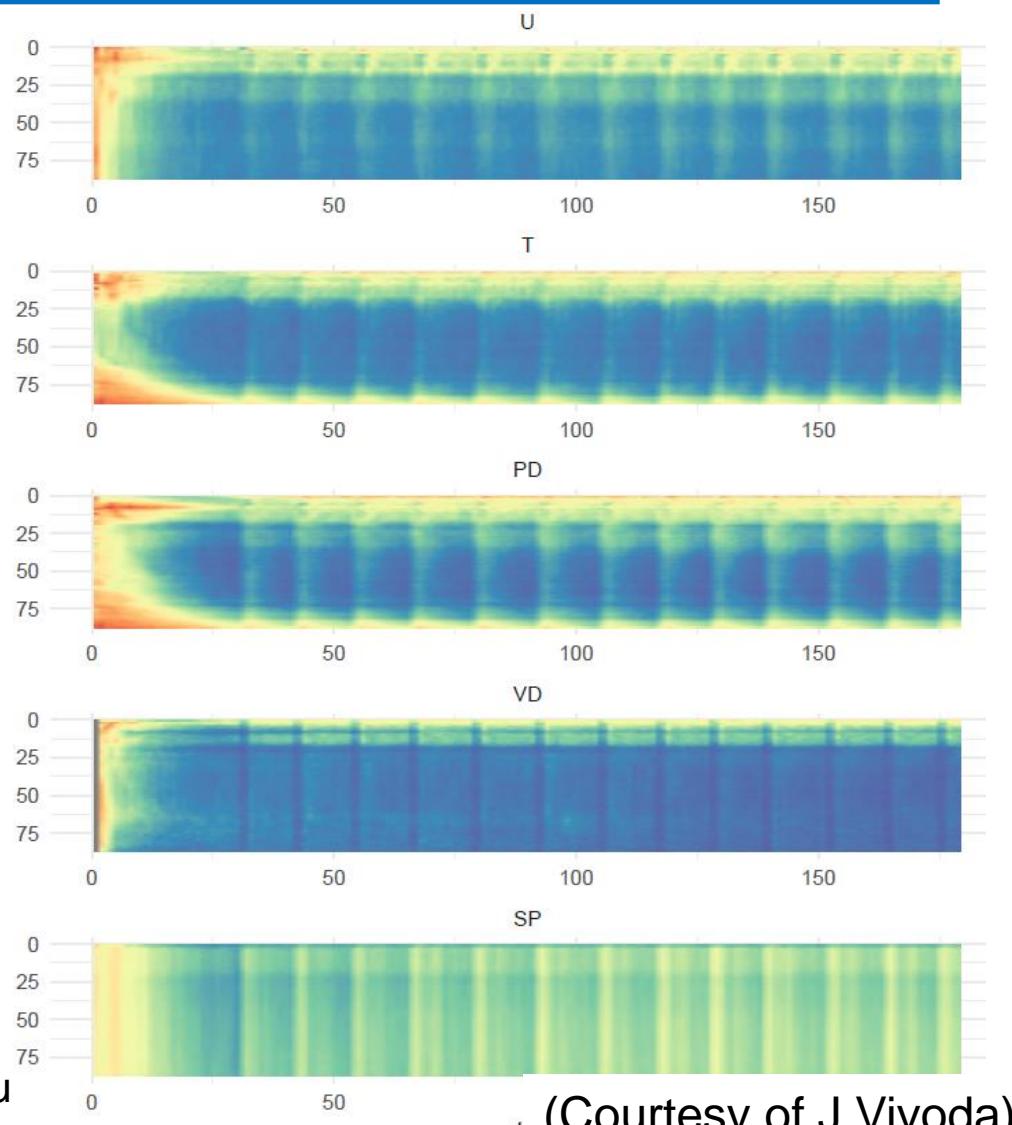
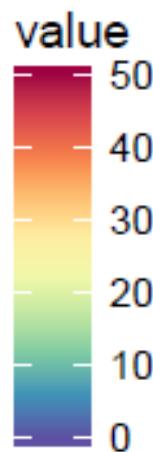
Number of points
with NESCI
calculation:



Experiments on small domain

Collect globally
information on the usage
of NESC; if NESC applied in
less than 10% of grid
points, skip corrector.

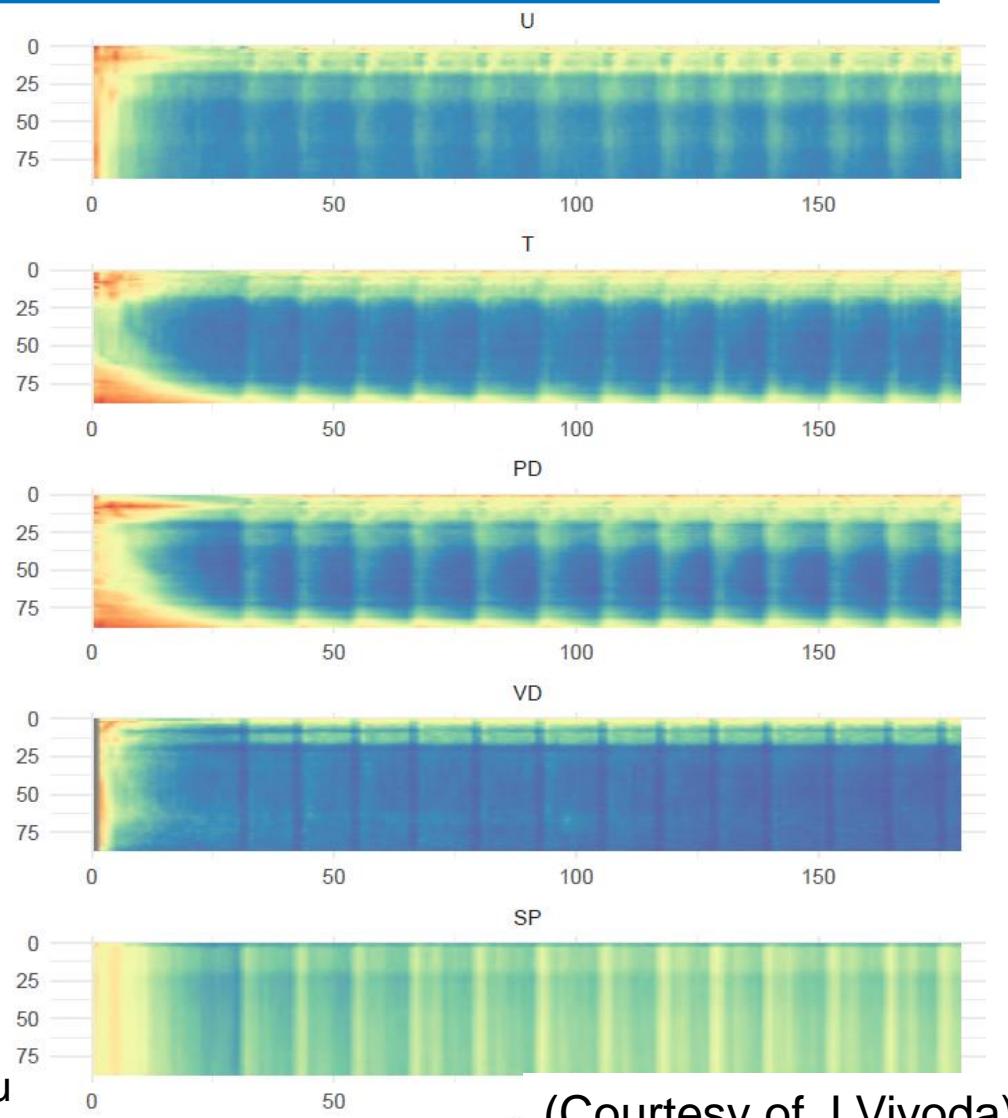
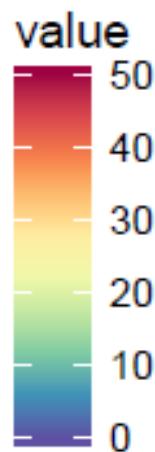
Number of points
with NESC
calculation:



Experiments on small domain

Hence corrector is applied only once per several time steps.

The accuracy and stability is restored.



Dynamic definition of the time scheme

Conclusions:

- SETTLS scheme is enabled in predictor step of the PC scheme (For LPC_CHEAP as well).
- Dynamic choice of the predictor used (SETTLS/NESC) and of the number of correctors applied may be an efficient answer to stability/accuracy trade-off.
- When using dynamic definition of the time scheme we are not able to predict exactly in advance the time to results, some threshold may be established.

Finite elements in the vertical in NH (Jozef Vivoda)

Phased to CY45T1: still many parameters, correct namelist setting needed, as

LVERTFE=.T.

NVFE_TYPE=3

LVFE_FIX_ORDER=.T.

LVFE_DELNHPRE=.T.

LVFE_ECMWF=.F.

LVFE_LAPL=.T.

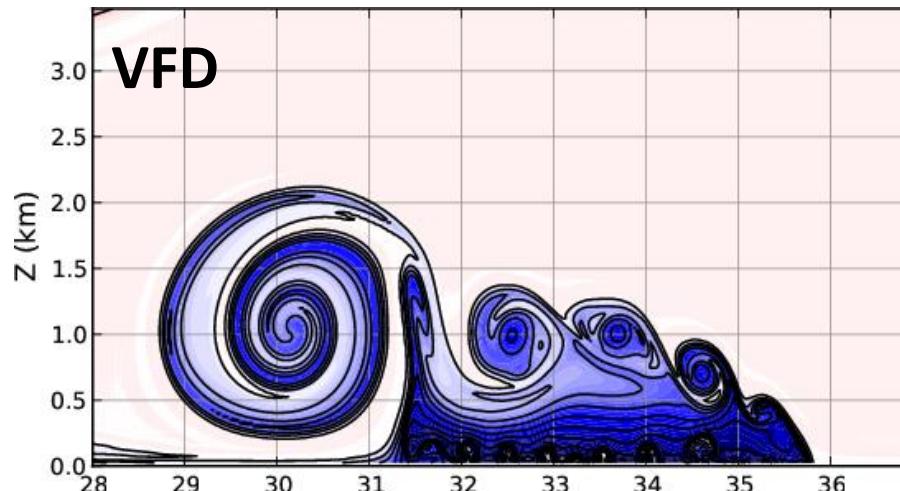
LVFE_X_TERM=.T.

NVFE_DERBC=2

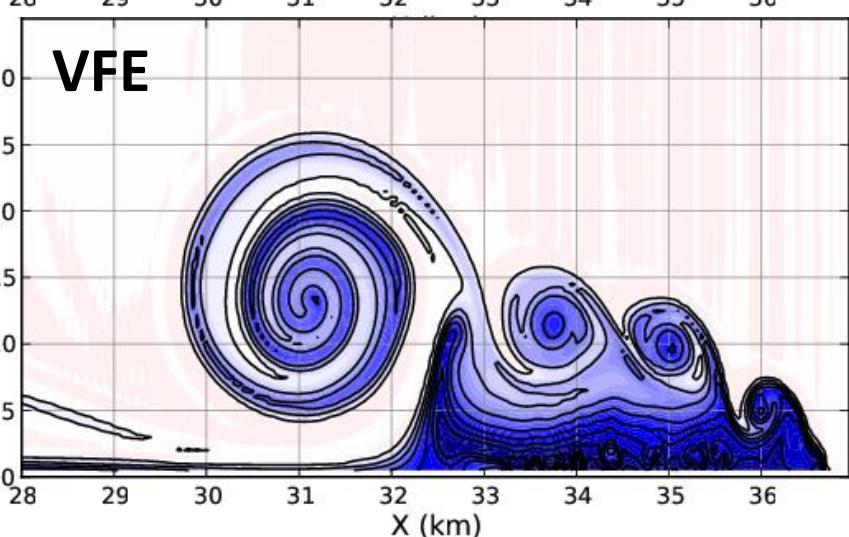
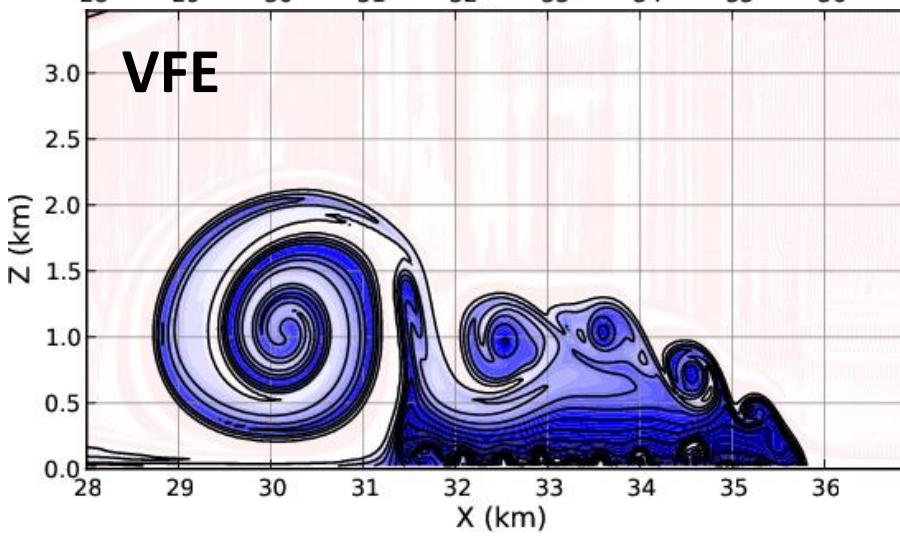
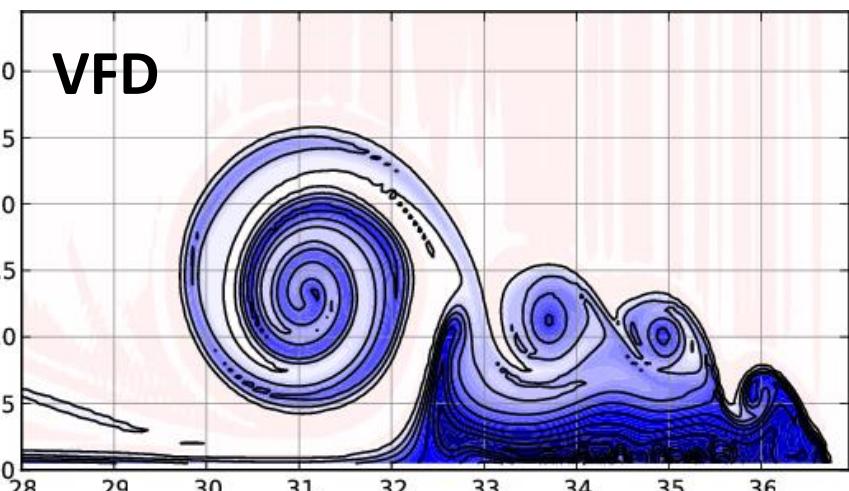
NVFE_INTBC=2

Finite elements in the vertical in NH

small Δz , short Δt

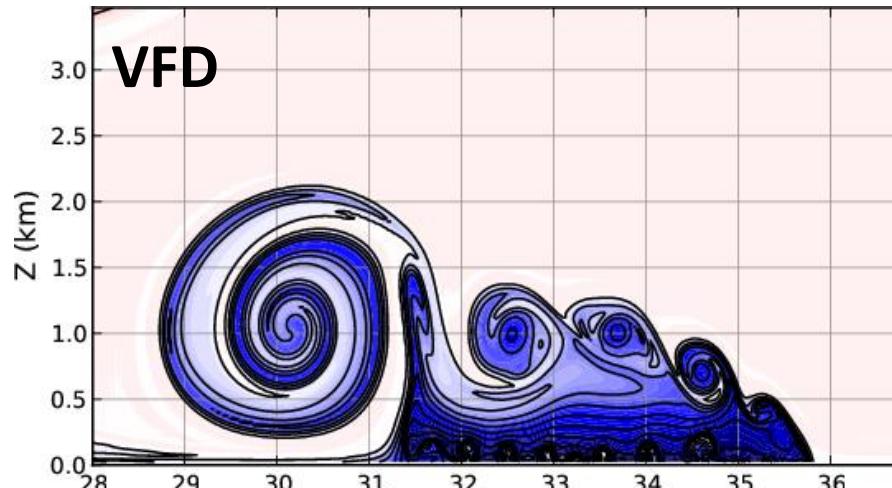


2x longer Δz

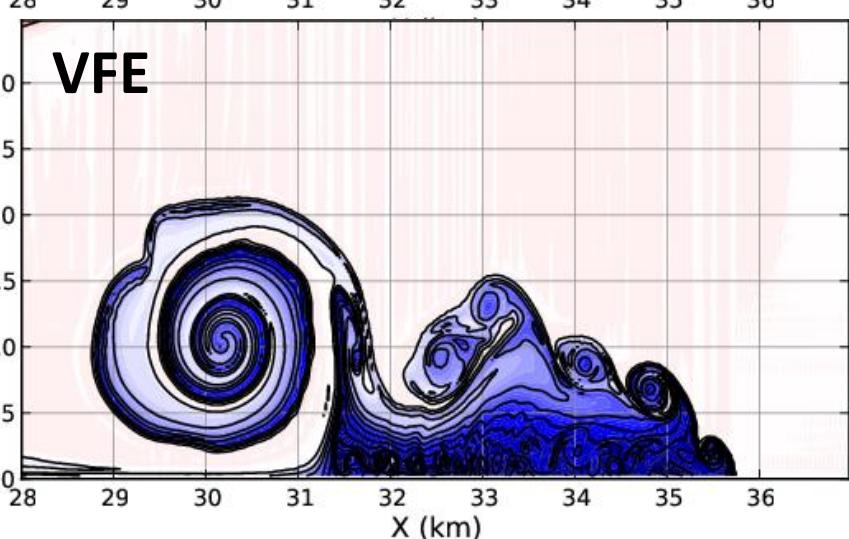
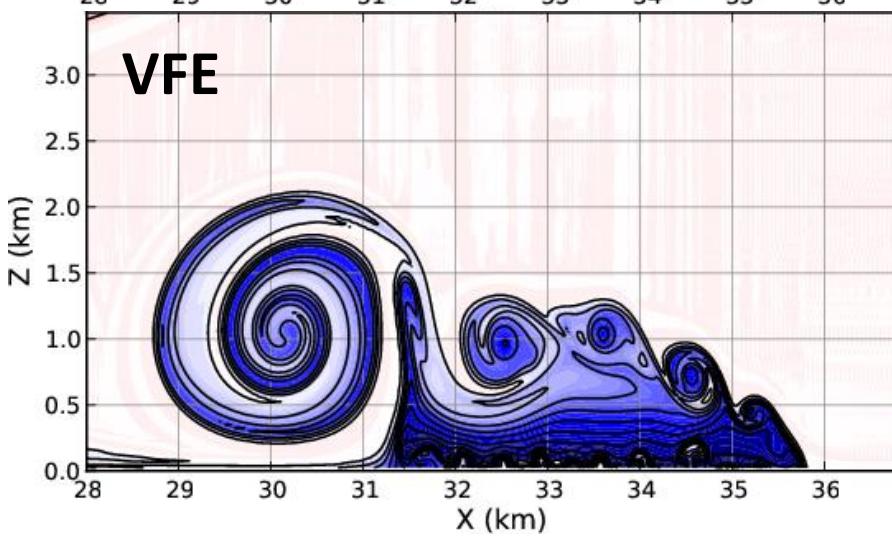
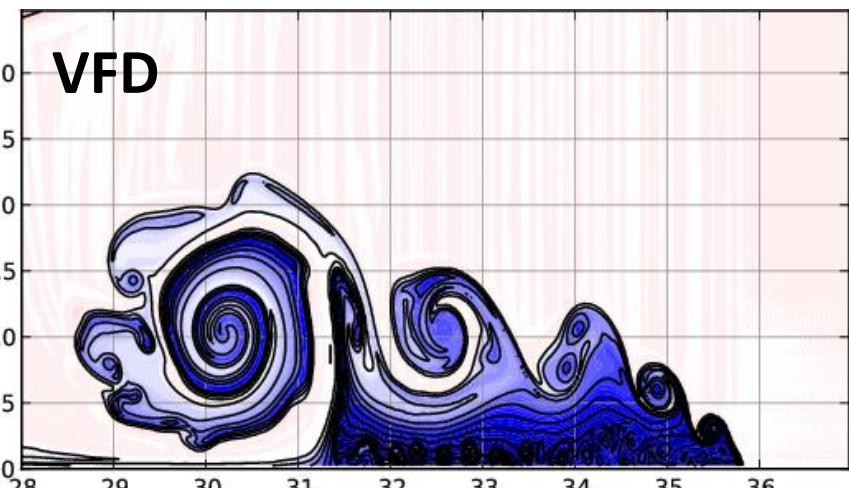


Finite elements in the vertical in NH

small Δz , short Δt



10x longer Δt



Finite elements in the vertical in NH

Phased to CY45T1:

New calculation of full level A,B through simplified process.

$$\text{We want: } \underline{\mathbf{I}_0^1} \cdot \frac{\delta \mathbf{A}}{\delta \boldsymbol{\eta}} = 0 \quad \text{We guess: } \widehat{\delta \mathbf{A}_l} = \tilde{A_l} - \widetilde{A_{l-1}}$$

$$\underline{\mathbf{I}_0^1} \cdot \frac{\delta \mathbf{B}}{\delta \boldsymbol{\eta}} = 1 \quad \widehat{\delta \mathbf{B}_l} = \tilde{B_l} - \widetilde{B_{l-1}}$$

$$\text{We calculate: } \frac{1}{p_0} \frac{\delta \mathbf{A}_l}{\delta \boldsymbol{\eta}_l} + 1 = \frac{1}{\beta} \left(\frac{1}{p_0} \frac{\widehat{\delta \mathbf{A}_l}}{\delta \boldsymbol{\eta}_l} + 1 \right) \quad \underline{\mathbf{I}_0^\eta} \cdot \frac{\delta \mathbf{A}}{\delta \boldsymbol{\eta}} = \mathbf{A}$$

$$\delta \mathbf{B}_l = \frac{\widehat{\delta \mathbf{B}_l}}{\underline{\mathbf{I}_0^1} \cdot \frac{\delta \mathbf{B}}{\delta \boldsymbol{\eta}}} \quad \underline{\mathbf{I}_0^\eta} \cdot \frac{\delta \mathbf{B}}{\delta \boldsymbol{\eta}} = \mathbf{B}$$

Finite elements in the vertical in NH

Phased to CY45T1:

New idea of Fabrice Voitus **for the Helmholtz solver** – we may eliminate all the variables but D (horizontal divergence) and **get rid of the iterative process** used up to now.

$$\begin{pmatrix} \underline{\mathbf{E}} & -\underline{\mathbf{F}} \\ -\underline{\mathbf{B}} & \underline{\mathbf{A}} + \underline{\mathbf{C}} \end{pmatrix} \begin{pmatrix} \mathbf{d} \\ \mathbf{D} \end{pmatrix} = \begin{pmatrix} \mathbf{R}_d \\ \mathbf{R}_D \end{pmatrix}$$

$$(\underline{\mathbf{B}}\underline{\mathbf{E}}^{-1}\underline{\mathbf{F}} + \underline{\mathbf{A}} + \underline{\mathbf{C}})\mathbf{D} = \mathbf{R}_D + \underline{\mathbf{B}}\underline{\mathbf{E}}^{-1}\mathbf{R}_d$$

May be solved directly even if C \neq 0.

The trajectory search in the SL advection scheme

- ▶ PC scheme with reiteration of SL trajectories produces noisy solution in some cases.
- ▶ If model horizontal resolution increased => local divergence may increase => Lipschitz criteria may be broken locally => divergent algorithm for searching SL origin point => increase in the number of iterations may lead to even less accurate solutions.
- ▶ Similar problems have been identified at ECMWF in IFS and fixed by local change of the computation of the half level wind.

The trajectory search in the SL advection scheme

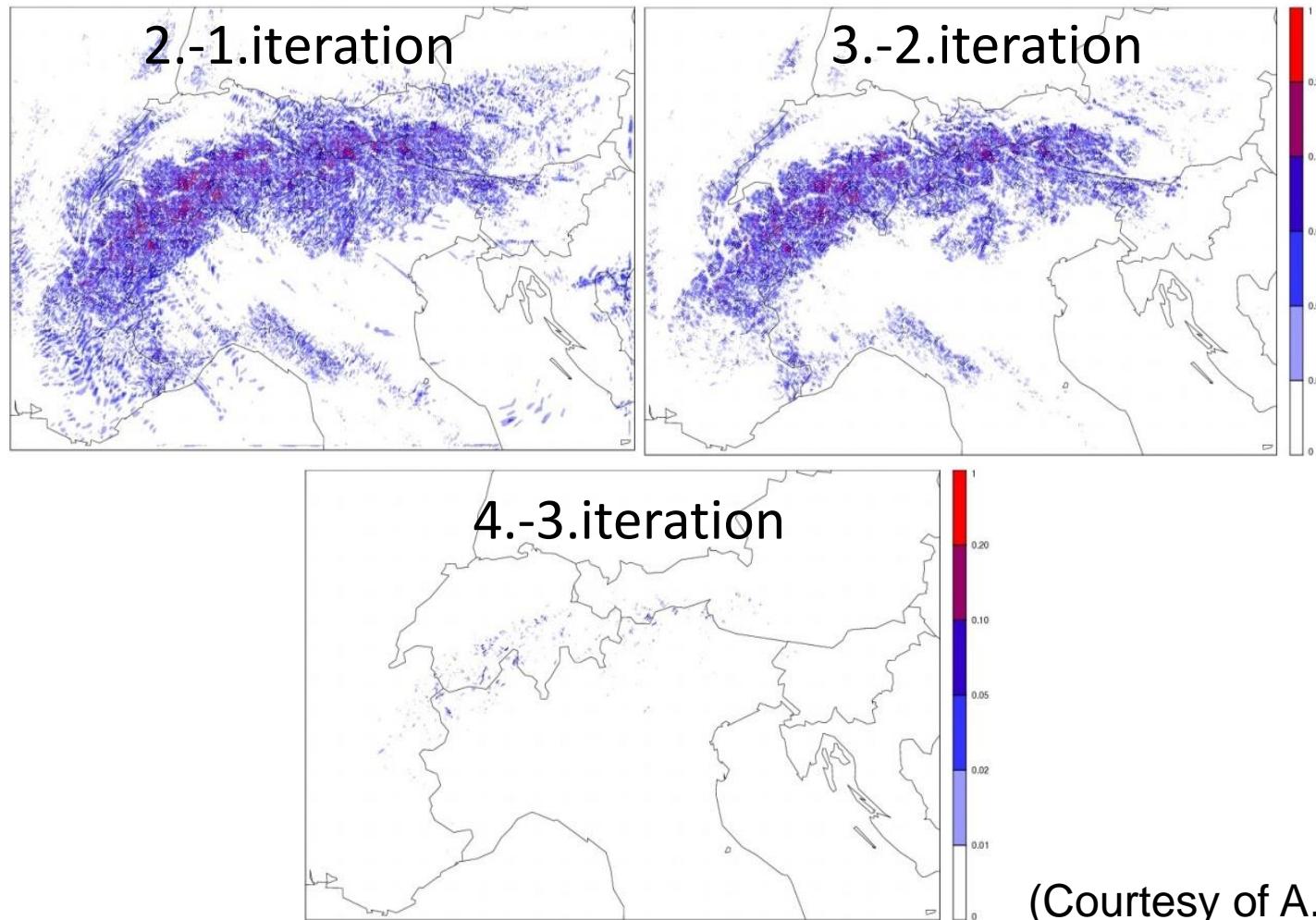
Current work:

- ▶ to calculate distances between two points representing estimations of the origin point from two successive iterations
- ▶ applied separately for horizontal and vertical components
- ▶ applied on several real cases

Conclusions:

- ▶ Second iteration already very close to the first one.
- ▶ Depending on the criteria we may find some divergent grid points, but the origin points are not moved by more than dms.
- ▶ More systematic testing on longer period is needed.

The trajectory search in the SL advection scheme



(Courtesy of A.Craciun)

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JE VOUDRAIS VOUS
REMERCIER POUR
VOTRE ATTENTION!



► 24, ALADIN/HIRLAM WS 2018, Toulouse



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ZAMG

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