Part V
Moist downdraught computation

1 General context

1.1 Principles

The downdraught starts following the local cooling induced by the preciptation flux. Precipitation can cool the environment in three ways:

- by taking the local temperature;
- by evaporating;
- by melting.

The most efficient effect is melting, but it is localized in a shallow layer below the triple-point-temperature isotherm. Else precipitation evaporation is the main process.
For the downdraught to remain subsaturated (and capable of evaporating something), the parametrization assumes that no condensate resides in it. So precipitation must be at the border of the downdraughts. Its fall velocity should be different from the air in the immediate environment, to allow the evacuation of the water vapour inducing further evaporation.
The falling drop is surrounded by air moving relatively to it in opposite direction to replace the mass above it. This air being cooled by the evaporation process may then feed a downdraught.
At the beginning, there is no downdraught, precipitation falls faster than the air in its environment. Gradually, the vertical velocity of the air around the precipitation becomes downwards-oriented and the precipitation accelerates, being less braked. Evaporation diminishes slightly.
The return current is then concentrated a little farther but it can still be in the precipitation area. There precipitation is slowed down while the evaporation is still important.
The downdraught might be then deplaced to this area, and so on. Practically, the precipitation is not homogeneous horizontally, so it is where the precipitation is the most intense that the downdraught will settle (under the condition there is an supply of dry air), while the less intense precipitation around will be in the return current and disappear below some level.
This means that the precipitation fall velocity is increased by the downdraught, but also that the precipitation area would be reduced downwards.
Dry air could come for above, possibly above the cloud, or be entrained at the same level. The supply of dry air could be a limiting factor, as was the moisture convergence for the updraught: a kind of dryness convergence towards the downdraught core.
But the adiabatic sinking of unsaturated air causes its temperature to raise, reducing the saturation and allowing further evaporation: the limiting factor is maintaining the negative buoyancy of this air with respect to the environment. This depends of the environment virtual temperature profile.
It seems that at the interface between the downdraught and the precipitation, the latter should fall faster. If the downdraught has a core of dryer air, this will become warmer than the borders, hence a smaller sink velocity so that the downdraught will split in several others to maximize the interface with the precipitation.

1.2 Downdraught environment

The downdraught occurs within the precipitation area. This is itself outside the updraught mesh fraction, in the so-called 'updraught environment'. We wrote for the updraught:

$$\psi_e = \frac{\overline{\psi} - \sigma_u \psi_u}{1 - \sigma_u} \tag{1}$$

For the vertical velocity, this distinction is essential:

$$\omega_e = \frac{\overline{\omega} - \sigma_u \omega_u}{1 - \sigma_u} \tag{2}$$

because this vertical veocity can be close to zero, while the updraught makes that the mean grid box $\overline{\omega}$] takes significant values.

If we consider areas of different properties: updraught, downdraught and environment:

$$\psi'_e = \frac{\overline{\psi} - \sigma_u \psi_u - \sigma_d \psi_d}{1 - \sigma_u - \sigma_d} \tag{3}$$

We need to estimate which is the velocity of the air in the immediate environment of the downdraught, because this air is entrained in the downdraught and must be accelerated to ω_d. Air entrained into the downdraught must have a smaller velocity than the precipitation, to allow its further evaporation. It seems reasonable to assume its velocity be ω_e, e.g. to be at rest while the precipitation falls across it.
The properties of the air which is entrained into the downdraught and of the air around it, are close to wet bulb conditions. The original temperature and moisture should be those outside the updraught and outside the cloud (we are below the cloud, the case of penetrative downdraught from the top of the clouds should be added later).
If σ_u becomes big (e.g. > 0.5), the mean values ψ_e in the environment can depart significantly from the mean grid-box values $\overline{\psi}$.
This distinction is presently not done, and while the MaC package includes the evolution of internal variables, updated by each parametrization, they still represent mean grid box values. For the microphysics we simply assumed that the cloud condensates are concentrated over a the "equivalent mesh fraction", while the mean grid box values of $\overline{q}$, $\overline{T}$, $\overline{T_w}$, $\overline{q_w}$,... were used.
If we really want to take into account significant mesh fractions σ_u, we should update these variables after accvud using Eq. 1.
Seen the updraught is warmer and moister than its environment, taking this into account would yield $T_e < \overline{T}$, $q_e < \overline{q}$ hence the downdraught negative buoyancy would be reduced.
The values passed to the downdraught are presently mean grid-box values, it could be more accurate to use updraught-environment values.

2 Initial calculation and start conditions

The advected prognostic variables:

- PDDOM $= \omega_d$ (the absolute downdraught velocity) is multiplied by $\triangle t$. The resulting value is compared to the routine argument POME $= \omega_e \triangle t$ corresponding to the updraught environment. We impose that

$$\omega_d \geq \max(0, \omega_e)$$

- The positive values of PDDAL are averaged over the vertical, to yield ZSIG9 $= \sigma_d$.

The microphysical variables passed to the routines are noted T, q, q_i, q_ℓ.
The downdraught occurs in the neighbourhood of precipitating condensed water, we assume that the conditions there correspond to the blue point of the environment – or of the mean grid box:

$$T_e = \overline{T}_w \quad , \qquad q_e = \overline{q}_w \tag{4}$$

These conditions are simply generated by the variability inside the grid box, and do not require to modify the mean values, or the local condensate content. To start the downdraught, we may simply take these same values as starting point for a descent.
The ice fraction for evaporation should not be the one computed using fonice (which concerns condensation), but well the phase partition of the precipitation, $\alpha^{\bar{l}}_{\text{snow}} =$ ZSNP$^{\bar{l}}$ returned by acsnp (alaro-0 version) or passed in argument PZSNP (MaC version). A more accurate estimation of α_{snow} can be done by considering the actual precipitation phase partition.
The local latent heat is then

$$L(\overline{T}^l_w, \alpha^l_{\text{snow}})$$

which supposes to interpolate α_{snow} to the full levels.

It would be preferable to diagnose α_{snow} from the actual input precipitation fluxes instead of the temperature profile as in acsnp

The dry static energy of the environment is used at the end of the routine to compute th transport flux of dry static energy by the downdraught. For this, we take the mean grid box value:

$$s_e = c_p \overline{T} + \phi$$

3 Evaporating descent

- the downdraught values are initialized to the blue point at the top level: $T_d^1 = \overline{T_w}^1$, $T_d^1 = \overline{q_w}^1$.
- The total precipitation from the microphysics is stored in array ZFP. In the downwards loop, we store into the array ZFEVP the (positive) moisture increments associated to the evaporation at the interface levels:

$$\triangle q_{ev}^{\bar{l}} = (q_d^l - q_d^b) \cdot \delta_{\text{act}}^l$$

The available precipitation at current level is the difference between the initial precipitation flux and the downdraught-evaporation flux, accumulated from the top level. To evaluate this, we would need to know the downdraught mass flux, i.e; σ_d. At this time, we only have the value σ_d^- advected from the previous time step. So we use a first estimation:

$$\mathsf{ZFEVP}^{\overline{l-1}} = \frac{1}{g\triangle t} \sum_{k=1}^{k=l-1} \triangle q_{ev}^{\bar{k}} \sigma_d^k \omega_d^k$$

And the available precipitation is

$$\mathsf{ZFPO}^{\overline{l-1}} = \mathsf{ZFP}^{\overline{l-1}} - \mathsf{ZFPE}^{\overline{l-1}}$$

This quantity is interpolated to the full levels, yielding the value $\mathcal{P}^{l-1} = \mathsf{ZFPB}$. The availability of precipitation to evaporate from level l-1 to level l is then expressed by the index δ_{av}:

$$\delta_{\text{av}} \equiv \mathsf{INAV} = \begin{cases} 1 & \text{if } \mathsf{ZFPB} > \epsilon \\ 0 & \text{otherwise} \end{cases}$$

- The starting point ψ_b of a descent segment, is obtained by mixing the downdraught properties ψ_d^{l-1} from the previous descent with the environment (entrainment process).
The entrainment coefficient variable ZENTRD(JLON) $\equiv \lambda_d$ is initialized to the namelist parameter TENTRD, and stays constant.
It could be made dependent of the location (as in the earlier scheme it was taken from the local updraught) - and also be reduced along the downwards loop following insufficient precipitation availability...

The non dimensional entrainment coefficient ξ may be expressed in the same way as for the updraught:

$$\mathsf{ZRMIX}^{\overline{l-1}} \equiv \xi'^{\overline{l-1}} = \lambda_d(\phi^{l-1} - \phi^l) = \frac{\triangle M_d}{M_d} = \frac{E_d \triangle p}{M_d} > 0$$

If λ_d is constant over the vertical, the increase of the mass flux due to entrainment is proportional to the layer thickness and the mass flux.

The entrainment is applied at once at l-1 *before* the descent, but the physical process is distributed from l-1$\rightarrow l$, so we must consider $\xi^{\overline{l-1}}$:

$$\psi_d^b = \psi_d^{l-1} + \xi'^{\overline{l-1}}(\psi_e^{l-1} - \psi_d^b) \quad \Longrightarrow \psi_d^b = \psi_d^{l-1} + \xi^{\overline{l-1}}(\psi_e^{l-1} - \psi_d^{l-1}) \qquad , \qquad \mathsf{ZMIX} \equiv \xi = \frac{\xi'}{1+\xi'}$$

For the environment values we take the blue point $(\overline{T_w}, \overline{q_w})$.

- If $\delta_{av} = 0$, we assume no entrainment hence $\xi = 0$. Since there is no condensate available, it is useless to compute a saturated descent: instead, a dry adiabat is followed, with $q_d^l = q_d^b$.
- Otherwise, a saturated descent is followed from level $b = l - 1$ to l, the iterative process of the Newton loop yields successive ZDELQ $= q^{k+1} - q^k$.
The calculation is slightly alleviated by the fact that the phase partition δ no longer varies along the Newton loop. We consider the phase of the precipitation at the interface level between $l-1$ and l, i.e. $\mathsf{PZSNP}^{\overline{l-1}}$.

- The virtual temperature of the downdraught and of the environment are required to estimate buoyancy. Keeping the hypothesis of no remaining condensate at the arrival point,

$$T_{vd} = T_d(1 + \frac{R_v - R_a}{R_a} q_d) \qquad \text{and} \qquad T_{ve} = T_w(1 - (q_i + q_\ell) + \frac{R_v - R_a}{R_a} q_w)$$

  But taking into account the condensate in the environment tends to decrease its temperature – hence reduce the negative buoyancy. In reality, there is all the time some condensate present in the downdraught, and the simplest is to neglect condensate differences between downdraught and environment, and write

$$\ell_e = (q_i + q_\ell), \qquad T_{vd} = T_d(1 - \ell_e + \frac{R_v - R_a}{R_a} q_d) \qquad \text{and} \qquad T'_{ve} = T_w(1 - \ell_e + \frac{R_v - R_a}{R_a} q_w)$$

  This use of cloud condensate is a little far-fetched.

- The downdraught activity ($\mathsf{INACT}^l \equiv \delta^l_{\text{act}} = 1$) is decreed where $\delta^l_{\text{av}} = 1$ *and* at least one of the following conditions is realised:

  – negative buoyancy: $T_{vd} < T'_{ve}$,

  – or downward downdraught velocity as advected from previous time step: $(\omega^l_d)^- > \max(0, \omega^l_e)$,

  – or downward downdraught velocity in the layer above: $\omega^{l-1}_d > \max(0, \omega^{l-1}_e)$.

  The velocity test should concern: $\omega_d > \max(0, \omega_e)$.
  What about and $0 \geq \omega_d > \omega_e$*: envt is rising, while dd is at rest or rising... is that no active downdraught ? – But* $\omega_d < 0$ *complexifies the calculation. Or should we compute on* $\omega_d - \omega_e$ *?*
  What about $\omega_e > 0$*: it means air is subsiding in the whole grid box.*

- If the downdraught is warmer than the blue point of the environment, its properties are reset to this blue point. The buoyancy term ZKUO1 is adapted accordingly.

- The final downdraught properties are stored in arrays $\mathsf{ZTDN} \equiv T_d$, $\mathsf{ZQDN} \equiv q_d$, $\mathsf{ZSDN} \equiv s_d$.
  After this, we can evaluate $\mathsf{ZDELQ} = q^l_d - q^b_d$: *it has to be strictly positive*, else we must reset it together with δ^l_{act} to zero !
  The difference of moist static energy between the downdraught and its environment is (assuming no difference in geopotential)

$$\mathsf{ZHDMHE}^l \equiv h^l_d - h'^l_e = \frac{c_p(q^l_e)(T^l_d - T^l_{we}) + L(T^l_e, \alpha_i)(q^l_d - q^l_{we})}{1 - \sigma'^l_d} \qquad (5)$$

  As we do not yet have σ^l_d (at the full level l), the division is postponed. Also beware that presently we use $\overline{h}$ instead of h_e.

- The prognostic downdraught velocity at the arrival level ω^l_d is then computed (see §4).

- The highest active level, ITOP, is estimated, allowing to limit the subsequent vertical loops.

- For the treatment by connected active segments (C.A.S.) of the momentum profile, a transition array INBAS is calculated as

$$\mathsf{INBAS}^l = \begin{cases} 1 & \text{at the highest active level of each segment} \\ 0 & \text{elsewhere} \end{cases}$$

4 Prognostic velocity profile

Unlike the updraught, we cannot neglect the environment velocity, which is independent of the downdraught velocity. For this reason, we choose the absolute downdraught velocity ω_d for the prognostic model variable. The complete evolution equation for this variable is then

$$\frac{\partial \omega_d}{\partial t} + (\mathbf{V}\cdot\nabla)_\eta \omega_d + \dot{\eta}_d \frac{\partial \pi}{\partial \eta}\frac{\partial \omega_d}{\partial \pi} = \text{source}(\omega_d)$$

$$\frac{\partial \omega_d}{\partial t} + (\mathbf{V}\cdot\nabla)_\eta \omega_d + \dot{\eta} \frac{\partial \pi}{\partial \eta}\frac{\partial \omega_d}{\partial \pi} + \left(\dot{\eta}_d \frac{\partial \pi}{\partial \eta} - \dot{\eta}\frac{\partial \pi}{\partial \eta}\right)\frac{\partial \omega_d}{\partial \pi} = \text{source}(\omega_d) \tag{6}$$

From this equation, the model dynamics sees only

$$\left.\frac{\partial \omega_d}{\partial t}\right|_{\text{dyn}} + (\mathbf{V}\cdot\nabla)_\eta \omega_d + \dot{\eta} \frac{\partial \pi}{\partial \eta}\frac{\partial \omega_d}{\partial \pi} = 0 \tag{7}$$

and treats it by the semi-lagrangian scheme. It represents a simple passive advection, without any source term (same as for humidity, hydrometeors, ozone, etc.).
The physics then solves the remaning part locally, at fixed vertical coordinate:

$$\left.\frac{\partial \omega_d}{\partial t}\right|_{\Phi} + (\dot{\eta}_d - \dot{\eta})\frac{\partial \pi}{\partial \eta}\frac{\partial \omega_d}{\partial \pi} = \text{source}(\omega_d) \tag{8}$$

To express the source term, we assume $\omega_d = -\rho_d g w_d$, $\frac{\partial \rho_d}{\partial t} \approx 0$:

$$\left.\frac{\partial \rho_d w_d}{\partial t}\right|_{\Phi} + (\dot{\eta}_d - \dot{\eta})\frac{\partial \pi}{\partial \eta}\frac{\partial \rho_d w_d}{\partial \pi} = -\frac{1}{g}\text{source}(\omega_d)$$

$$\underline{\left.\frac{\partial w_d}{\partial t}\right|_{\Phi} + (\dot{\eta}_d - \dot{\eta})\frac{\partial \pi}{\partial \eta}\frac{\partial w_d}{\partial \pi}} + (\dot{\eta}_d - \dot{\eta})\frac{\partial \pi}{\partial \eta}\left(\frac{w_d}{\pi} - w_d\frac{\partial \ln T_v}{\partial \pi}\right) = -\frac{1}{\rho g}\text{source}(\omega_d)$$

where the underlined term is equal to the source of the w_d equation. Introducing this in Eq. 8 yields

$$\left.\frac{\partial \omega_d}{\partial t}\right|_{\Phi} + (\dot{\eta}_d - \dot{\eta})\frac{\partial \pi}{\partial \eta}\left(\frac{\partial \omega_d}{\partial \pi} - \frac{\omega_d}{\pi} + \omega_d\frac{\partial \ln T_v}{\partial \pi}\right) = -\rho g \cdot \text{source}(w_d) \tag{9}$$

Similarly to the updraught, we can also consider that

$$(\dot{\eta}_d - \dot{\eta})\frac{\partial \pi}{\partial \eta} \approx (\omega_d - \overline{\omega})$$

4.1 Relative and absolute velocity

The braking effect is normally related to the difference between the downdraught velocity and its immediate environment, which we do not know: it could be an updraught, or a still updraught environment, or an environment moved vertically in a large scale motion. More likely, the downdraught is associated to precipitation, which has a fall speed $w_{\mathcal{P}}$, with respect to the environment. The entrained falling precipitation could pull the downdraught air downwards.
The mean downdraught environment velocity is

$$\omega'_e = \frac{(1-\sigma_u)\omega_e - \sigma_d\omega_d}{1-\sigma_u-\sigma_d} \quad , \qquad \omega_e = \frac{\overline{\omega} - \sigma_u\omega_u}{1-\sigma_u}$$

It needs the downdraught mesh fraction at current level.
Considering a downdraught occurring in the precipitation flux, the difference between the downdraught velocity and its direct environment might be rather

$$\omega_d - \omega_{\mathcal{P}} = \omega_d - (\rho g w_{\mathcal{P}} + \omega'_e) \approx \omega_d - (\rho g w_{\mathcal{P}} + \omega_e) \tag{10}$$

For simplicity, we assimilated here ω'_e to ω_e.
But the expression (10) is quite exaggerated: the falling precipitation itself induces a return upwards air current which is intertwined with it, so that it would be excessive to assume that the entire mass has the fall speed of the precipitation.

Doing so leads to excessive downdraught velocities, as no braking effect may be applied while the mean advected downdraught velocity is below the precipitation fall speed.
We introduced then a possible modulation of the expression of $\omega_\mathcal{P}$, with the parameter GDDWPF, between $\omega_\mathcal{P} = \omega_e$ for GDDWPF = 0 to $\omega_\mathcal{P}$ given by the equation (10) for GDDWPF = 1.
The precipitation flux velocity is obtained assuming – like in the microphysics routines – constant fall speeds for snow ($w_s = 0.9 m/s$) and rain ($w_r = 5 m/s$). We interpolate the array PZSNP to the full levels to determine which of them to use:

$$\mathsf{ZMELTED} = 1 - \frac{\mathsf{PZSNP}^{\overline{l-1}} + \mathsf{PZSNP}^{\bar{l}}}{2}$$

$$\mathsf{ZOMP} \equiv \omega_\mathcal{P} \triangle t = \omega_e \triangle t + \mathsf{GDDWPF} \cdot \frac{p}{R_a T} g \triangle t \left\{ w_r \cdot \mathsf{ZMELTED} + w_s \cdot (1 - \mathsf{ZMELTED}) \right\} \tag{11}$$

Presently, we think that the best choice is to ignore the fall speed and take GDDWPF = 0.

Now the drag term would change its sign, depending on whether the downdraught velocity is bigger or smaller than the "precipitating environment velocity" $\omega_\mathcal{P}$.

- Far from the surface, the case of $\omega_d < \omega_\mathcal{P}$ is not realistic, as the downdraught originates in air that has already the velocity $\omega_\mathcal{P}$: no physical process could speed it down, from this.
- When approaching the surface, the downdraught absolute velocity must go back to zero, while the precipitation fall speed is not affected. At this place, the downdraught is likely to be slowed down below the precipitation velocity.

Anyway, it is clear that a braking effect only occurs where $\omega_d > \omega_\mathcal{P}$. If it is not the case, we will simply neglect the interaction with the surrounding fluid and keep only the buoyancy term.

Introducing these source terms in Eq. 9 yields

$$\begin{aligned} \left.\frac{\partial \omega_d}{\partial t}\right|_\Phi + (\omega_d - \overline{\omega}) \left(\frac{\partial \omega_d}{\partial \pi} - \frac{\omega_d}{\pi} + \omega_d \frac{\partial \ln T_v}{\partial \pi} \right) &= -\frac{g^2}{1+\gamma'} \frac{\pi}{R_a} \frac{T_{vd} - T_{ve}}{T_{ve} T_{vd}} \\ &\quad - \delta_{d\mathcal{P}} \left\{ (\lambda_d + \mathcal{K}_{dd}/g) \frac{R_a T_{vd}}{\pi} \right\} (\omega_d - \omega_\mathcal{P})^2 \end{aligned} \tag{12}$$

where the coefficient

$$\delta_{d\mathcal{P}} = \begin{cases} 1 & \text{where } \omega_d > \omega_\mathcal{P} \\ 0 & \text{elsewhere} \end{cases}$$

4.2 Interaction with the surface

To pass from the material derivative of w to the tendency of ω, we had to consider:

$$w = \frac{dz}{dt} = \left(\frac{\partial z}{\partial t} \right)_\pi + \mathbf{V}_\pi \cdot \nabla_\pi z - \frac{\omega}{\rho g}$$

and neglecting the local tendency of z but also the horizontal advection term associated to the slope difference between the isobaric and the equipotential surfaces, we got

$$w \approx -\frac{\omega}{\rho g}$$

But when the downdraught approaches the surface, the flow has to bend, under the influence of the local high created by the accumulation of air near the surface, and eventually take the horizontal direction. In this case, the isobaric and equigeopotential may no longer be confounded, and the building up of the local high is accompanied by a non negligible tendency of the surface pressure.
The expression of the acceleration also becomes more complicated.

$$\begin{aligned} \frac{dw}{dt} &= \frac{\partial^2 z}{\partial t^2} + \frac{\partial}{\partial t} \{ \mathbf{V} \cdot \nabla z \} + \frac{\partial}{\partial t} \left\{ -\frac{\omega}{\rho g} \right\} \\ &\quad + \mathbf{V} \cdot \nabla \left\{ \frac{\partial z}{\partial t} \right\} + \mathbf{V} \cdot \nabla \{ \mathbf{V} \cdot \nabla z \} + \mathbf{V} \cdot \nabla \left\{ -\frac{\omega}{\rho g} \right\} + \omega \frac{\partial^2 z}{\partial t \partial z} + \omega \frac{\partial}{\partial \pi} \{ \mathbf{V} \cdot \nabla z \} + \omega \frac{\partial}{\partial \pi} \left\{ -\frac{\omega}{\rho g} \right\} \\ &\approx \frac{\partial}{\partial t} \{ \mathbf{V} \cdot \nabla z \} + \mathbf{V} \cdot \nabla \{ \mathbf{V} \cdot \nabla z \} + \mathbf{V} \cdot \nabla \left\{ -\frac{\omega}{\rho g} \right\} + \omega \frac{\partial}{\partial \pi} \{ \mathbf{V} \cdot \nabla z \} + \frac{\partial}{\partial t} \left\{ -\frac{\omega}{\rho g} \right\} + \omega \frac{\partial}{\partial \pi} \left\{ -\frac{\omega}{\rho g} \right\} \end{aligned}$$

if we neglect the transient tendencies. It appears quite difficult to estimate the different contributions in this equation, and we will take a schematic approach considering that the vertical equation (12) may simply be complemented by an additional term representing the effect of the local high situated near the surface: this term has some similarity to the "dynamic pressure" term

$$\frac{\omega_d^2}{\pi} - \frac{1}{2}\frac{\partial \omega_d^2}{\partial \pi}$$

but there the derivative implies a positive contribution to the tendency of ω_d, i.e. a downwards increase. We propose to add a term

$$-\mathsf{GDDDP}\frac{|\omega_d|\omega_d}{(\pi_{\rm surf}-\pi)^\beta} = -\delta_d \frac{\mathsf{GDDDP}}{(\pi_{\rm surf}-\pi)^\beta}\,\omega_d^2 \qquad , \qquad \delta_d = {\rm sign}(\omega_d)$$

whose impact gradually increases as $(\pi_{\rm surf}-\pi)$ decreases when approaching the surface. Presently we use $\beta = 2$, so GDDDP is a reference pressure thickness above the surface for the decrease of the downdraught velocity ($\mathsf{GDDDP} \sim 100$ hPa). Speaking of a downdraught, we should always have $\delta_d = 1$.
Eq. 12 becomes:

$$\begin{aligned}\left.\frac{\partial \omega_d}{\partial t}\right|_\Phi &= -(\omega_d - \overline{\omega})\left(\frac{\partial \omega_d}{\partial \pi} - \frac{\omega_d}{\pi} + \omega_d \frac{\partial \ln T_v}{\partial \pi}\right) \\ &\quad - \frac{g^2}{1+\gamma'}\frac{\pi}{R_a}\frac{T_{vd}-T_{ve}}{T_{ve}T_{vd}} \underbrace{- \delta_{d\mathcal{P}}\left\{(\lambda_d + \mathcal{K}_{dd}/g)\frac{R_a T_{vd}}{\pi}\right\}}_{\equiv \chi}(\omega_d - \omega_{\mathcal{P}})^2 - \frac{\mathsf{GDDDP}\delta_d}{(\pi_{\rm surf}-\pi)^\beta}\,\omega_d^2\end{aligned}$$

where χ is positive where $\omega_d > \omega_{\mathcal{P}}$, zero elsewhere.
The final expression is then

$$\begin{aligned}\left.\frac{\partial \omega_d}{\partial t}\right|_\Phi &= \omega_d^2\left\{\frac{1}{\pi} - \frac{\partial \ln T_v}{\partial \pi} - \chi - \frac{\mathsf{GDDDP}\delta_d}{(\pi_{\rm surf}-\pi)^\beta}\right\} + \omega_d\left\{2\omega_{\mathcal{P}}\chi - \overline{\omega}\left(\frac{1}{\pi} + \frac{\partial \ln T_v}{\partial \pi}\right)\right\} + (\overline{\omega} - \omega_d)\frac{\partial \omega_d}{\partial \pi} \\ &\quad - \omega_{\mathcal{P}}^2\chi - \frac{g^2}{1+\gamma'}\frac{\pi}{R_a}\frac{T_{vd}-T_{ve}}{T_{ve}T_{vd}}\end{aligned} \qquad (13)$$

4.3 Discretization

Eq. 13 may be written as

$$\left.\frac{\partial \omega_d}{\partial t}\right|_\Phi = A\omega_d^2 + B\omega_d + (\overline{\omega} - \omega_d)\frac{\partial \omega_d}{\partial p} + C \qquad (14)$$

with

$$\begin{aligned} A &= \frac{1}{p^l} - \frac{\partial \ln T_v}{\partial p} - \chi - \frac{\mathsf{GDDDP}\delta_d}{(p^{\overline{L}} - p^l)} \\ B &= 2\omega_{\mathcal{P}}\chi - \overline{\omega}\left(\frac{1}{p^l} + \frac{\partial \ln T_v}{\partial p}\right) \\ C &= -\omega_{\mathcal{P}}^2\chi - \frac{g^2}{1+\gamma'}\frac{p^l}{R_a}\frac{T_{vd}-T_{ve}}{T_{ve}T_{vd}} \end{aligned}$$

The discretization of the advection term follows the ideas of i Geleyn *et* al. (1982). Noting

$$F_{l,n} = \omega_d^l \triangle t \qquad \text{for level } l \text{ and time step } n,$$

$$c^{\bar{l}} \equiv \frac{(\overline{\omega}^l - \omega_d^l) + (\overline{\omega}^{l+1} - \omega_d^{l+1})}{2}\triangle t \qquad \text{at time step } n,$$

$$\begin{aligned} F_{l,n+1} - F_{l,n} &= AF_{l,n+1}^2 + BF_{l,n+1}\triangle t + C(\triangle t)^2 \\ &\quad + \frac{1}{p^{\bar{l}} - p^{\overline{l-1}}}\left[-c^{\bar{l}}\left(F_{l,n+1} + \frac{F_{l,n} - F_{l+1,n}}{2}\right) + c^{\overline{l-1}}\left(F_{l-1,n+1} - \frac{F_{l-1,n} - F_{l,n}}{2}\right)\right] \end{aligned}$$

or

$$a^l F_{l,n+1}^2 + b^l F_{l,n+1} + c^l = 0$$

with, neglecting the term in $(\ln T_v)$,

$$a = A \approx \frac{1}{p^l} - \chi - \frac{\mathsf{GDDDP}\delta_d}{(p^{\overline{L}} - p^l)} \qquad \text{must be } < 0$$

$$b = B\triangle t - \frac{c^{\bar{l}}}{\triangle p^l} - 1 \approx -1 - \frac{c^{\bar{l}}}{\triangle p^l} + 2\omega_{\mathcal{P}}\triangle t\chi - \frac{\overline{\omega}\triangle t}{p^l} \qquad \text{must be } < 0$$

$$c = C^l(\triangle t)^2 + F_{l,n} + \frac{1}{\triangle p^l}\Big\{-c^{\bar{l}}\big(\frac{F_{l,n} - F_{l+1,n}}{2}\big) + c^{\overline{l-1}}\big(F_{l-1,n+1} - \frac{F_{l-1,n} - F_{l,n}}{2}\big)\Big\}$$

$$\approx F_{l,n} - \chi\big(\omega_{\mathcal{P}}\triangle t\big)^2 - \frac{g^2}{(1+\gamma')R_a}p^l\Big(\frac{T_{vd} - T_{ve}}{T_{ve}T_{vd}}\Big) + \frac{1}{\triangle p^l}\Big\{-c^{\bar{l}}d^{\bar{l}} + c^{\overline{l-1}}\big(F_{l-1,n+1} - d^{\overline{l-1}}\big)\Big\} \qquad \text{better } > 0$$

with

$$d^{\bar{l}} = \frac{F_{l,n} - F_{l+1,n}}{2}$$

Then

$$F_{l,n+1} = \frac{-b^l - \sqrt{{b^l}^2 - 4a^lc^l}}{2a^l} \qquad \text{which must be } \geq 0$$

A positive value of a must be forbidden. We better retain only the cases where $c > 0$ i.e. the buoyancy term is predominent. In all other cases (or when the square square root does not exist) we reset ω_d to $\max(0, \omega_e)$.

> *must be: to* $\max(0, \omega_e)$ *! because the flux uses* $\omega_d - \omega_e$*.*
> *Moreover, should we set* INND*=0 ?*

The adiabatic descent $b \to l$ yielding T_d^l, q_d^l, and as the entrainment λ_d is taken constant, we can compute directly the downdraught velocity ω_d^+ at level l, which we subsequently use to evaluate the evaporation flux.

5 The downdraught closure

5.1 Principle

To express a closure of the downdraught, we need expressions for

- the "external" driving force, i.e. the input of energy which feeds the downdraught activity;
- the "direct consumption" of this energy by the downdraught (i.e. with no storage);
- the storage of this energy through the increase of the downdraught mesh fraction.

5.1.1 Energy input

The downdraught results form the cooling associated to the falling precipitation.
This cooling results from precipitation evaporation and melting, but also from bringing it to the local layer temperature.
The subroutine cpfhpfsl computes the resulting heat flux $F_{h\mathcal{P}}$ (PZFHP in acmodo), associated to the total precipitation flux PFPFPL + PFPLSN. In its definition, $F_{h\mathcal{P}}$ increases downwards in case of evaporation (or melting), so its downwards increment represents the external heat sink, which may generate the downdraught activity.

It is assumed actually that only a fraction $\varepsilon =$ GDDEVF of the heat sink benefits to the downdraught, because the downdraught area is smaller than the precipitation area (room must be left for an interface between the downdraught and the precipitation). We could think that

$$\mathsf{GDDEVF} \approx \frac{\sigma_d}{\sigma_{\mathcal{P}}}$$

The corresponding parameter GDDEVA of the diagnostic earlier scheme (accvimpd) was tuned to GDDEVA = 0.25. The downdraught must keep a sufficient interface surface, and splits when it becomes too wide; we assume that the return upwards currents take place into the same grid box, and they could also be within the

precipitation area. This limits the downdraught area to only a part, say half of the precipitation area. The value 0.25 would represent a case where the downdraught covers a quarter of the precipitating area. Presently the default value GDDEVF=0.5.
The energy source is

$$\varepsilon \cdot \int_{p_t}^{p_b} -g \frac{\partial F_{h\mathcal{P}}}{\partial p} \frac{dp}{g} \qquad [Wm^{-2}]$$

5.1.2 Consumption

We have seen that the external force driving the downdraught is the cooling, which induces negative buoyancy. It is difficult to conceive, as in the updraught, that water vapour transport would express the consumption of this energy by the downdraught activity. However, this was the original option taken up to now in experimental runs. It provided a kind of closure, because it includes the transport of mass by the downdraught; but the dependency on water vapour gradients is less easy to justify. The expression was then:

$$\int_{p_t}^{p_b} L\sigma_d(\omega_d - \omega_e') \frac{\partial q_e}{\partial p} \frac{dp}{g} \qquad [Wm^{-2}]$$

Actually, this came by symmetry with the updraught closure. The big difference with the updraught, is that the driving force of the updraught was an expternal source of water vapour (the "Moisture Convergence"), which could be supposed to vary slowly compared to the updraught activity. The Kuo closure expressed that all this vapour either converted to condensate (and precipitation) or detrained from the updraught, or contributed to the storage of moist static energy through an increase of the updraught mesh fraction.
Here the external source (the cooling by precipitation) comes from the same grid box, and can vary as fast as the downdraught activity. The downdraught does not react to the moistening by precipitation, but to the cooling: it is not possible to express a balance simply through water vapour. We spoke in the introduction of the supply of dry air as a possible limiting factor, similar to the moisture convergence for downdraught. This seems however quite difficult to estimate.
The cooling induces negative buoyancy: the downdraught "consumes" the energy associated to the temperature difference by creating its mass flux. The prognostic downdraught velocity writes the (im)balance between the negative buoyancy and the braking forces: the difference results in downdraught acceleration. In other words, the energy associated to the creation of buoyancy is either dissipated or used to accelerate the fluid parcels. Noting F_b (N kg^{-1}) the buoyancy force, the consumption of energy is

$$\int_{p_t}^{p_b} F_b \sigma_d \frac{\omega_d}{\rho g} \frac{dp}{g} \qquad [Wm^{-2}]$$

The symmetric of this closure for the updraught, would be a "prognostic cape closure": but for the updraught, the use of moisture convergence is more justified because it is an important limiting factor.

5.1.3 Storage

If the energy input is not equal to the consumption, the variation of storage of energy in the downdraught takes/brings the difference. Energy is stored as moist static energy, but also as kinetic energy, because the air in the additional area must be accelerated. When the mesh fraction decreases, the moist static energy can be consumed by the downdraught, while the kinetic energy is at risk to be lost (?).

$$\frac{\partial \sigma_d}{\partial t} \cdot \int_{p_t}^{p_b} (h_d - h_e') + \frac{\omega_d^2 - \omega_e^2}{2(\rho g)^2} \frac{dp}{g}$$

where h_e' is the moist static energy of the downdraught environment.

5.1.4 Prognostic equation

With a constant mesh fraction over the vertical (which seems very acceptable for the downdraught), the prognostic closure equation is written

$$\underbrace{\frac{\partial \sigma_d}{\partial t}\cdot\int_{p_t}^{p_b}(h_d-h_e')+\frac{\omega_d^2-\omega_e^2}{2(\rho g)^2}\frac{dp}{g}}_{\text{storage}}=\underbrace{\sigma_d\int_{p_t}^{p_b}F_b\frac{\omega_d}{\rho g}\frac{dp}{g}}_{-\text{consumption}}+\underbrace{\varepsilon\cdot\int_{p_t}^{p_b}-g\frac{\partial F_{h\mathcal{P}}}{\partial p}\frac{dp}{g}}_{input} \tag{15}$$

5.2 Implementation

The mesh fraction is obtained by discretizing equation (15).
We compute:

$$\begin{aligned}
\mathsf{ZFB} &= -\frac{g^2}{(1+\gamma')R_a}p^l\Big(\frac{T_{vd}-T_{ve}}{T_{ve}T_{vd}}\Big) & \\
\mathsf{ZS2} &= \sum_1^L \delta_{\rm act}^l(\mathsf{PFHP}^{l-1}-\mathsf{PFHP}^l)\triangle t & \leq 0\\
\mathsf{ZS3} &= \sum_1^L \delta_{\rm act}^l\cdot\frac{\mathsf{ZFB}^l\cdot(\omega_d^l\triangle t)}{(\rho g\triangle t)^2}\frac{\triangle p}{g} & >0\\
\mathsf{ZS4} &= \sum_1^L \delta_{\rm act}^l\mathsf{ZHCMHE}^l\cdot\frac{\triangle p}{g} & \leq 0\\
\mathsf{ZS10} &= \sum_1^L \delta_{\rm act}^l\frac{(\omega_d\triangle t)^2-(\omega_e\triangle t)^2)}{2(\rho g\triangle t)^2}\frac{\triangle p}{g} & > \text{ or } <0
\end{aligned}$$

yielding

$$\mathsf{ZSIG}\equiv\sigma_d^+=\frac{\sigma_d^-(\mathsf{ZS4}+\mathsf{ZS10})+\mathsf{GDDEVF}\cdot\triangle t\cdot\mathsf{ZS2}}{\mathsf{ZS4}+\mathsf{ZS10}-\mathsf{ZS3}\triangle t} \tag{16}$$

If ZS3 is smaller than $1.E-12$, we reset the mesh fraction to zero, since it would mean there is no sufficient downdraught activity.
After this, ZSIG is copied to all active levels in PDDAL. The downdraught relative mass flux at the interface levels is calculated as

$$\mathsf{ZFORM}^{\bar l}=\left\{\sigma_d^l(\omega_d^l-\omega_e^l)\triangle t+\sigma_d^{l+1}(\omega_d^{l+1}-\omega_e^{l+1})\triangle t\right\}/2$$

then it is protected against nonlinear instability.
In the tests, we added a parameter to limit the decrease of the downdraught mass flux between consecutive levels:

$$F^{\bar l'}=\max\big(F^{\bar l},F^{\overline{l-1}'}-\mathsf{GDDFXM}\triangle p^l\big)$$

By default, we set GDDFXM=1.E5 so that this limitation is inoperant.
The old formulation of the protection against non-linear instability was:

$$F^{\overline{l-1}'}=\max\Big(0,F^{\bar l'}+(F^{\overline{l-1}}-F^{\bar l'})\frac{(1+\frac{F^{\bar l'}}{\triangle p^l})}{1+\frac{|F^{\overline{l-1}}-F^{\bar l'}|}{\triangle p^l}}\Big)$$

This has been replaced a few years ago, by:

$$F^{\bar l'}=\max\Big(0,F^{\overline{l-1}'}+\frac{(F^{\bar l}-F^{\overline{l-1}'})}{1+\frac{\max(0,F^{\bar l}-F^{\overline{l-1}'})}{\triangle p^l}}\Big) \tag{17}$$

6 Horizontal momentum profile and vertical budget

The downdraught momentum profile is expressed as

$$\mathbf{V}_d^l = \beta^l \mathbf{V}_d^t + (1 - \beta^l)\widehat{\mathbf{V}}^l$$

where t is the top of the downdraught. We use:

$$(\mathsf{ZUM}, \mathsf{ZVM}) \equiv \widehat{\mathbf{V}}, \qquad \mathsf{ZBET} \equiv \beta, \qquad (\mathsf{ZA13}, \mathsf{ZA14}) \equiv \mathbf{V}_d^t$$

There is no change in the expressions of β and $\widehat{\mathbf{V}}$.

- *In inactive layers,* $\mathsf{ZBET} = 0$ *and* $(\mathsf{ZUM}, \mathsf{ZVM}) = (\mathsf{PU}, \mathsf{PV})$.
- *At the top of each connected active layer (*INBAS=1*),* $\mathsf{ZBET} = 1$ *and* $(\mathsf{ZUM}, \mathsf{ZVM}) = (\mathsf{PU}, \mathsf{PV})$.
- *In the active layers below the top:*

$$\begin{aligned}
\mathsf{ZBET}^l &= \mathsf{ZBET}^{l-1} \cdot (1 - \xi'^l) \\
(\mathsf{ZUM}, \mathsf{ZVM})^l &= \widehat{\mathbf{V}}^l = \widehat{\mathbf{V}}^{l-1} + \xi'^l(\mathbf{V}_e^{l-1} - \widehat{\mathbf{V}}^{l-1}) + \mathcal{G}_d(\mathbf{V}_e^l - \mathbf{V}_e^{l-1})
\end{aligned}$$

At the top of the downdraught and in all inactive layers, the "downdraught" velocity is set equal to the environment velocity.
After this, the final velocity profile is stored into

$$(\mathsf{ZUM}, \mathsf{ZVM})^l = \mathbf{V}_d^l = \beta^l (\mathsf{ZA13}, \mathsf{ZA14})^l + (1 - \beta^l)(\mathsf{ZUM}, \mathsf{ZVM})^l$$

7 Output fluxes and variables

7.1 Downdraught diffusion fluxes

7.1.1 Explicit calculation

This is done for tests, setting LLEXPL=.TRUE.. Noting $\mathsf{ZF}^{\bar{l}} = \mathsf{ZFORM}^{\bar{l}}/(g\triangle t) = (\sigma_d \omega_d)^{\bar{l}}/g$,

$$\begin{aligned}
(\mathsf{ZSTRCUD}, \mathsf{ZSTRCVD}) \equiv J_V^{\mathrm{dd}\,\bar{l}} &= \mathsf{ZF}^{\bar{l}} \frac{(\mathbf{V}_d^l + \mathbf{V}_d^{l+1}) - (\mathbf{V}_e^l + \mathbf{V}_e^{l+1})}{2} \\
(\mathsf{ZDIFCQD}) \equiv J_q^{\mathrm{dd}\,\bar{l}} &= \mathsf{ZF}^{\bar{l}} \frac{(q_d^l + q_d^{l+1}) - (q^l + q^{l+1})}{2} \\
(\mathsf{ZDIFCQID}) \equiv J_{qi}^{\mathrm{dd}\,\bar{l}} &= \mathsf{ZF}^{\bar{l}} \frac{(q_{id}^l + q_{id}^{l+1}) - (q_i^l + q_i^{l+1})}{2} \\
(\mathsf{ZDIFCQLD}) \equiv J_{q\ell}^{\mathrm{dd}\,\bar{l}} &= \mathsf{ZF}^{\bar{l}} \frac{(q_{\ell d}^l + q_{\ell d}^{l+1}) - (q_\ell^l + q_\ell^{l+1})}{2} \\
(\mathsf{ZDIFCSD}) \equiv J_s^{\mathrm{conv}\,\bar{l}} &= \mathsf{ZF}^{\bar{l}} \frac{(s_d^l + s_d^{l+1}) - (s_e^l + s_e^{l+1})}{2}
\end{aligned}$$

7.1.2 Implicit calculation

This is the normal treatment, to ensure stability. The principle has been explained in part IV, for the updraught. Here it is written:

$$\mathsf{zaux} = \frac{(\sigma_d \omega_d \triangle t^{\bar{l}})}{\triangle p^l + (\sigma_d \omega_d \triangle t)^{\bar{l}}}$$

$$\mathsf{ZDIFCQD}^{\bar{l}} = \mathsf{zaux}\{\mathsf{ZDIFCQD}^{\overline{l-1}} + \frac{\triangle p^l}{2g\triangle t}(\frac{(q_d^l - q^l) + (q_d^{l+1} - q^{l+1})}{2})\}$$

$$\mathsf{ZDIFCQID}^{\bar{l}} = \mathsf{zaux}\{\mathsf{ZDIFCQID}^{\overline{l-1}} - \frac{\triangle p^l}{2g\triangle t}(\frac{(q_i^l) + (q_i^{l+1})}{2})\}$$

$$\mathsf{ZDIFCQLD}^{\bar{l}} = \mathsf{zaux}\{\mathsf{ZDIFCQLD}^{\overline{l-1}} - \frac{\triangle p^l}{2g\triangle t}(\frac{(q_\ell^l) + (q_\ell^{l+1})}{2})\}$$

$$(\mathsf{ZSTRCUD}, \mathsf{ZSTRCVD})^{\bar{l}} = \mathsf{zaux}\{(\mathsf{ZSTRCUD}, \mathsf{ZSTRCVD})^{\overline{l-1}} + \frac{\triangle p^l}{2g\triangle t}(\frac{(\mathbf{V}_d^l - \mathbf{V}^l) + (\mathbf{V}_d^{l+1} - \mathbf{V}^{l+1})}{2})\}$$

Check this wrt theory , see updraught !!

7.2 Downdraught deactivation

Setting thThe test switch LLBO to True allows to let the downdraught work in open loop. In this case, INND is set to zero everywhere.
Where INND=0:

- all the fluxes are reset to zero;
- the evaporation increments are set to zero;
- the downdraught velocity and mesh fraction are set to zero.

Is it right to reset both prognostic variables to zero ?

Case of the pseudo-return: if no downdraught activity has been found (ITOP $\geq$ KLEV, not used if NPHYREP $\notin \{0, -1\}$), the flux calculation is skipped, however, we must reset both prognostic variables, to zero.

to zero or ω_e ?

7.3 Precipitation evaporation

While building the profile, positive evaporation increments $\triangle q_{ev}^{\bar{l}}$ at the interface levels were stored into ZFEVP. Now we put in ZFEVP the precipitation evaporation flux:

$$\mathsf{ZFEVP} = \mathcal{E}^{\bar{l}} = \mathcal{E}^{\overline{l-1}} + \frac{\sigma_d \omega_d^{\bar{l}} \triangle t \triangle q_{ev}^{\bar{l}}}{g\triangle t}$$

The flux is then partitioned between snow and rain evaporation:

$$\mathsf{PFESL} = (1 - \alpha_{\mathrm{snow}}) \cdot \mathsf{ZFEVP}, \qquad \mathsf{PFESN} = \mathsf{ZFEVP} - \mathsf{PFESL}$$

In the MaC scheme, we update the water vapour here:

$$q^l = q^l + \frac{\sigma_d \omega_d^l \triangle t \triangle q_{ev}^l}{\triangle p^l}$$

MaC: in the code, ZFORM *used: should be interpolated !!*

7.4 Output values (MaC package)

The microphysical values of q, q_i, q_ℓ are updated after the action of the downdraught diffusion fluxes. When this results into a negative specific content, a correction flux is computed and added to the corresponding turbulent diffusion flux.

$$q^l = q^l + \frac{g\triangle t}{\triangle p^l}(J_q^{dd\overline{l-1}} - J_q^{dd\overline{l}}) \qquad q_i^l = q_i^l + \frac{g\triangle t}{\triangle p^l}(J_{qi}^{dd\overline{l-1}} - J_{qi}^{dd\overline{l}}) \qquad q_\ell^l = q_\ell^l + \frac{g\triangle t}{\triangle p^l}(J_{q\ell}^{dd\overline{l-1}} - J_{q\ell}^{dd\overline{l}})$$

$$(\triangle J_q^{\mathrm{cor}})^l = \frac{\triangle p^l}{g\triangle t}\min(0, q^l) \qquad \text{and} \qquad J_q^{\mathrm{tur}} = J_q^{\mathrm{tur}} + J_q^{\mathrm{cor}}$$

and similarly for q_i and q_ℓ.
We must combine the fluxes associated to the sole downdraught with the contribution of other parameterisations. Therefore we have to split them between liquid and solid, subgrid and resolved. The downdraught worked on the total precipitation flux, and computed a total evaporation/sublimation flux.

$$\mathcal{P} = \max(0, \mathcal{P} + \mathcal{E}) \qquad \mathcal{P}_s = \mathcal{P}\cdot\alpha_{\mathrm{snow}} \qquad \mathcal{P}_r = \mathcal{P} - \mathcal{P}_s$$

Finally, the downdraught diffusion fluxes are added to the convective diffusion fluxes:

$$\begin{aligned} &\mathsf{PDIFCQ} = \mathsf{PDIFCQ} + \mathsf{ZDIFCQD}, && \mathsf{PDIFCS} = \mathsf{PDIFCS} + \mathsf{ZDIFCSD}, \\ &\mathsf{PDIFCQI} = \mathsf{PDIFCQI} + \mathsf{ZDIFCQID}, && \mathsf{PDIFCQL} = \mathsf{PDIFCQL} + \mathsf{ZDIFCQLD}, \\ &\mathsf{PSTRCU} = \mathsf{PSTRCU} + \mathsf{ZSTRCUD}, && \mathsf{PSTRCV} = \mathsf{PSTRCV} + \mathsf{ZSTRCVD} \end{aligned}$$