Three time level iterative centered implicit scheme with Eulerian advection treatment

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August 2004, Vienna

1 Motivation

The three time level Eulerian scheme plays important role in the development of dynamics. Its existence allows inter-comparison between semi-Lagrangian (SL) and Eulerian version of time stepping procedures. It turned out to be the useful configuration in identifications of shortcomings in SL formulation.

2 Notation

The notation $\mathcal{X}$ is used for quantity

$$\mathcal{X} = \frac{p}{mR_dT}\vec{\nabla}\phi\frac{\partial\vec{v}}{\partial\eta},$$

which appears in the elastic term. This quantity plays important role in the model ALADIN. It affects the model stability and its implicit treatment is assured via prognostic variable d_4.

The name d variable is used for prognostic variable related to vertical divergence ($\frac{\partial gw}{\partial\eta}$). It represents one of the variables d_3 or d_4. At the places which are specific for each variable the exact variable is mentioned. The variables are defined as following

$$d_3 = -\frac{p}{mR_dT}\frac{\partial gw}{\partial\eta} \quad (1)$$

$$d_4 = -\frac{p}{mR_dT}\frac{\partial gw}{\partial\eta} + \mathcal{X} \quad (2)$$

The notation X is used for the model state at given time instant

$$X = (U, V, T, ln(\pi_s), \hat{q}, d).$$

The notation $\mathcal{M}$ is used for the nonlinear model defined by

$$\frac{dX}{dt} = \mathcal{M}(X).$$

The notation $\mathcal{B}$ denotes the linear operator. The operator $\mathcal{B}$ determines the model spectral solver around which the iterations are performed.

The notation $\mathcal{A}$ denotes the Eulerian advection operator

$$\mathcal{A}(X) = -\vec{v}\vec{\nabla}X - \dot{\eta}\frac{\partial X}{\partial \eta}.$$

In the text we used abbreviations:

3TL	three-time level
2TL	two-time level
SL	semi-Lagrangian
EUL	Eulerian
FIC	fully implicit centered
ICI	iterated centered implicit
SI	semi-implicit
LHS	left hand side
RHS	right hand side

3 Formulation

The 3TL ICI EUL scheme is an iterative approximation of 3TL FIC EUL scheme

$$X^{t+\Delta t} - \Delta t\mathcal{M}(X^{t+\Delta t}) = X^{t-\Delta t} + 2\Delta t\mathcal{A}(X^{t}) + \Delta t\mathcal{M}(X^{t-\Delta t}).$$

We put implicit terms on the LHS of the equation. The 3TL FIC EUL scheme can not be implemented directly because the inversion of $\left(X^{t+\Delta t} - \Delta t\mathcal{M}(X^{t+\Delta t})\right)$ is not tractable. Therefore the iterative approximation has to be used.

The traditional 3TL semi-implicit scheme is a predictor of the iterative procedure (we note it by superscript $t + \Delta t(0)$). Successive correctors are formulated with iterations around the traditional SI spectral solver (n denotes n-th iteration).

$$
\begin{aligned}
P: \qquad (I-\Delta t\mathcal{B})X^{t+\Delta t(0)} &= X^{t-\Delta t} + 2\Delta t\mathcal{A}(X^t) \\
&\quad +2\Delta t\mathcal{M}(X^t) + \Delta t\mathcal{B}\left(X^{t-\Delta t} - 2X^t\right) \\
C(n): \quad (I-\Delta t\mathcal{B})X^{t+\Delta t(n)} &= X^{t-\Delta t} + 2\Delta t\mathcal{A}(X^t) \\
&\quad +\Delta t\left(\mathcal{M}(X^{t+\Delta t(n-1)}) + \mathcal{M}(X^{t-\Delta t})\right) - \Delta t\mathcal{B}X^{t+\Delta t(n-1)}.
\end{aligned} \tag{3}
$$

The corrector is written in the simplest form in (3). But the corrector step has been put into the different form for the purpose of implementation into the model ALADIN. The corrector is expressed rather as a correction to the predictor.

$$
\begin{aligned}
P: \qquad (I-\Delta t\mathcal{B})X^{t+\Delta t(0)} &= X^{t-\Delta t} + 2\Delta t\mathcal{A}(X^t) \\
&\quad +2\Delta t\mathcal{M}(X^t) + \Delta t\mathcal{B}\left(X^{t-\Delta t} - 2X^t\right) \\
C(n): \quad (I-\Delta t\mathcal{B})X^{t+\Delta t(n)} &= X^{t-\Delta t} + 2\Delta t\mathcal{A}(X^t) \\
&\quad +2\Delta t\mathcal{M}(X^t) + \Delta t\mathcal{B}\left(X^{t-\Delta t} - 2X^t\right) \\
&\quad +\Delta t\left(\mathcal{R}(X^{t+\Delta t(n-1)}) + \mathcal{R}(X^{t-\Delta t}) - 2\mathcal{R}(X^t)\right)
\end{aligned} \tag{4}
$$

We introduced here notation $\mathcal{R}(X)$. It is referred to as the iterated nonlinear residual. Formulation (4) is the generalization of (3), because in (3) we assume that $\mathcal{R}(X) = \mathcal{M}(X) - \mathcal{B}X$, while the choice of $\mathcal{R}(X)$ operator in (4) is arbitrary. This allows to test 3TL EUL ICI scheme with various $\mathcal{R}(X)$ operators. However in practice, we use always $\mathcal{R}(X) = \mathcal{M}(X) - \mathcal{B}X$ and the model data flow reflects (4).

The advection is excluded from the iterative process and it is evaluated always as $\mathcal{A}(X^t)$. This results from the analysis of stability of linear advection problem.

The terms on the RHS of (4) can be grouped and written as

$$
\begin{aligned}
P: \qquad (I-\Delta t\mathcal{B})X^{t+\Delta t(0)} &= RHS_P \\
C(n): \quad (I-\Delta t\mathcal{B})X^{t+\Delta t(n)} &= RHS_C + \Delta t\mathcal{R}(X)^{t+\Delta t(n-1)}
\end{aligned} \tag{5}
$$

with

$$
\begin{aligned}
RHS_P &= X^{t-\Delta t} + 2\Delta t\mathcal{A}(X^t) + 2\Delta t\mathcal{M}(X^t) + \Delta t\mathcal{B}\left(X^{t-\Delta t} - 2X^t\right) \\
RHS_C &= RHS_P + \Delta t\left(\mathcal{R}(X^{t-\Delta t}) - 2\mathcal{R}(X^t)\right).
\end{aligned} \tag{6}
$$

The quantity RHS_C is computed during the predictor and stored, because it depends only on the state at time instants t and $t-\Delta t$. Only the term $\Delta t\mathcal{R}(X)^{t+\Delta t(n-1)}$ is added to the stored quantity during the corrector.

3.1 Formulation specificities for d_4 prognostic variable

Prognostic equation for d_4 is not treated according (4) exactly. There are the two exceptions:

1. the treatment of $\mathcal{X}$ tendency

   The evolution of d_4 is given as

   $$\frac{dd_4}{dt} = \frac{dd_3}{dt} + \frac{d\mathcal{X}}{dt}. \tag{7}$$

   The term $\frac{dd_3}{dt}$ is

   $$\frac{dd_3}{dt} = -\frac{g^2 p}{mR_dT}\frac{\partial}{\partial\eta}\left(\frac{1}{m}\frac{\partial(p-\pi)}{\partial\eta}\right) + \frac{gp}{mR_dT}\vec{\nabla}w\frac{\partial\vec{v}}{\partial\eta} - d_3D_3 + d_3D, \tag{8}$$

   and it is included in the $\mathcal{M}$ and treated accordingly.
   The term $\frac{d\mathcal{X}}{dt}$ is not evaluated directly but (7) is rewritten as

   $$\frac{\partial d_4}{\partial t} + \vec{v}\vec{\nabla}d_4 + \dot{\eta}\frac{\partial d_4}{\partial\eta} = \frac{dd_3}{dt} + \frac{\partial\mathcal{X}}{\partial t} + \vec{v}\vec{\nabla}\mathcal{X} + \dot{\eta}\frac{\partial\mathcal{X}}{\partial\eta}. \tag{9}$$

   When advection of $\mathcal{X}$ is subtracted from the advection of d_4, it yields

   $$\mathcal{A}(d_4) = -\vec{v}\vec{\nabla}d_3 - \dot{\eta}\frac{\partial d_3}{\partial\eta}. \tag{10}$$

   The local change of $\mathcal{X}$ is computed with final differences as

   $$\begin{array}{ll} P: & \frac{\partial\mathcal{X}}{\partial t} \approx \frac{\mathcal{X}^t - \mathcal{X}^{t-\Delta t}}{\Delta t} \\ C(n): & \frac{\partial\mathcal{X}}{\partial t} \approx \frac{\mathcal{X}^{t+\Delta t(n-1)} - \mathcal{X}^{t-\Delta t}}{2\Delta t} \end{array} \tag{11}$$

   To arrive to the predictor approximation in (11), the value $\mathcal{X}^{t+\Delta t}$ has been estimated as $\mathcal{X}^{t+\Delta t} \approx 2\mathcal{X}^t - \mathcal{X}^{t-\Delta t}$.

2. the computation of $\vec{\nabla}\mathcal{X}$ term

   This term is required to compute $\vec{\nabla}w$ in (8) and $\vec{\nabla}d_3$ in (10).

   To compute $\vec{\nabla}\mathcal{X}$ we would have to compute $\vec{\nabla}^2\phi$ and it would require to compute the second derivatives of all prognostic variables. This approach would be very computationally expensive in the spectral model. To compute directly nonlinear $\mathcal{X}$ from the prognostic equations in the spectral space is also impossible.

   Therefore the computation of $\vec{\nabla}\mathcal{X}$ in model ALADIN is approximative. Pseudo-prognostic equation is used to compute the estimate of $\mathcal{X}^{t+\Delta t}$. This estimate (denoted with tilde as

$\tilde{\mathcal{X}}^{t+\Delta t}$) is then transformed into the spectral space where it is used to compute $\tilde{d_3}^{t+\Delta t} = {d_4}^{t+\Delta t} - \tilde{\mathcal{X}}^{t+\Delta t}$ and its derivatives $\vec{\nabla}\tilde{d_3}^{t+\Delta t}$.

Formulation of pseudo-prognostic equation is controlled with the variable ND4SYS in the model ALADIN. We found out that for the 3TL ICI EUL scheme existing implementations (ND4SYS=1 or 2) caused model instability. We assume that this is due to inconsistency between computation of $\tilde{\mathcal{X}}^{t+\Delta t}$ in (11) and in the pseudo-prognostic equations. To assure consistency we therefore tested formulation

$$\begin{aligned} P: \quad & \tilde{\mathcal{X}}^{t+\Delta t} = 2\mathcal{X}^{t} - \mathcal{X}^{t-\Delta t} \\ C(n): \quad & \tilde{\mathcal{X}}^{t+\Delta t} = \tilde{\mathcal{X}}^{t+\Delta t(n-1)}. \end{aligned} \tag{12}$$

This formulation (ND4SYS=5) is stable.

3.2 Asselin filter in 3TL ICI EUL scheme

We are using the Asselin filter in model ALADIN to suppress the numerical oscillations in the 3TL scheme. It is computed in two steps as:

$$\begin{aligned} timestep(k): \quad & \tilde{X}^{t} = \alpha X^{t-\Delta t} + (1-\alpha-\beta)X^{t} \\ timestep(k+1): \quad & X^{t-\Delta t} = \tilde{X}^{t-\Delta t} + \beta X^{t} \end{aligned} \tag{13}$$

In the 3TL ICI EUL scheme we have to evaluate $\mathcal{R}(X)$ at time instants $t-\Delta t$,t and $t+\Delta t$. To compute $\mathcal{R}(X^{t-\Delta t})$ we had three possibilities:

(A) to compute $\mathcal{R}(X^{t-\Delta t})$ from $X^{t-\Delta t}$ during predictor and then to store $\mathcal{R}(X^{t-\Delta t})$

(B) to skip first part of Asselin filter during predictor. To store $X^{t-\Delta t}$ and X^{t} and its derivatives and then during the corrector to evaluate $\mathcal{R}(X^{t-\Delta t})$. The first part of Asselin filter would than be computed during the last corrector.

(C) to compute $\mathcal{R}(X^{t-\Delta t})$ from $\mathcal{R}(X^{t})$ and to store it. This is computationally and memory less consuming solution. But in this case we have to perform Asselin filter directly on $\mathcal{R}(X)$ quantity as

$$\begin{aligned} timestep(k): \quad & \tilde{\mathcal{R}}(X)^{t} = \alpha\mathcal{R}(X)^{t-\Delta t} + (1-\alpha-\beta)\mathcal{R}(X)^{t} \\ timestep(k+1): \quad & \mathcal{R}(X)^{t-\Delta t} = \tilde{\mathcal{R}}(X)^{t-\Delta t} + \beta\mathcal{R}(X)^{t} \end{aligned} \tag{14}$$

We decided to compute $\mathcal{R}(X^{t-\Delta t})$ from $\mathcal{R}(X^{t})$ directly (method C) to minimize the computational expenses.

The same problem mentioned for $\mathcal{R}(X)^{t-\Delta t}$ exists for quantity $\mathcal{X}^{t-\Delta t}$. It is also computed from $\mathcal{X}^{t}$ by means of Asselin time filter as

$$
\begin{aligned}
timestep(k): \quad & \tilde{\mathcal{X}}^{t} = \alpha \mathcal{X}^{t-\Delta t} + (1-\alpha-\beta)\mathcal{X}^{t} \\
timestep(k+1): \quad & \mathcal{X}^{t-\Delta t} = \tilde{\mathcal{X}}^{t-\Delta t} + \beta \mathcal{X}^{t}
\end{aligned}
\tag{15}
$$

3.3 Treatment of passive scalar variables

The scalar variable s is evolving according

$$\frac{ds}{dt} = F_s,$$

and only tendency source is the physical parameterizations contribution F_s. The time discretization of this equation in 3TL ICI EUL scheme manner is

$$s^{t+\Delta t} = s^{t-\Delta t} + 2\Delta t \mathcal{A}(s)^{t} + F_s^{t-\Delta t}. \tag{16}$$

The scalar variable equations are not iterated (if the physics is not iterated). For the purpose of the iterative scheme, the RHS of equation (16) has to be stored during predictor and retrieved during the corrector. Stored quantity is

$$RHS_s = s^{t-\Delta t} + 2\Delta t \mathcal{A}(s)^{t} + F_s^{t-\Delta t}. \tag{17}$$

3.4 Physical-Dynamical Interface

The physical tendencies of thermodynamics variables are evaluated in 3TL schemes at time instant $t - \Delta t$. They are computed only during the predictor. The physical tendency is added to the quantity RHS_P in (6) as

$$RHS_P = RHS_P + 2\Delta t F^{t-\Delta t}.$$

Because the stored RHS_C is a function of RHS_P it contains the physical contribution as well.

Contrary to the SL iterative scheme the physical contribution is not stored in the independent buffers (GFLPC and GMV). It is rather included in the precomputed RHS_C.

4 Implementation in CY29T2

4.1 Setup

- **YOMGFL, TYPE_GFLS, GFL_SUBS**

  Added GFLPC buffer used to keep RHS_s quantities according (17).

  Buffer GFLPC is defined in YOMGFL.

  To introduce this buffer one needs to define in TYPE_GFLS

NUMFLDSPC	Number of GFL fields (predictor/corrector)
NDIMPC	Dimension of iterative scheme auxiliary array (GFLPC)
LPC	Field in predictor/corrector time stepping (GFLPC)
MPPC	Predictor/corrector auxiliary array "pointer"

This variables are then initialized by means of functions defined in GFL_SUBS. These functions were adjusted to take into account the buffer GFLPC. These functions are then called from SUDYN.

- **TYPE_GMVS and GMV_SUBS**

  New arrays were added to GMV and GMVS. They are used as a storage for quantity RHS_C defined in (6). The pointers for this new arrays are

New GMV pointers	
MCURHS	keeps RHS_C for U equation
MCVRHS	keeps RHS_C for V equation
MCTRHS	keeps RHS_C for T equation
MCSPDRHS	keeps RHS_C for PD equation
MCSVDRHS	keeps RHS_C for VD equation
New GMVS pointers	
MCSPRHS	keeps RHS_C for $ln(\pi_s)$ equation

Two new arrays were added to GMV for the purpose of Asselin filtering of $\mathcal{X}$ according (15). This computation is needed for ND4SYS=5 only.

New GMV pointers	
MD3YNL	in P stored $\tilde{\mathcal{X}}^{t-\Delta t}$
MCD3YNL	in P stored filtered $\mathcal{X}^{t-\Delta t}$

The pointers and GMV and GMVS dimensions are set in GMV_SUBS. For the purpose of 3TL ICI EUL scheme we initialize following additional pointers

Initialized GMV pointers	
MC[X]RHS	storage of RHS_C for U,V,T,SPD,SVD equations
MD3YNL	storage of $\tilde{\mathcal{X}}^{t-\Delta t}$ for ND4SYS=5
MCD3YNL	storage of $\mathcal{X}^{t-\Delta t}$ for ND4SYS=5
M[X]NL	storage of $\tilde{\mathcal{R}}(X)^{t-\Delta t}$ for U,V,T,SPD,SVD equations
MC[X]NL	storage of $\mathcal{R}(X)^{t-\Delta t}$ for U,V,T,SPD,SVD equations
Initialized GMVS pointers	
MCSPRHS	storage of RHS_C for $ln(\pi_s)$ equation
MSPLN	storage of $\tilde{\mathcal{R}}(X)^{t-\Delta t}$ for $ln(\pi_s)$ equation
MCSPLN	storage of $\mathcal{R}(X)^{t-\Delta t}$ for $ln(\pi_s)$ equation

The arrays M[X]NL and MC[X]NL are initialized for the purpose of Asselin filter computation according (14).

- **SUDIM1**

  Default value of LPC attributes of GFL arrays set to .FALSE..

- **SUDYN**

  Initialization of GFLPC array.

- **SUDYNA**

  ND4SYS=5 is introduced and it is set as a default value for EUL scheme with d_4 prognostic variable. ND4SYS=5 is coded according (12).

- **SUSC2B**

  Allocation of GFLPC buffer.

4.2 Integration

The routines CPDYN (hydrostatic Eulerian calculations) and GNHDYN (non-hydrostatic) calculations have been merged into the routine CPEULDYN. The 3TL ICI EUL scheme can be therefore used for the hydrostatic model as well.

- **GPINISLB**

  Initialize GMV(YT1) and PGFLT1 buffers.

P:	$X^{t+\Delta t} = X^{t-\Delta t}$
C(n):	$X^{t+\Delta t} = 0$

- **CPEULDYN**

  The computations are done in the following order

  1. Vertical and horizontal advection (LSPRT taken into account)

P:	$Z[X]ADV = \mathcal{A}(X)^t$
C(n):	not computed

  2. semi-implicit correction

P:	$P[X]SIC = \Delta t \mathcal{B}\left(X^{t-\Delta t} - 2X^t\right)$
C(n):	$P[X]SIC = -\Delta t \mathcal{B} X^{t+\Delta t(n-1)}$

  3. semi-implicit correction (for the purpose of $\mathcal{R}$ terms evaluation used in RHS_C)

P:	$Z[X]SI0 = \Delta t \mathcal{B} X^t$
C(n):	$Z[X]SI0 = \Delta t \mathcal{B} X^{t+\Delta t(n-1)}$

  4. nonlinear-tendency term

     The Coriolis force and curvature terms are considered in this part of computations for the momentum equation. In the CY28T1, this terms were computed in the horizontal advection part.

P:	$Z[X]M = \Delta t \mathcal{M}(X)^t$
C(n):	$Z[X]M = \Delta t \mathcal{M}(X)^{t+\Delta t(n-1)}$

  5. temporal advance (non-linear residuals $\mathcal{R}$ computation)

P:	$Z[X]RES = Z[X]M - Z[X]SI0 = \Delta t \mathcal{R}(X)^t$
C(n):	$Z[X]RES = Z[X]M - Z[X]SI0 = \Delta t \mathcal{R}(X)^{t+\Delta t(n-1)}$

  6. **GPTF2PC** second part of Asselin filter

P:	$PC[X]NLT9 = \Delta t \mathcal{R}(X)^{t-\Delta t}$
	$PCD3YNLT9 = \mathcal{X}^{t-\Delta t}$
C(n):	not called

  7. temporal advance (computations of RHS_C for each equation according (6))

P:	$P[X]RHS = RHS_C(X)$
C(n):	not computed

8. temporal advance (final preparation of RHS)

P:	$P[X]T1 = X^{t-\Delta t} + 2\Delta t F_X + 2\Delta t \mathcal{A}(X)^t + 2\Delta t \mathcal{M}(X)^t$
C(n):	$P[X]T1 = RHS_C(X) + \Delta t \mathcal{R}(X)^{t+\Delta t(n-1)}$

   The RHS of the predictor is not yet complete. Only the explicit guess is computed, because lagged physics is called with it. The SI correction stored in P[X]SIC is added later in GPENDTR routine.

   The RHS of the corrector is completed at this place. At the end of this routine we subtract P[X]SIC. We obtain then the quasi-explicit guess, which can be theoretically used to called lagged physics even during the corrector step. Term P[X]SIC is than again added in GPENDTR routine.

9. **GPTF1PC** first part of Asselin filter

P:	$P[X]NLT9 = \Delta t \tilde{\mathcal{R}}(X)^t$
	$PD3YNLT9 = \tilde{\mathcal{X}}^t$
C(n):	not called

10. temporal advance (computations related to $\mathcal{X}$)

   Here the tendency of $\mathcal{X}$ is computed according to ND4SYS option (more in Smolikova report).

   The array PSVDAUXT1 contains after computation the estimate of $\mathcal{X}$ (more precisely $-\mathcal{X}$) , used later in the spectral space for gradient computations.

   To the PSVD1 term the tendency of $\mathcal{X}$ is added computed with finite differences.

   For the option ND4SYS=5 we compute

P:	$PSVDAUXT1 = -(2\mathcal{X}^t - \mathcal{X}^{t-\Delta t})$
	$PSVDT1 = PSVDT1 + (2\mathcal{X}^t - \mathcal{X}^{t-\Delta t}) - \mathcal{X}^{t-\Delta t}$
C(n):	$PSVDAUXT1 = -\mathcal{X}^{t+\Delta t(n-1)}$
	$PSVDT1 = PSVDT1 + \mathcal{X}^{t+\Delta t(n-1)} - \mathcal{X}^{t-\Delta t}$

11. temporal advance (subtraction of SI term)

P:	no action
C(n):	$P[X]T1 = P[X]T1 - P[X]SIC$

12. scalar array storage

P:	$PGFLPC = PGFLT1 = s^{t-\Delta t} + 2\Delta t\mathcal{A}(s)^t + F_s^{t-\Delta t}$
C(n):	$PGFLT1 = PGFLPC$

- **GPENDTR**
  Missing linear terms are added to the explicit guess and RHS is completed. This action is not done in CPEULDYN, because we keep in the arrays the explicit guess for the call of lagged physics.

5 Tests

5.1 Linear Hydrostatic Flow regime

This is the one of few cases with known analytical solution. We have run this test to validate the implementation of 3TL EUL ICI scheme. The setup of experiment is the same as given in Bubnova at al, 1995,

Variable	Value
Height of Agnesi hill	$1m$
Half width of Agnesi hill	$16000m$
Surface pressure	$101325Pa$
Horizontal wind	$8ms^{-1}$
Isothermal temperature	$239.5K$
Brunt-Vaisala frequency	$0.02s^{-1}$
NDGL, NDGUX (quadratic trunc)	192
NFLEVG	99
EDELX	$3200m$
Position of middle of the hill	$185600m$
REPONTAU	$1200s$
REPONTP	$19400m$
REPONBT	$10000m$

The experiment has been run with the sponge layer at the top to avoid internal reflections of waves from the rigid lid at the top of model domain. The experiments were run with $SITR = SITRA = 239.5K$.

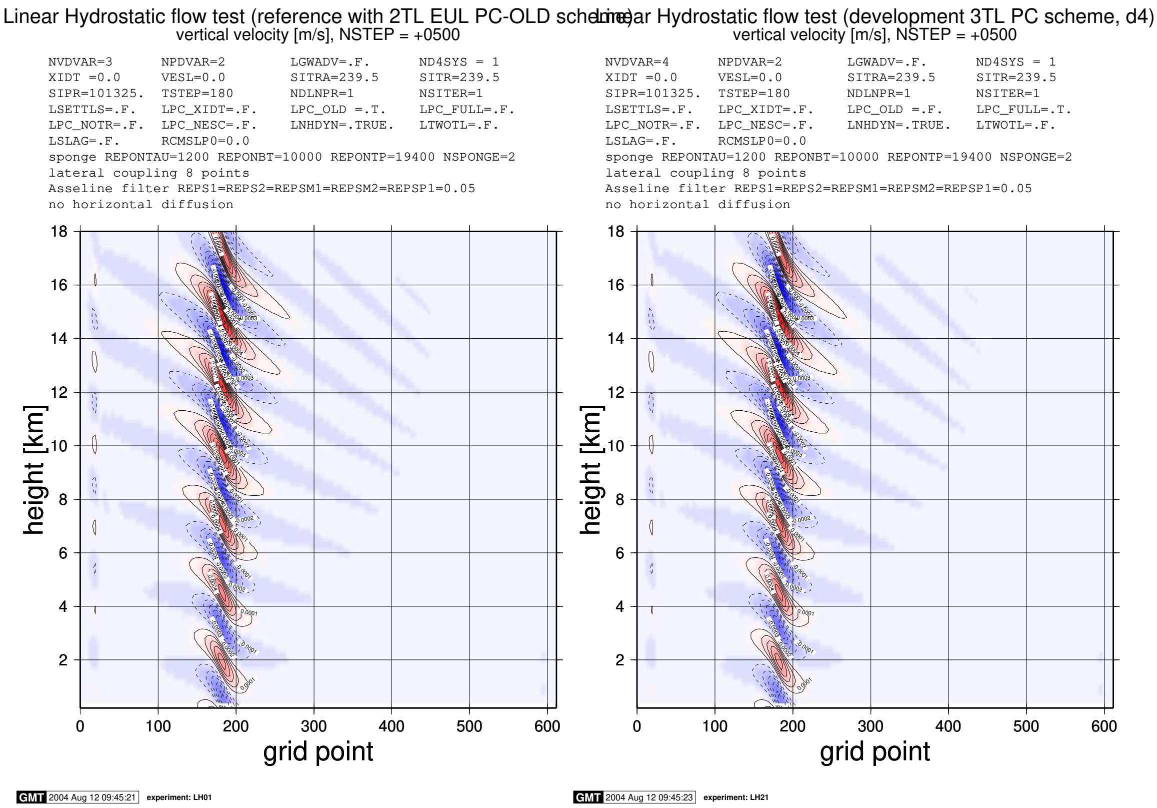

Figure 1: LIHY: the state after the 500 time steps with $\Delta t = 180s$. The vertical velocity is plotted with step $0.0001ms^{-1}$. The results on the left picture are obtained with the LPC_OLD scheme with 1 iteration and d_3 and on the right with the 3TL EUL ICI scheme with 1 iteration with d_4 prognostic variable.

min: -0.001124
max: 0.00117
step: 0.0001

min: -0.001125
max: 0.00117
step: 0.0001

5.2 Non-linear Non-hydrostatic Flow regime

This is the idealized case which mimics the forcing at the scales as it appears in the real cases. It is non-linear regime (very close to wave breaking) and the non-hydrostatic part of the model is very active. The conclusions drawn for this case are usually valid for the real cases as well.

The setting of experiment are taken from Masek (NLNH2 test from LACE reports)

Variable	Value
Height of Agnesi hill	$1000m$
Half width of Agnesi hill	$1000m$
Surface pressure	$101325Pa$
Horizontal wind	$10ms^{-1}$
Surface temperature	$293K$
Brunt-Vaisala frequency	$0.01s^{-1}$
NDGL, NDGUX (quadratic trunc)	128
NFLEVG	100
EDELX	$200m$
Position of middle of the hill	$6400m$
REPONTAU	$200s$
REPONTP	$29500m$
REPONBT	$12000m$

The results (figure 2 and 3) obtained with 3TL ICI EUL scheme applied in this test can be concluded as following

- with d_3 variable the test is stable only if we use 3 iterations. The SITRA variable doesn't help in this case, because the orography forcing is the source of instability.
- with d_4 variable and no horizontal diffusion the results are very noisy independently of ND4SYS.
- with d_4 variable and horizontal diffusion the results are accurate and the scheme is stable with 1 iteration.

The experiments were run with $SITRA = 100K$ and $SITR = 350K$.

5.3 ALPIA 3D pseudo-idealized case with physical forcing

With this test we validated the physical-dynamical interface.

This is idealized humid atmosphere above the real orography of Alps. The setup of idealized atmosphere is the same as for SCANIA experiment (Brozkova). The data were provided by Stadlbacher. The horizontal resolution of the test is 5km.

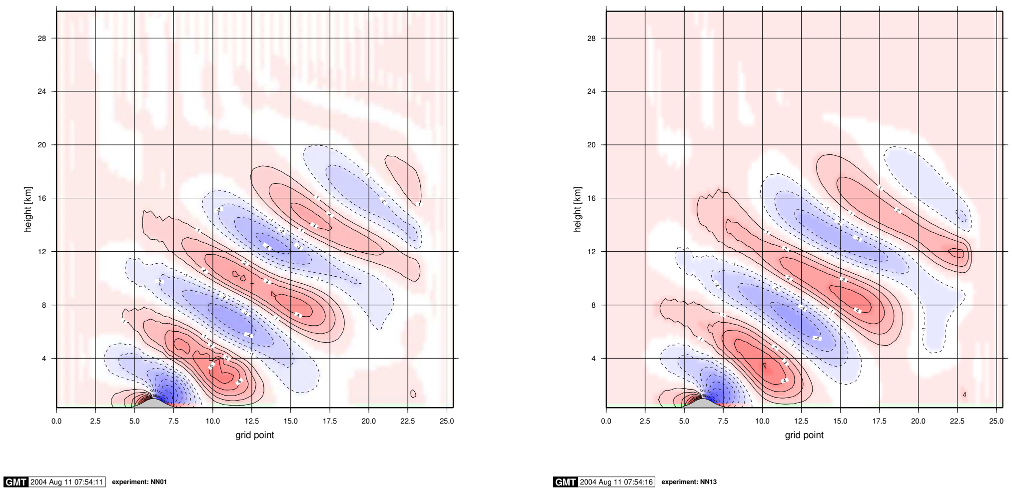

Figure 2: NLNH: the state after the 2000 time steps with $\Delta t = 2.5s$. The vertical velocity is plotted with step $1ms^{-1}$. (left) LPC_OLD scheme with 1 iteration and d_3 (right) 3TL EUL ICI scheme with 3 iteration with d_3. Experiments with 3TL EUL ICI scheme and d_3 with 1 and 2 iterations were unstable

We tested 3TL EUL ICI scheme with the physics with $\Delta t = 40s$. It was found that in this environment, the scheme with $ND4SYS = 1$ or 2 was unstable. Therefore we implemented next option $ND4SYS = 5$ according (12). This option is available only for Eulerian scheme, because $ND4SYS = 1$ or 2 were found stable for SL schemes.

On the figure 4 are shown 12h cumulated precipitation (ranges +0 - +12h). The 2TL SL SETTLS ICI scheme on the left was run with $\Delta t = 80s$, while the 3TL ICI EUL scheme was run with $\Delta t = 40s$.

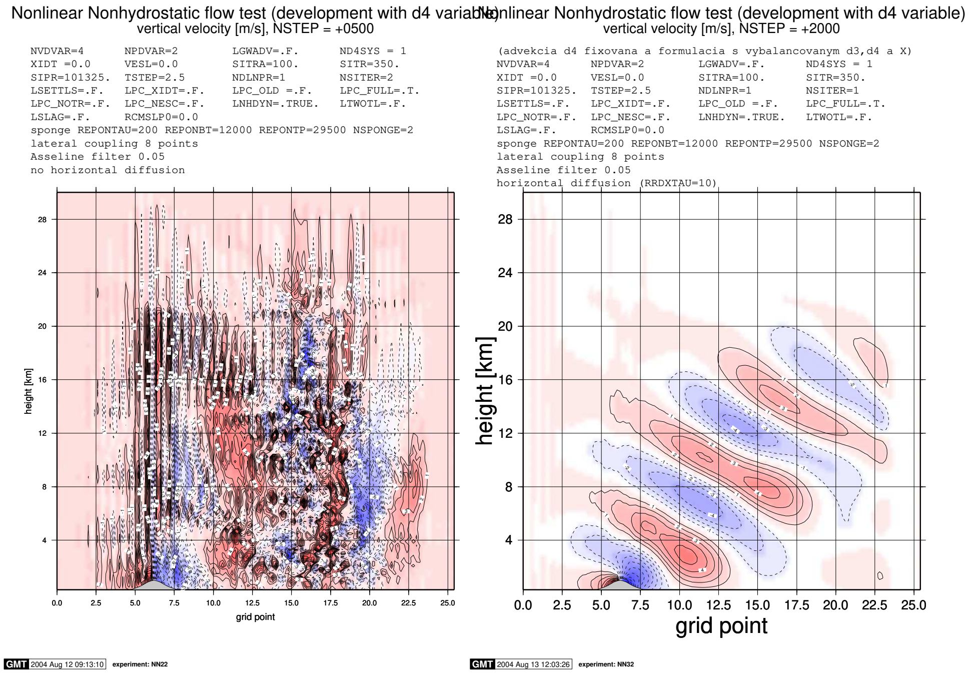

Figure 3: NLNH: the state after the 2000 time steps with $\Delta t = 2.5s$. The vertical velocity is plotted with step $1ms^{-1}$. (left) 3TL EUL ICI scheme with d_4 and no diffusion gives very noisy solution (tested with 1,2 and 3 iterations) (right) 3TL EUL ICI scheme with 1 iteration with d_4 with additional horizontal diffusion

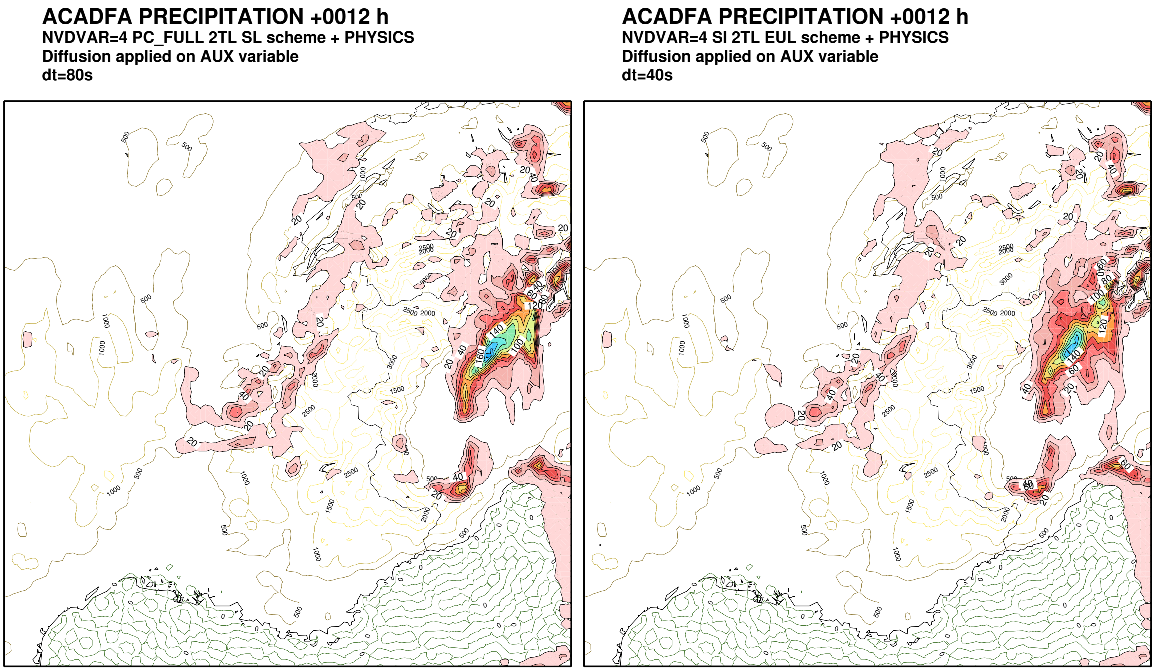

Figure 4: PHYS: Cumulated 12h precipitation since the start of the integration. Results were obtained with (left) 2TL SL ICI SETTLS scheme with d_4 and $\Delta t = 80s$ (right) 3TL EUL ICI scheme with 1 iteration with d_4 and $\Delta t = 80s$.