3D turbulence scheme for NWP models

Filip Váňa

with help from J. Mašek, L. Bengtsson-Sedlar, S. Tijm, I. Bašťák-Ďurán, S. Malardel,...

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ONPP / ČHMÚ - LACE

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- Dissipative - has to be constantly supplied by an energy
- In atmosphere it is responsible for momentum transport and scalar mixing, both several orders of magnitude greater than molecular diffusion

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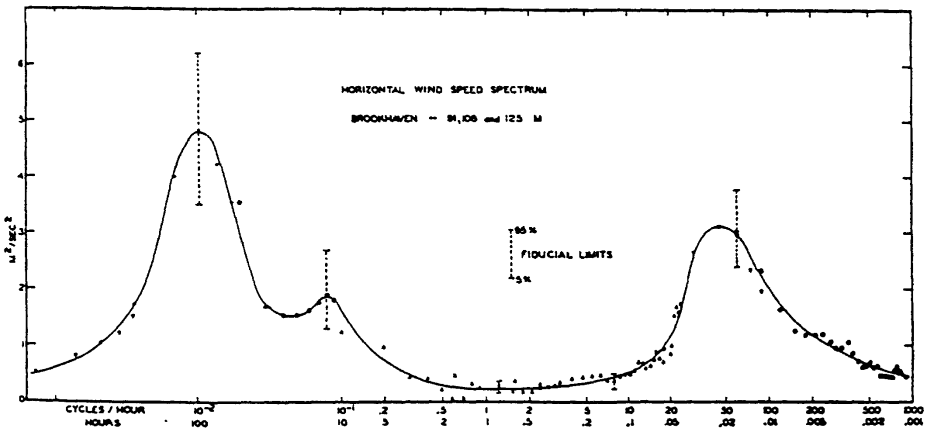
  - Jet-streams, CAT, ...
- Plays role to nearly all effects with timescale shorter than $\approx$ 1 hour



1D versus 3D approach

NWP models: $\Delta x \gg \Delta z$

- $\frac{\partial \Psi}{\partial x} < \frac{\partial \Psi}{\partial z}$
- quasi-horizontal homogenity is assumed ($\Psi = \langle \Psi \rangle_x$)
- horizontal and vertical scales are separated
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$\Rightarrow$ horizontal component can't be neglected - usually treated within the so called horizontal diffusion scheme

Horizontal diffusion in NWP

- Linear (super)diffusion with increased viscosity
  $F_h = K_h \nabla^r \Psi$ with $K_h = \nu + \nu_N =$ const., $r = 2, 4, ..$

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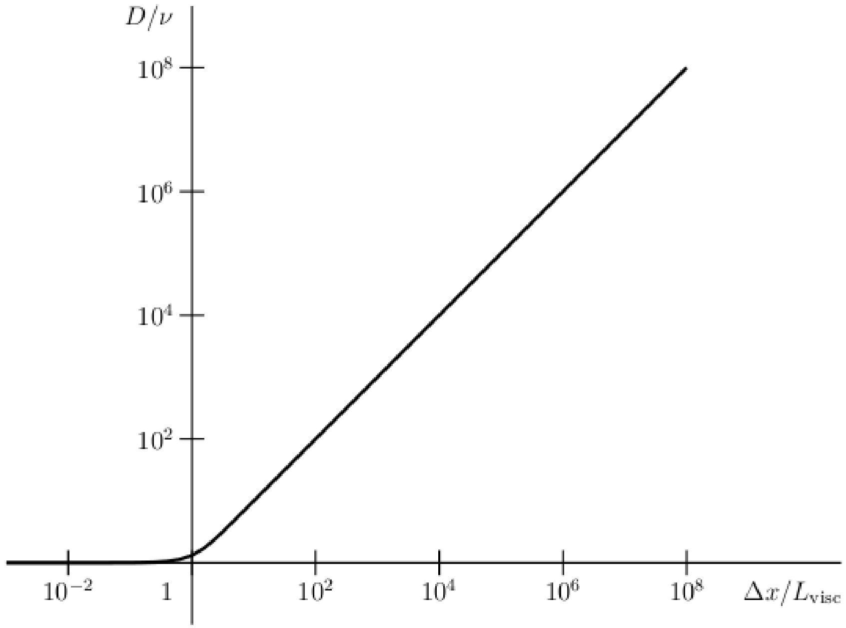

Figure 2: Dependency of numerical diffusion coefficient D on horizontal mesh size Δx for Burger's equation (ν – kinematic viscosity; L_{visc} – viscous length scale).

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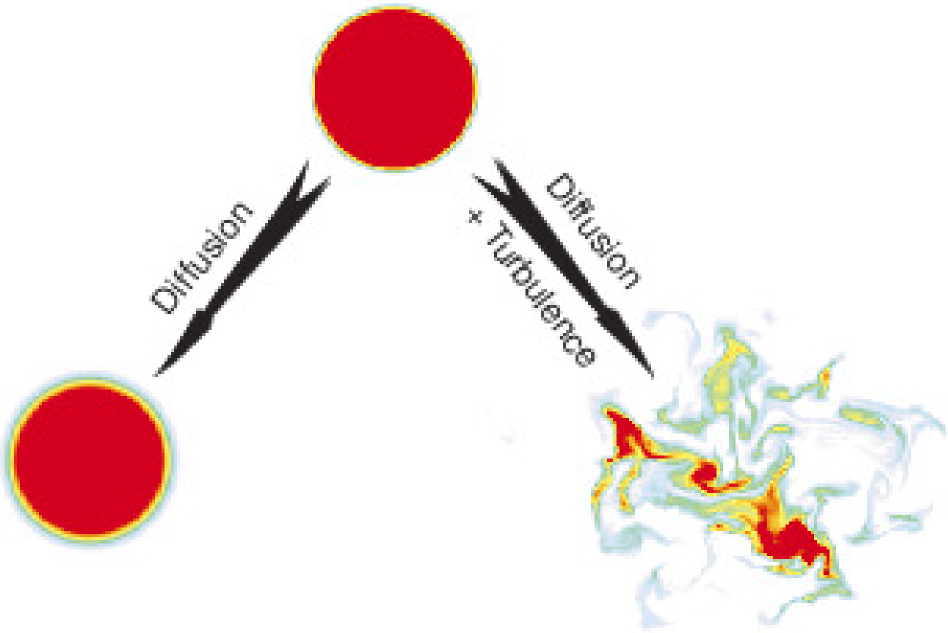

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- Smagorinsky (1963)
  $$F_x = (k_H \Delta)^2 \left(\frac{\partial}{\partial x}(|D| D_T) + \frac{\partial}{\partial y}(|D| D_S) \right)$$
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Not very consistent with the (vertical) turbulence parametrization

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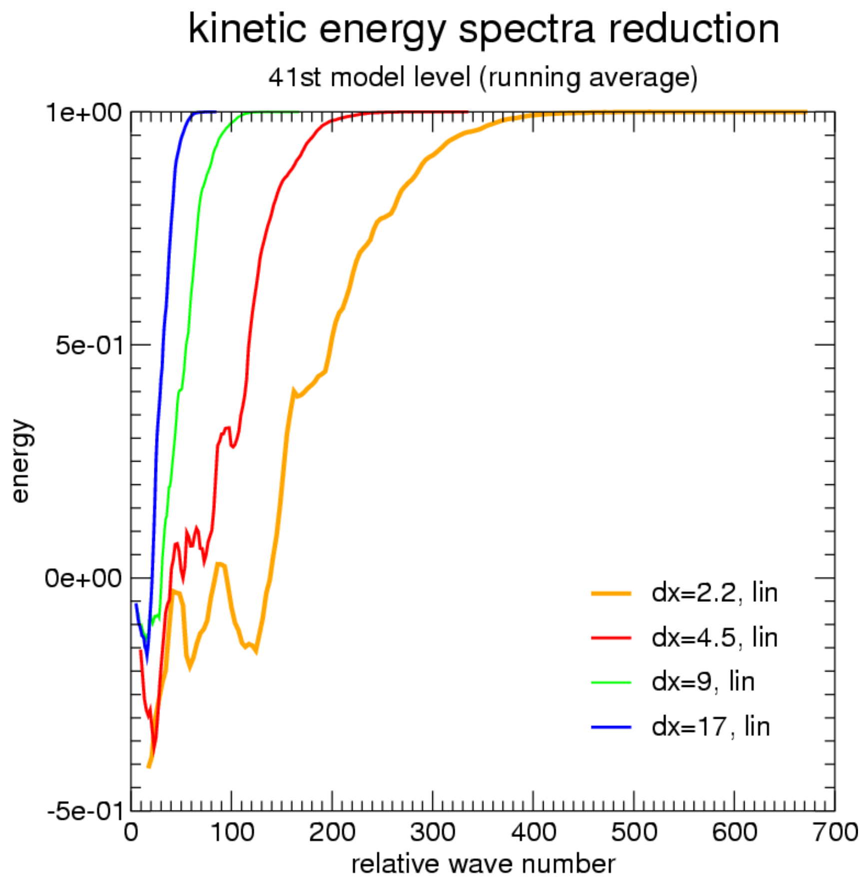

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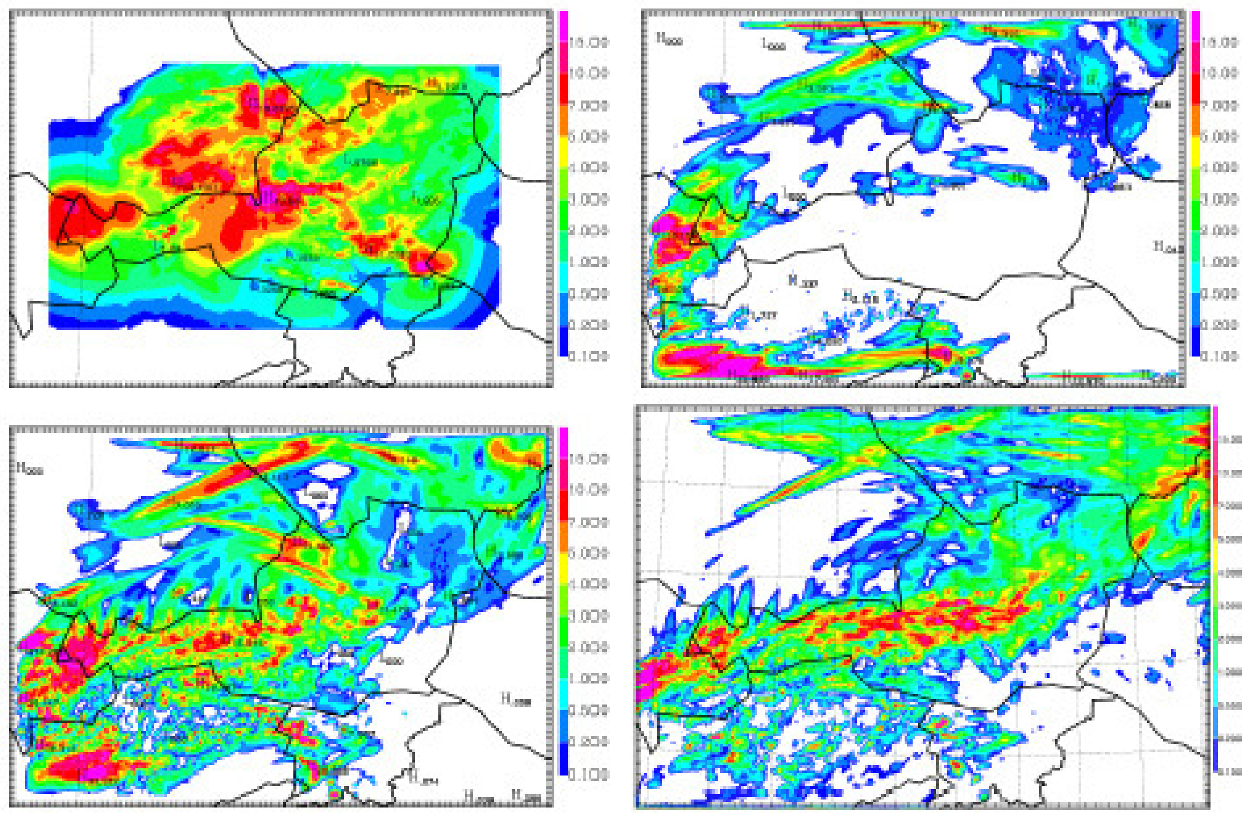
24h accumulated precipitation for the 23/06/2006



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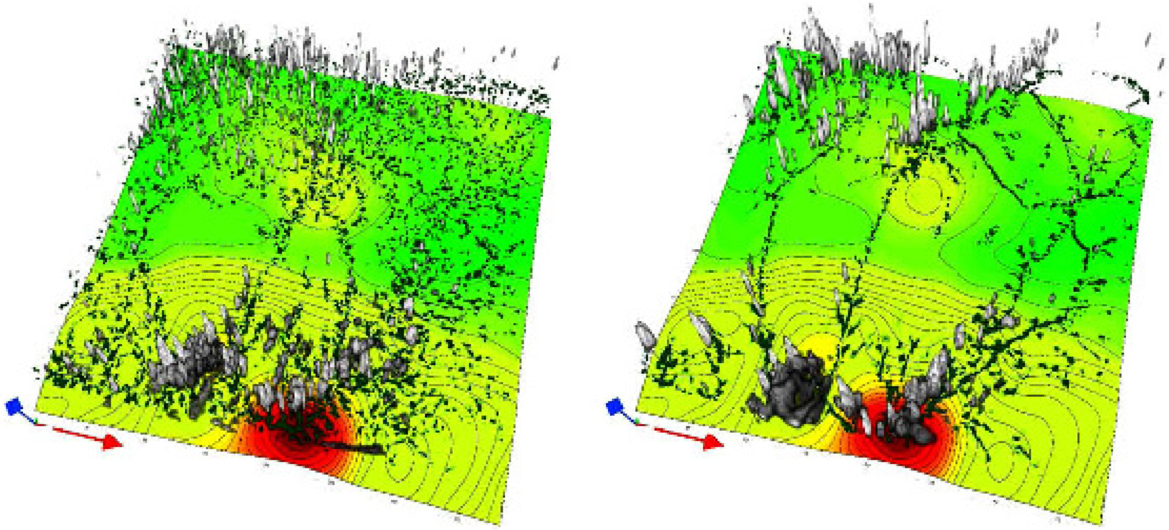

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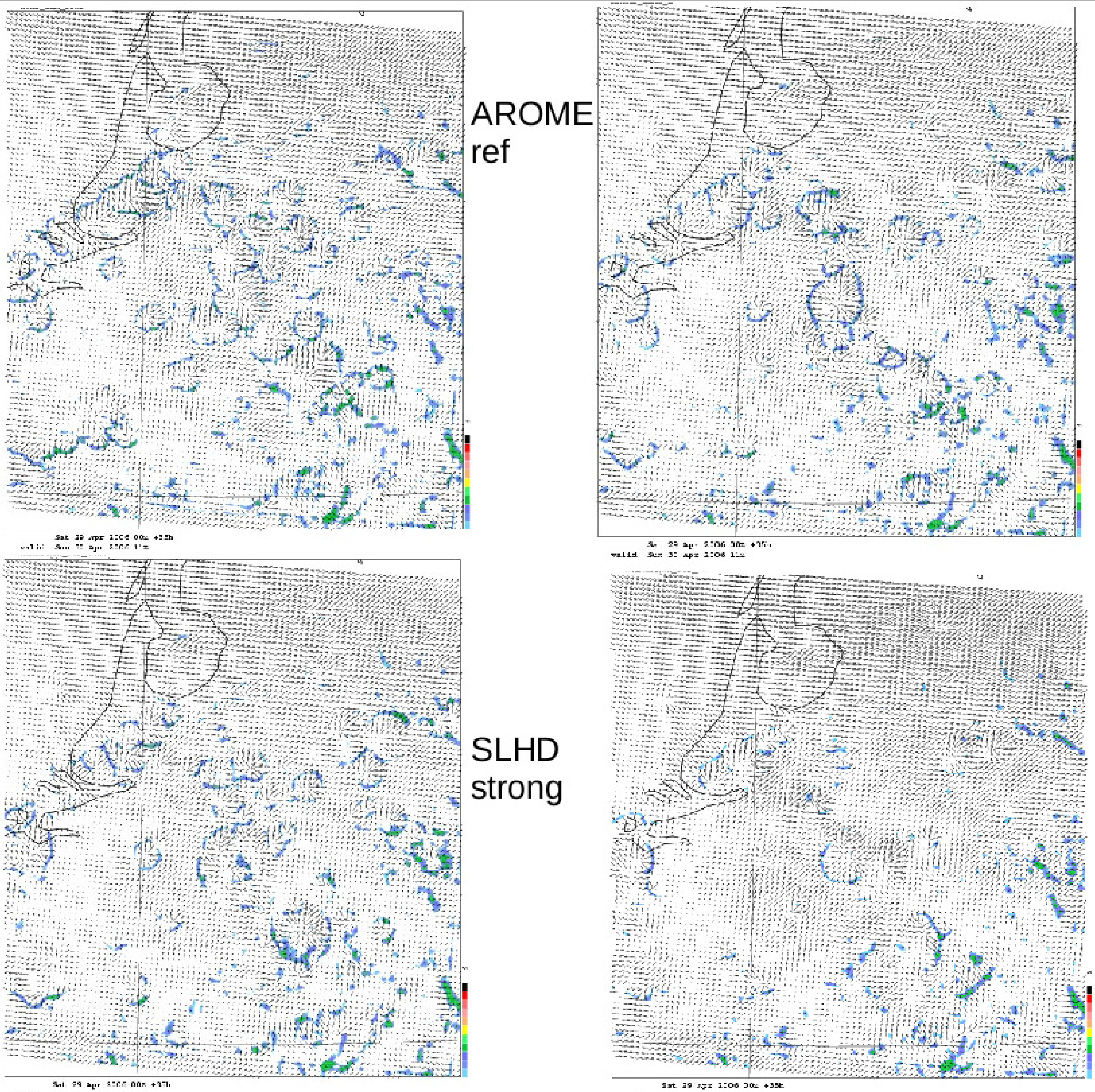
11 UTC

NOHD

AROME ref

SLHD weak

SLHD strong



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- Evidence that simulated convection is strongly related to the model viscosity
- Consistent 3D turbulence helps to reduce too intense precipitation maxims and timing of convection (UKMO experience)
- Deformations based models (i.e. Smagorinsky type) assume only balance between mechanical production and dissipation. What about the buoyancy?

More consistent 3D approach

Three dimensional K-theory

$$\frac{\partial \bar{\Psi}}{\partial t} + \ldots = \boxed{\frac{\partial \tau_{i\Psi}}{\partial x_i}} + \ldots$$

$$\tau_{i\Psi} = \begin{cases} K_m \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) & \text{when} \quad \Psi = u_j \\ K_\Psi \left(\frac{\partial \bar{\Psi}}{\partial x_i} \right) & \text{for any other case.} \end{cases}$$

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$\Rightarrow$ suitable for models with uniform and (truly) high resolution (LES,...)
To make it still applicable to less favorable model geometries (like those used for NWP) some extra work (or assumptions) are required...

3D approaches for NWP models

- Introduce a 1D sub-model
  Phenomenological model describing the 3D turbulence along a 1D line.
  ex: ODT (one dimensional turbulence) model defining $\frac{\partial u_i'}{\partial t} - \nu \frac{\partial^2 u_i'}{\partial x^2} = 0$ with $u_i'(x) = u_i'(f(x)) + c_i K(x)$ representing the influence of eddies reaching the location x and a momentum-conserving modification of the velocity profiles that implements energy transfer among velocity components. The 3D u_i is then corrected with respect of u_i' from the ODT.

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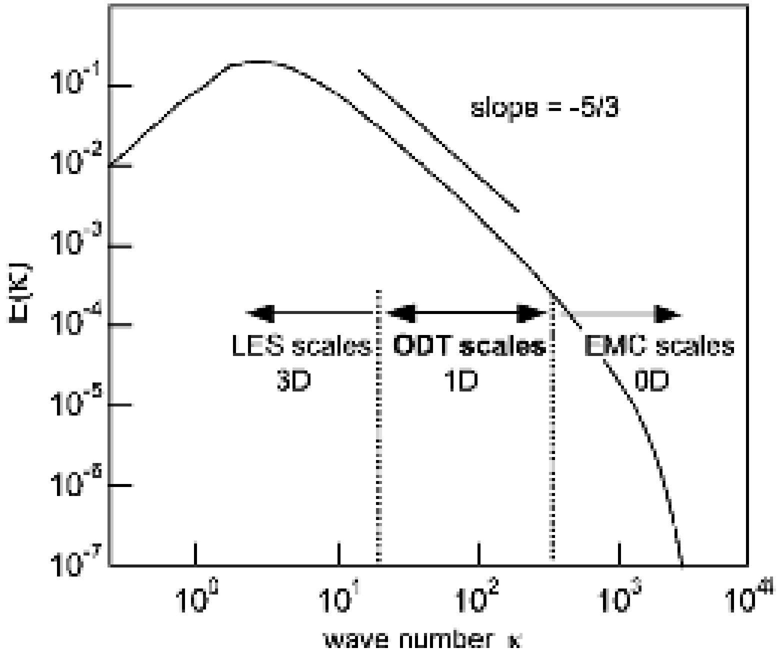

3D approaches for NWP models

- Introduce a 1D sub-model
- Dynamic modelling (to at least horizontal part)
  Introducing second filter $\langle ... \rangle_f = \alpha \cdot \overline{...}$ and assuming the turbulence model independence with respect to actual filtration, the exchange coefficients can be dynamically adjusted.

3D approaches for NWP models

- Introduce a 1D sub-model
- Dynamic modelling (to at least horizontal part)
- Assume something about the flow
  - stationary (invariant with respect to transition in time)
  - homogeneous (invariant with respect to translation in space)
  - isotropic (invariant with respect to rotation)

$\Rightarrow$ The aim is not to entirely apply all three of them.

Aladin: Spectral diffusion

General form of linear horizontal diffusion applied to Ψ (with r being even number and $K =$ const.):

$$\left.\frac{\partial \Psi}{\partial t}\right|_{\text{diff}} = -(-1)^{\frac{r}{2}} K \nabla^r \Psi$$

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In ALADIN it can be easily evaluated in spectral space. The discretized formula for diffusion then becomes:

$$\left.\frac{\Psi^+ - \Psi^-}{\Delta t}\right|_{\text{diff}} = -\frac{\exp(-0.5\pi i \boxed{r})}{(2\pi)^{\boxed{r}}} \left[\frac{L_x^2}{\mathcal{M}^2} + \frac{L_y^2}{\mathcal{N}^2}\right]^{\frac{\boxed{r}}{2}} \boxed{H}\,\boxed{g(l)}\,\nabla^{\boxed{r}} \Psi^+$$

with: $H = \dfrac{\text{RRDXTAU}}{(1+0.5 r_{nlginc})^{2.5} [\Delta X]_{gp} \quad \text{RDAMP}[\Phi]}$ and $r =$ REXPDH

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- Unconditionally stable
- Full freedom for tuning
- Efficient
- Preserves mean (conservativeness)
- Destroying atmospheric balance
- Affected by extension zone
- Can be used to just spectral fields
- Difficulty with sloped coordinate
- Domain dependent
- Not very physical in terms of representing turbulence

Aladin: SLHD

General form of model equation

$$\boxed{\frac{d\Psi}{dt} = \mathcal{L}\Psi + \mathcal{N} + \mathcal{F}}$$

To evaluate Ψ^+ using 2TL SISL scheme one needs to solve:

$$\Psi_F^+ = \left(1 - \frac{\Delta t}{2}\mathcal{L}\right)^{-1} \left[\underbrace{\left(1 + \frac{\Delta t}{2}\mathcal{L}\right)\Psi_O^0 + \Delta t \mathcal{F}_O^0 + \frac{\Delta t}{2}\mathcal{N}_O^*}_{I} + \frac{\Delta t}{2}\mathcal{N}_F^* \right]$$

SLHD grid point diffusion is defined when

$$\boxed{I = I_A + \kappa (I_D - I_A)}$$

Definition of kappa

$$\kappa = \kappa(|D|, DIV_H, \Delta t, \Delta x)$$

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Triggering by DIV_H:

- Triggering based on DIV_H physically important only for strongly incompressible cases.
- But it is a common trick to simulate the non-linear interactions between inertia-gravity waves and rotational motion, preventing the spurious accumulation of inertia-gravity wave energy near the cut-off wavenumber.
- Additionally it is believed to help for the convergence areas, where the origin of the SL trajectory is not defined unambiguously.

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The $|D|$ and DIV_H can be optionally evaluated along true p-surfaces (using chain rule to evaluate horizontal derivatives) in order to prevent spurious circulation above sloped terrain.

Aladin: SLHD

Definition of diffusive interpolator I_D

- General two-parametric interpolator
  - Restricted to at least 2^{nd} order accuracy, leaving just one tunable to control the interpolation property.
  - SLHD defined by making this tunable proportional to κ.
  - Stability within stability limits of the SL scheme.
  - Response slightly dependent to the O-point position with respect to model mesh.
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- Laplacian smoother transported to weights
  - $$\begin{aligned}\tilde{y} &= (1+\varepsilon\Delta x^2\partial_x^2)y \\ &= (1+\varepsilon\Delta x^2\partial_x^2)w_1(y_1-y_0)+w_2(y_2-y_0)+w_3(y_3-y_0) \\ &= \tilde{w_1}(\varepsilon)(y_1-y_0)+\tilde{w_2}(\varepsilon)(y_2-y_0)+\tilde{w_3}(\varepsilon)(y_3-y_0)\end{aligned}$$
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- Combination of both

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- More realistic (non-linear)
- Local and 3D character
- Applicable to any advected field
- Stability within the limits of SL stability
- Limited tuning
- Needs time (few time-steps) to develop an adequate response
- Control of orography triggered noise needs a special care

General rules for 3D turbulence in Aladin

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- Remain reasonably efficient and stable - stay bellow 10% of additional model cost (including implications to Δt)

Proposed 3D turbulence scheme

$$\frac{\partial \Psi}{\partial t} + ... = \boxed{-K_H \frac{\partial^2 \Psi}{\partial x^2} - K_H \frac{\partial^2 \Psi}{\partial y^2}} - \frac{\partial}{\partial z}\left(K_V \frac{\partial \Psi}{\partial z}\right) - K_{Num}\mathcal{D}(\Psi)$$

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where

- $K_H = K_H(x, y, z, t)$ but we assume $\frac{\partial K_H}{\partial x} + \frac{\partial K_H}{\partial y} = 0$
- $\nabla_H^2 \Psi$ is evaluated by the SLHD smoother
- $K_V \neq K_H$
- K_V and K_H are derived in a consistent way (QNSE?):

$$\begin{aligned} K_{m,V} &= L_K C_K \sqrt{e}\chi_3(Ri) \\ K_{h,V} &= L_K C_K C_3 \sqrt{e}\phi_3(Ri) \end{aligned} \Rightarrow \begin{aligned} K_{m,H} &= L_K^H C_K \sqrt{e}\chi_H(Ri) \\ K_{h,H} &= L_K^H C_K C_3 \sqrt{e}\phi_H(Ri) \end{aligned}$$

(*more in TOUCANS presentations later in this week...*)

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- Should we set $L_K = L^H_K$? How to then ensure the smooth 1D $\Leftrightarrow$ 3D transition?
- What to do with the numerical diffusion (SLHD)?

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- Time to include the diabatic tendency (from turb) to w.