SLHD

(Semi-Lagrangian Horizontal Diffusion)

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Horizontal diffusion

Model equation for $\vec{v}$:

$$\frac{d\vec{v}}{dt} + \underbrace{2\vec{\Omega} \times \vec{v}}_{\text{coriolis}} + \underbrace{RT\nabla \ln p + \nabla\Phi}_{\text{pressure force}} = \vec{\mathcal{F}}_{\vec{v}} + \vec{\mathcal{S}}_{\vec{v}} + \vec{K}_{\vec{v}}$$

with:

$\vec{\mathcal{F}}_{\vec{v}}$ representing diabatic processes
$\vec{\mathcal{S}}_{\vec{v}}$ representing sources/sinks
$\vec{K}_{\vec{v}}$ representing **horizontal diffusion**

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  $\Rightarrow$ **for scales » *o*(1km) typically neglected**

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  removing the accumulated energy from the end of a model resolved spectrum and filtration of the numerical noise; with $\Delta x \approx$ 9.5 km the impact to u, v is of 10^{-2}-10^{0} m/s

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  - *Allows absolutely stable implicit formulation*
  - *Algorithmically efficient*
  - *Straightforward tuning through K and r*
  - *Almost exclusively used in atmospheric models*

Horizontal diffusion in ALADIN

Spectral diffusion

- linear diffusion $K =$ const.
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SLHD (since 2003)

- grid point space scheme
- non-linear scheme $\approx K(d)\nabla^r X$
- ∇^r is represented by sL interpolators ($r \approx$ 2-4)

SLHD design

$$X_F^+ = \left(1 - \frac{\Delta t}{2}\mathcal{L}\right)^{-1} \left[\underbrace{\left(1 + \frac{\Delta t}{2}\mathcal{L}\right) X_O^- + \Delta t \mathcal{F}_O^- + \frac{\Delta t}{2}\mathcal{N}_O^*}_{I} + \frac{\Delta t}{2}\mathcal{N}_F^* \right]$$

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$$\kappa = F(d, \Delta x, \Delta t), \quad d = \sqrt{\left(\underbrace{\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}}_{d_T}\right)^2 + \left(\underbrace{\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}}_{d_S}\right)^2}$$

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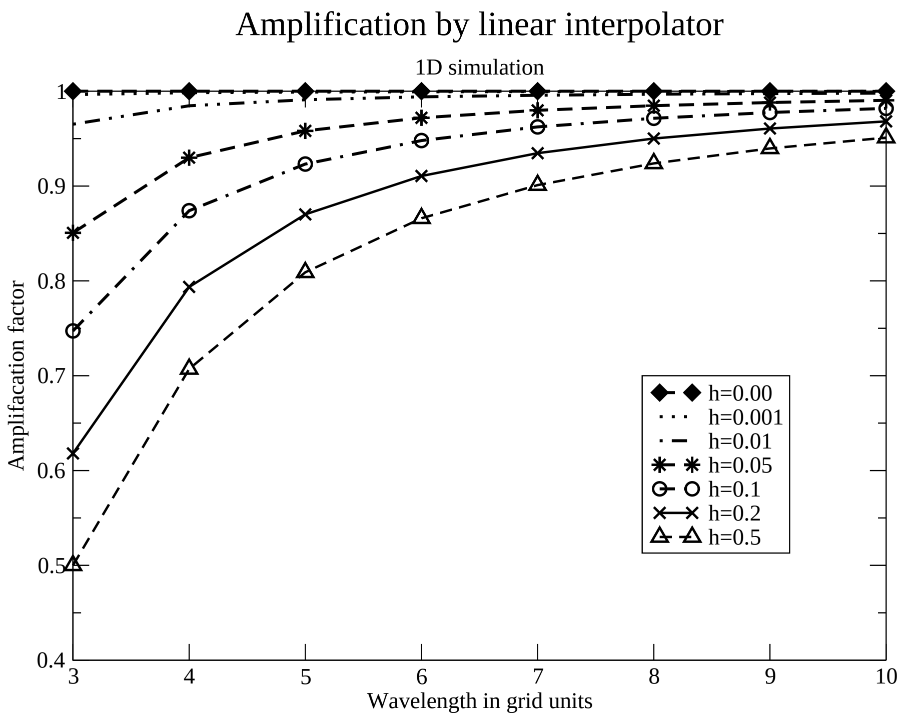
$a = 2$ SLHDA0 $\left(\frac{[\Delta x]_{ref}}{[\Delta x]}\right)^{\text{ZSLHDP1}}$

$d_0 = \frac{\text{SLHDD00}}{2} \left(\frac{[\Delta x]_{ref}}{[\Delta x]}\right)^{\text{ZSLHDP3}}$

Linear vs. homogeneous interpolation



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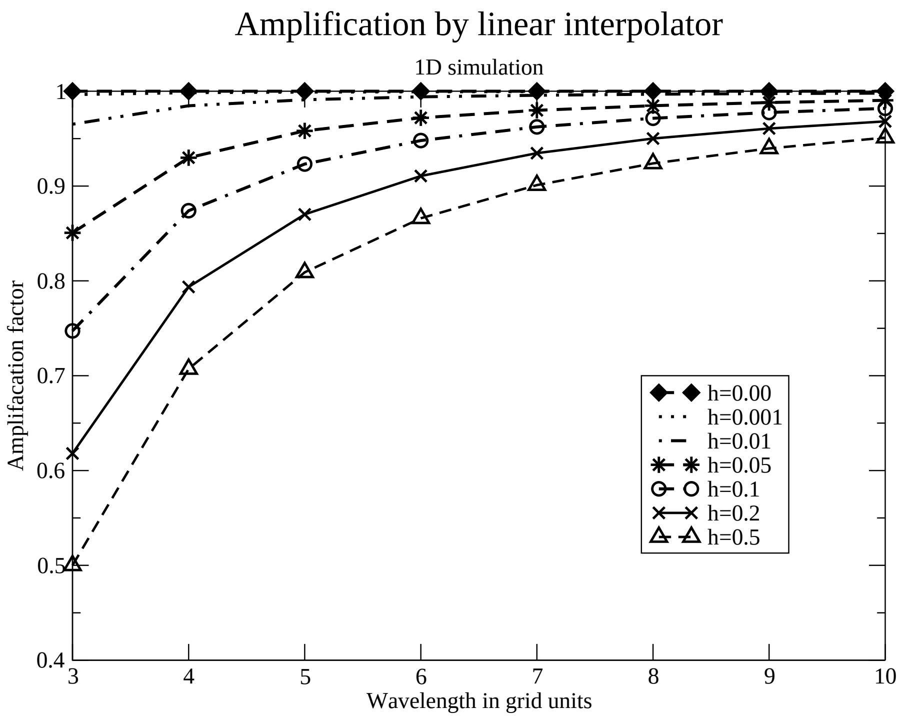
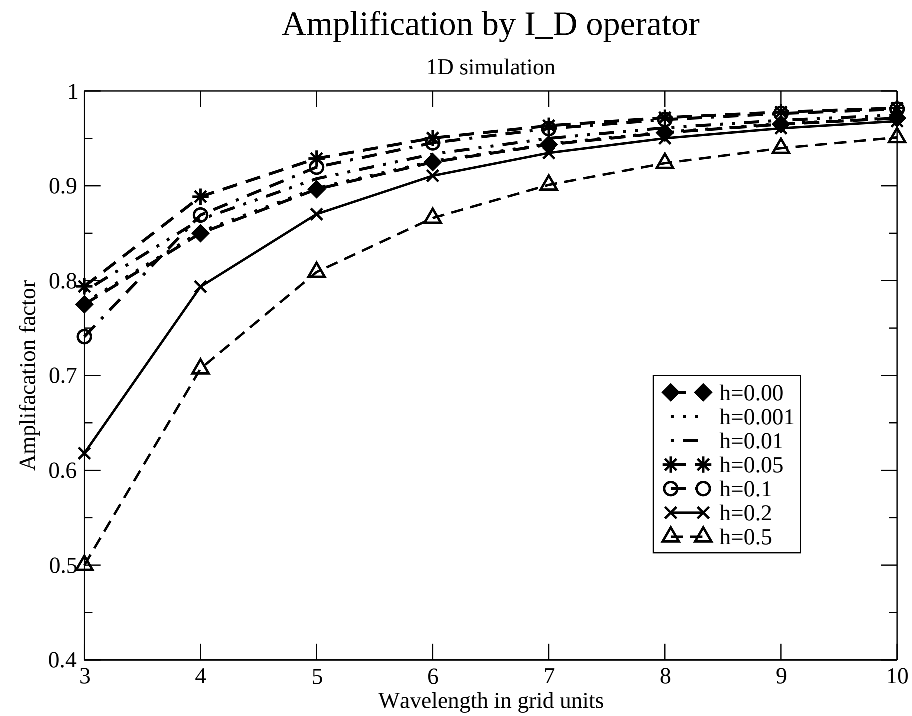
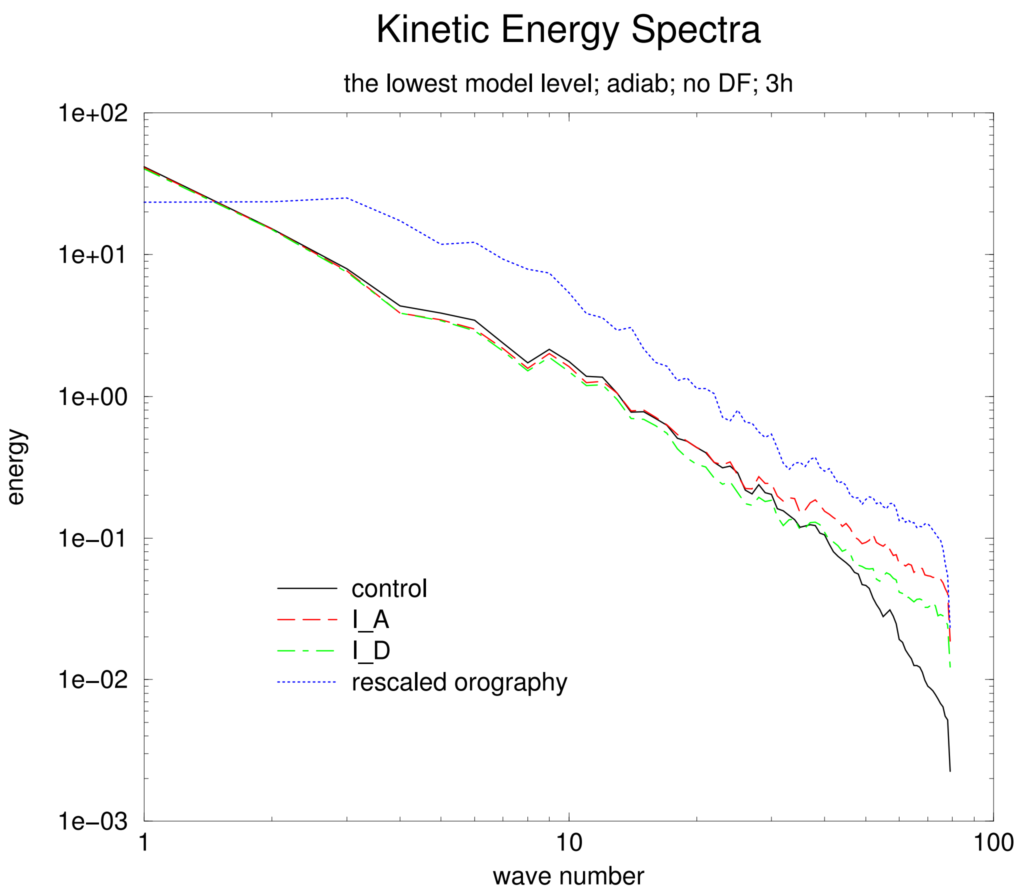




SLHD - implementation specificity



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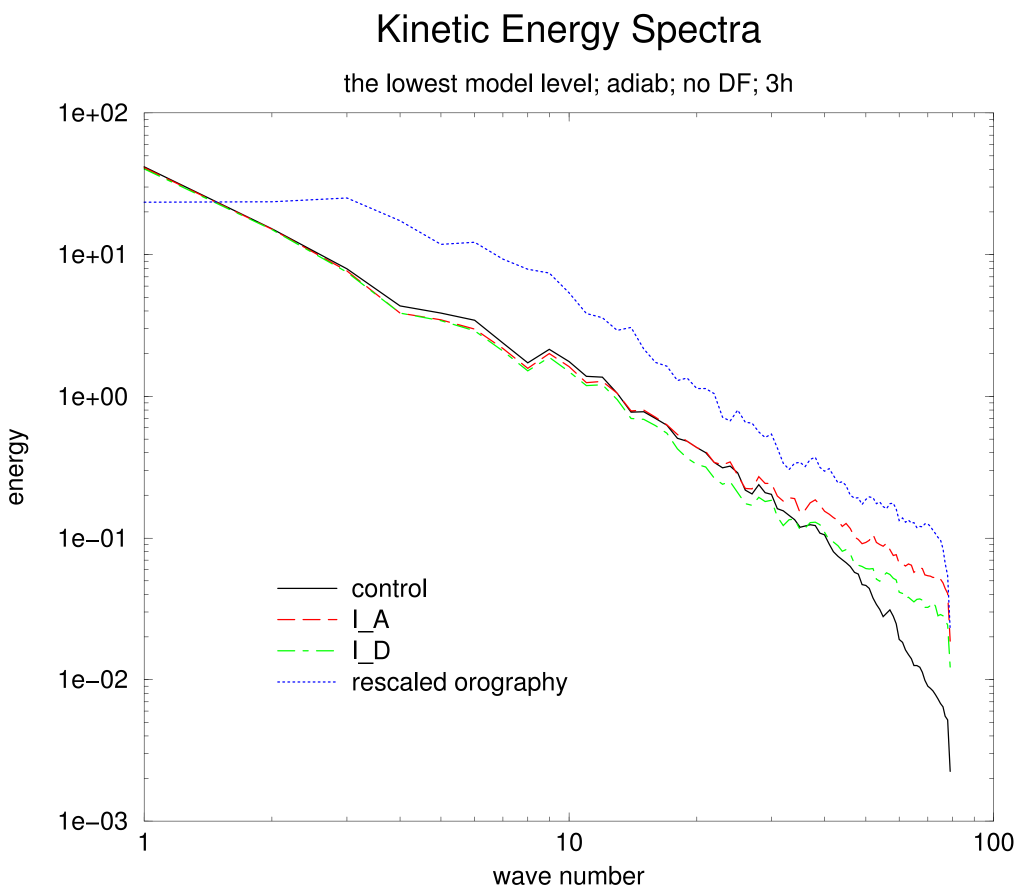

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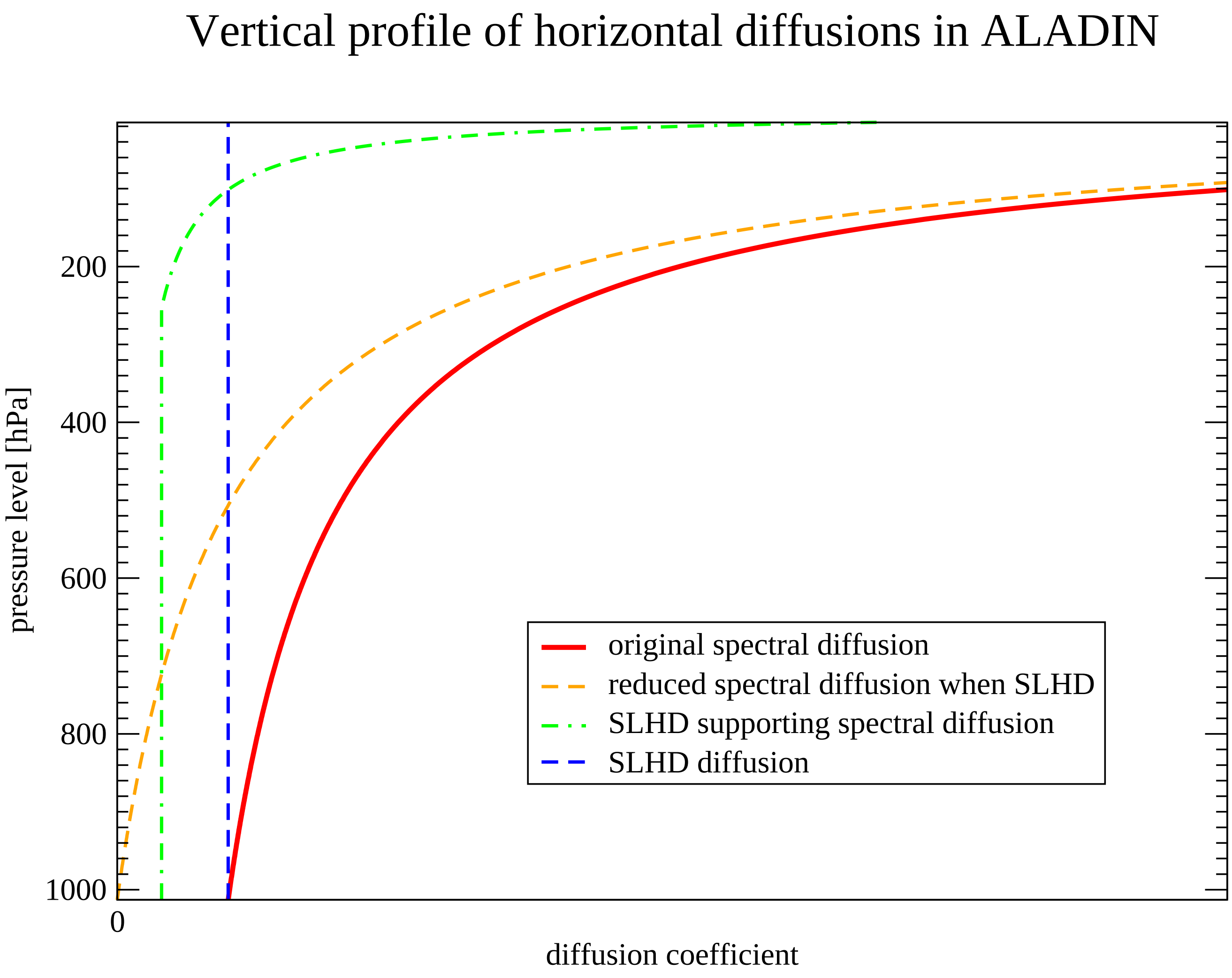
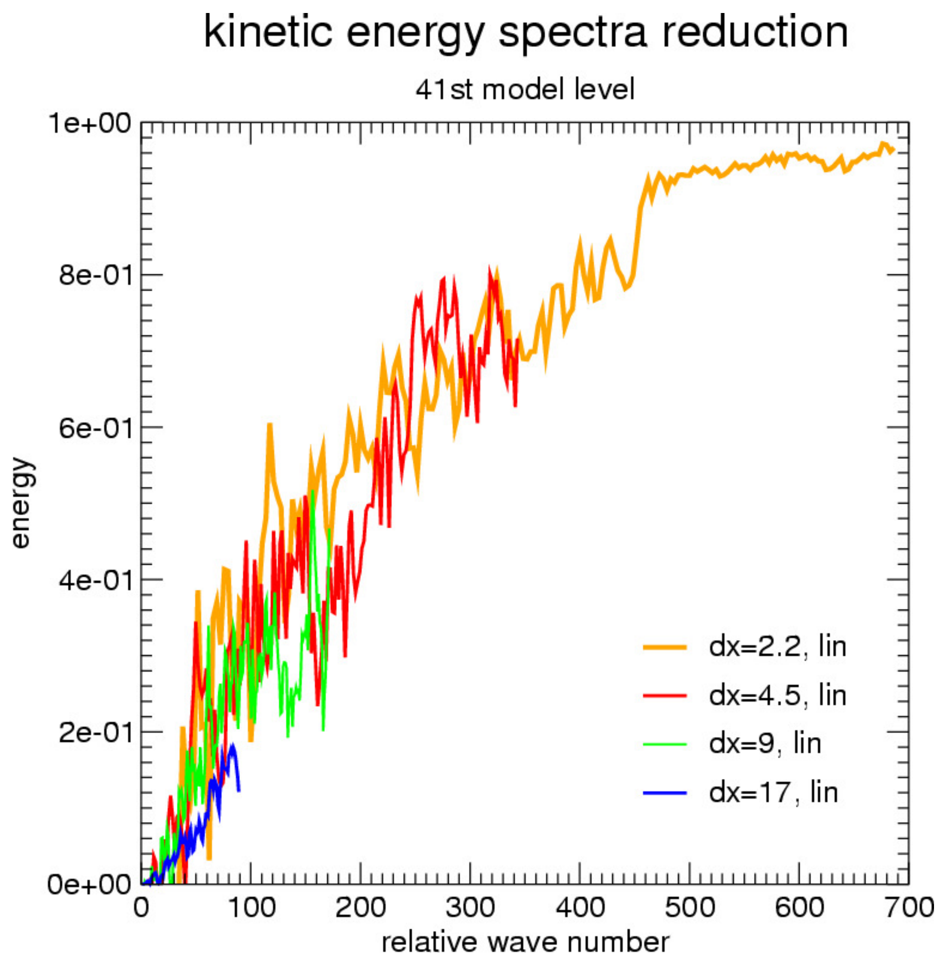
SLHD - ALADIN implementation

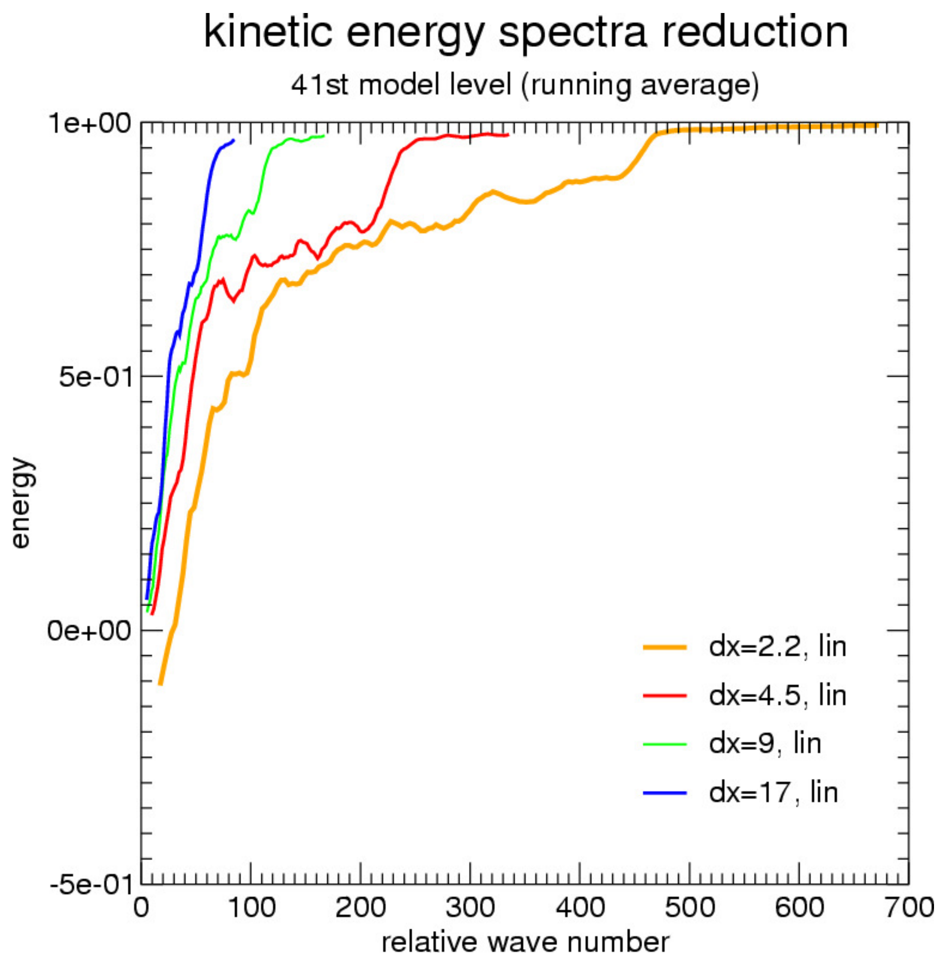


SLHD tuning



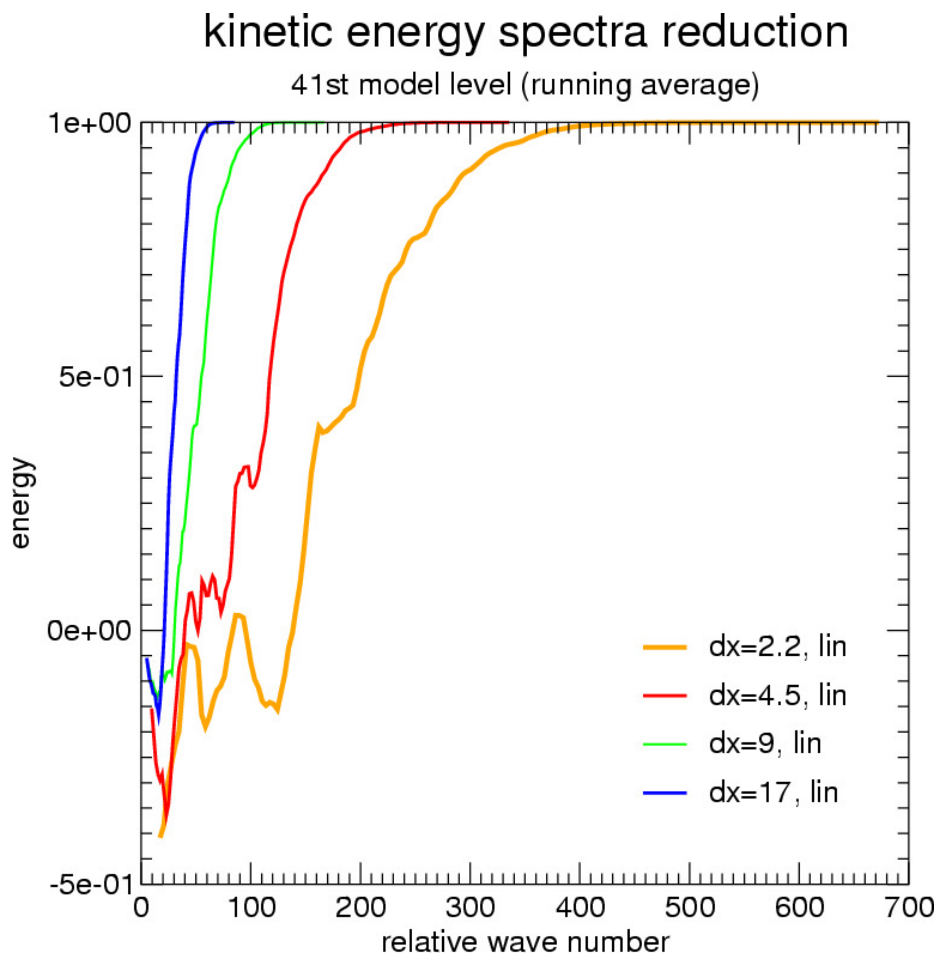
- Ideal tuning

SLHD tuning



- Ideal tuning
- Compromise tuning

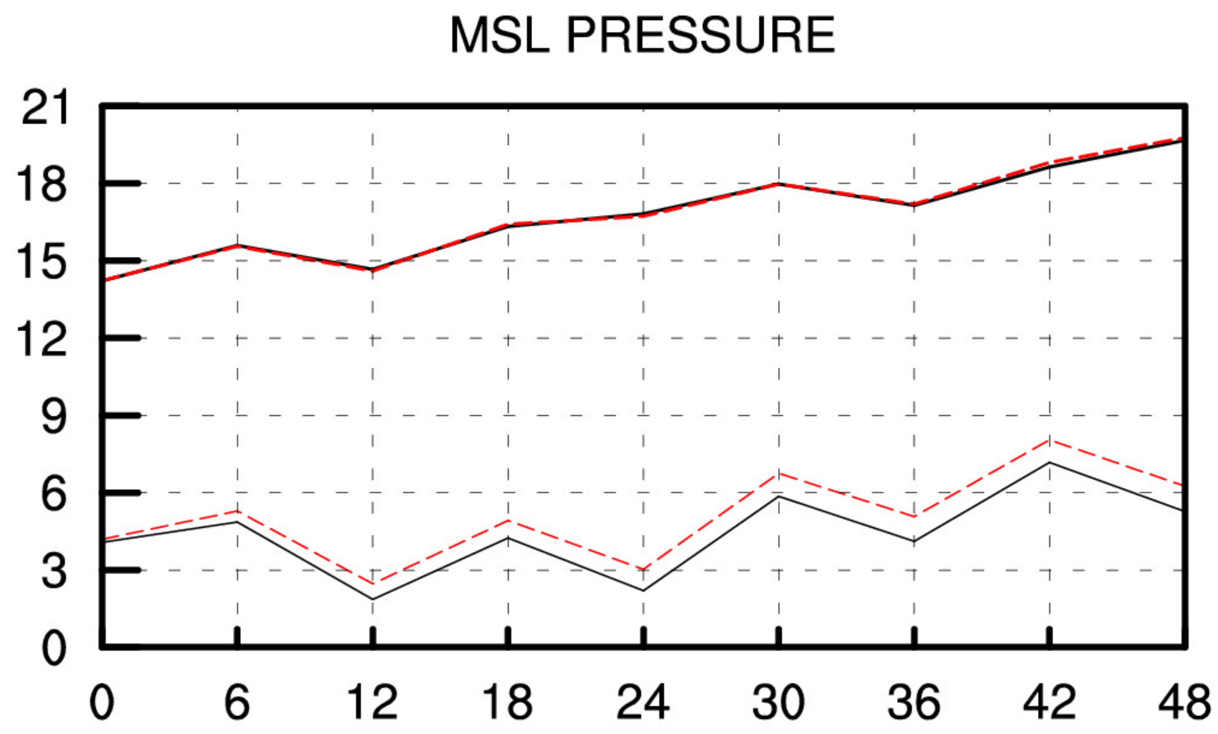
SLHD tuning



- Ideal tuning
- Compromise tuning
- Spectral diffusion

RMSE evolution of the MSL pressure

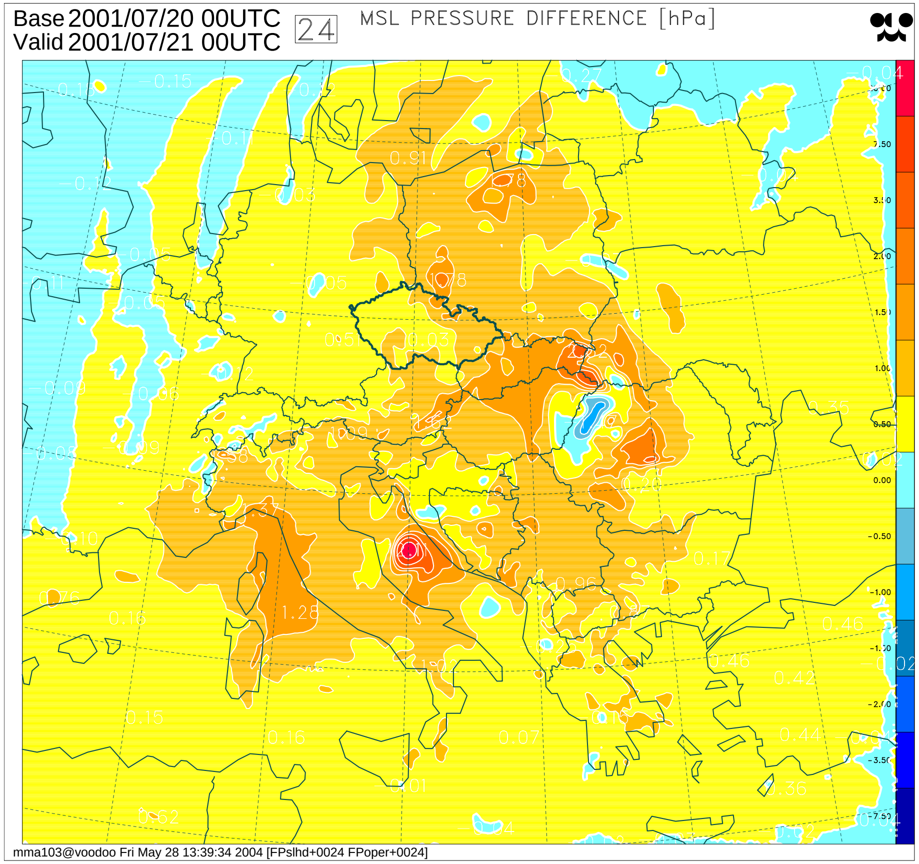
Parallel test, 19 days

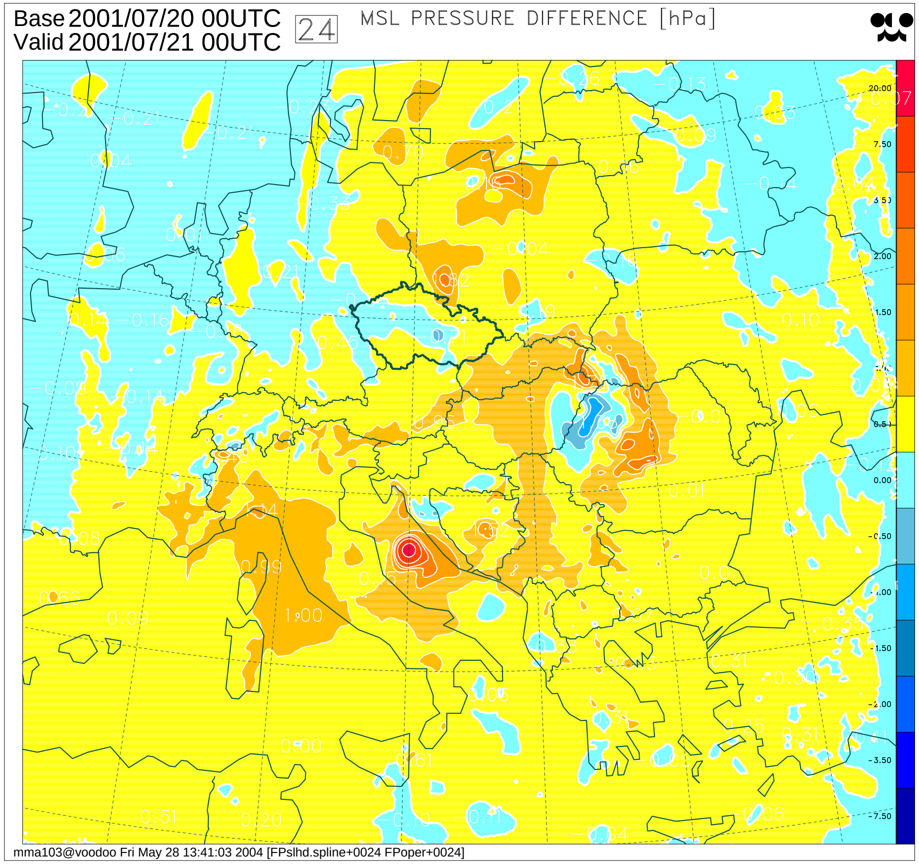


SLHD properties

Lagrangian cubic interpolation

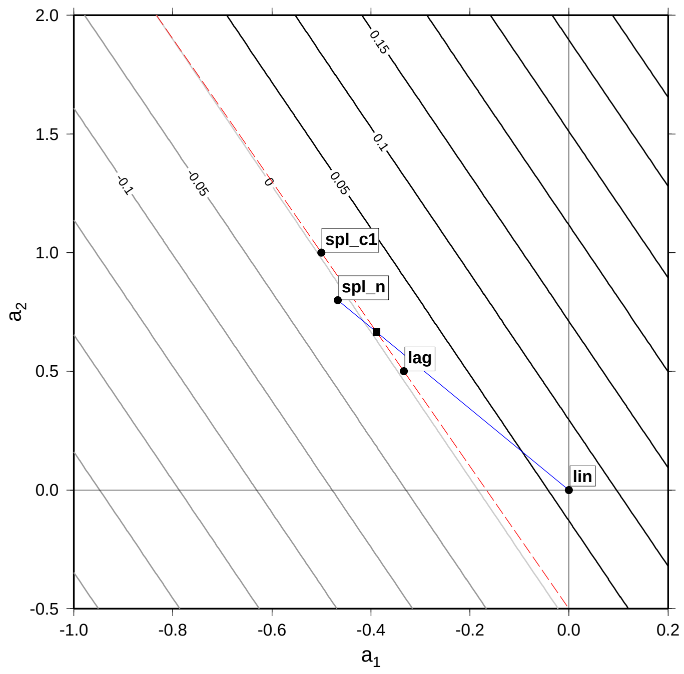
⇒ Natural 4 points cubic spline



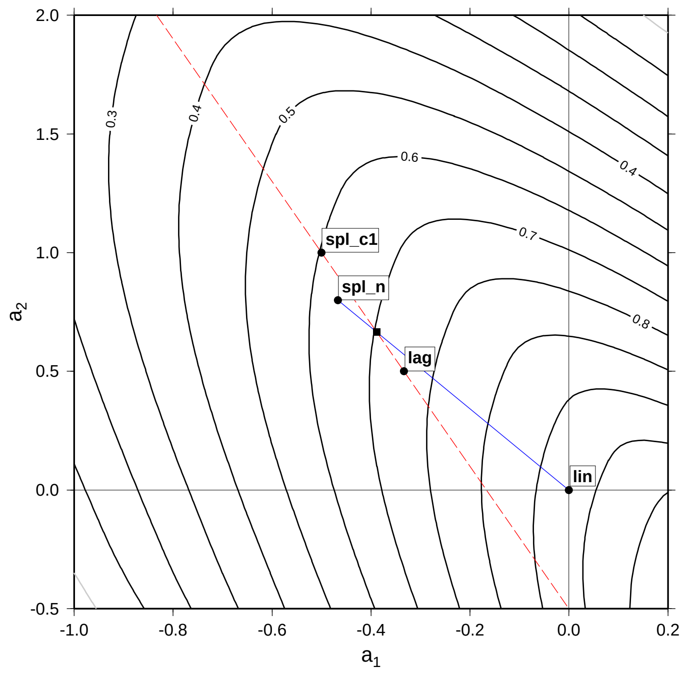


Dimensionless damping rate

Damping factor for N = 100, m = 10

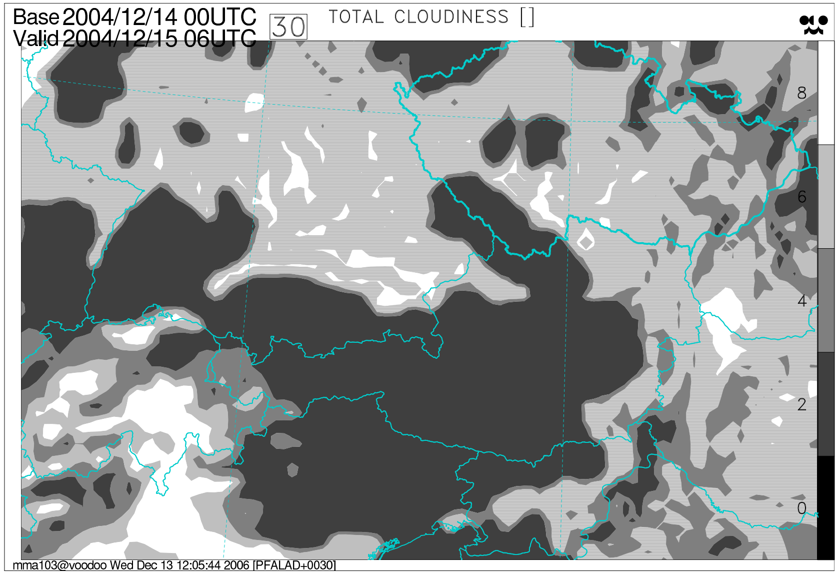


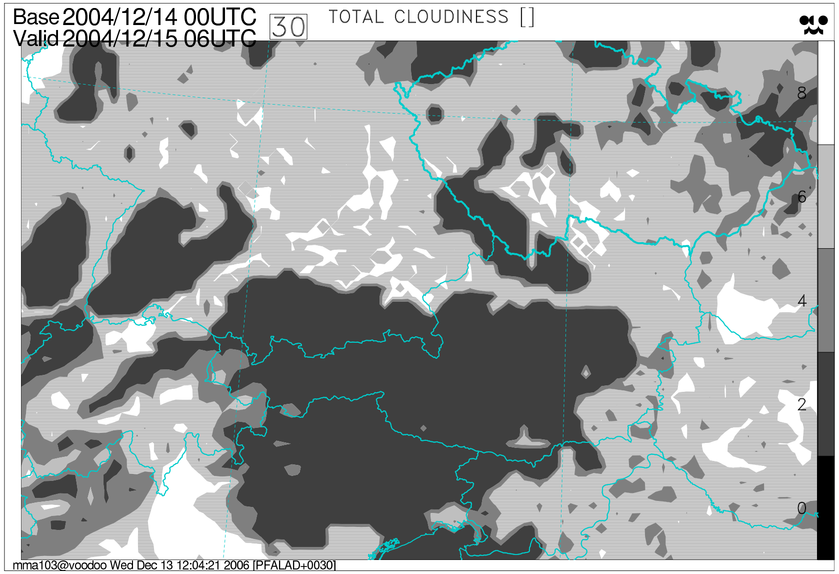
Damping factor for N = 100, m = 40



SLHD on moisture

Total cloudiness forecast for December 15th, 2004

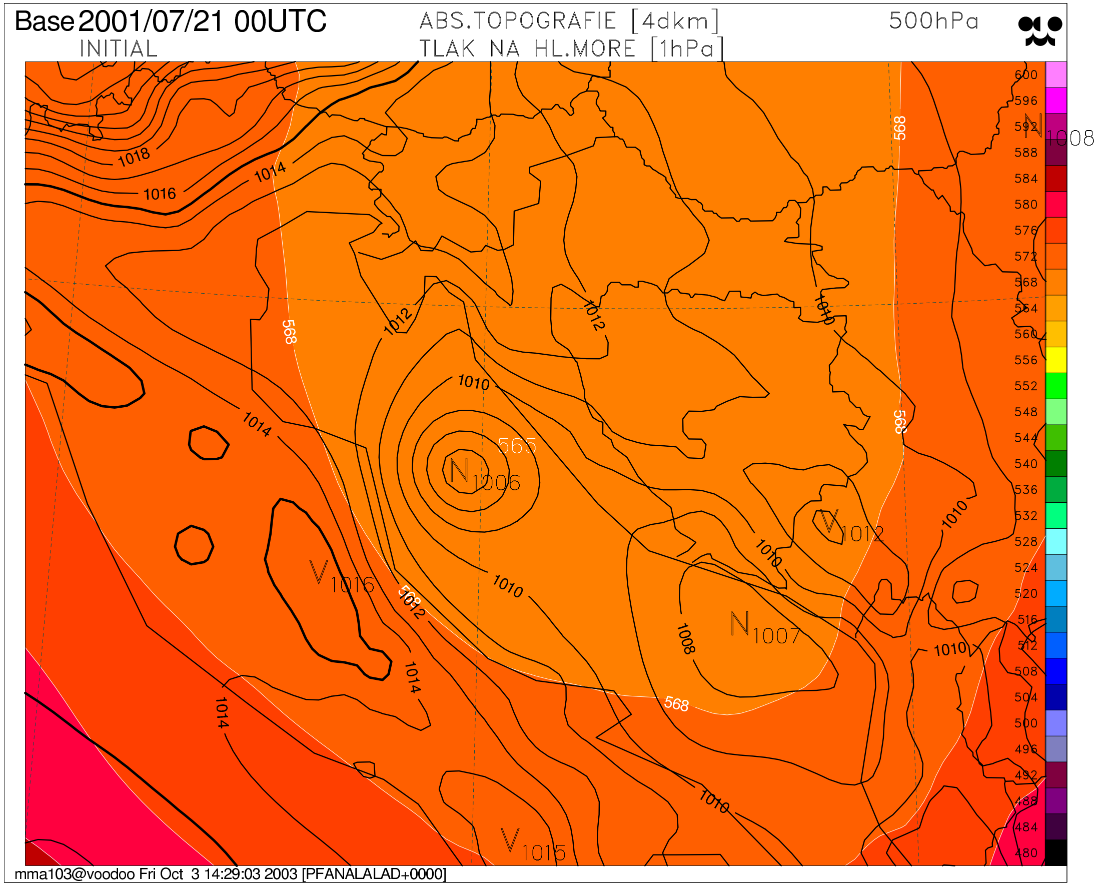




linear diffusion vs. SLHD

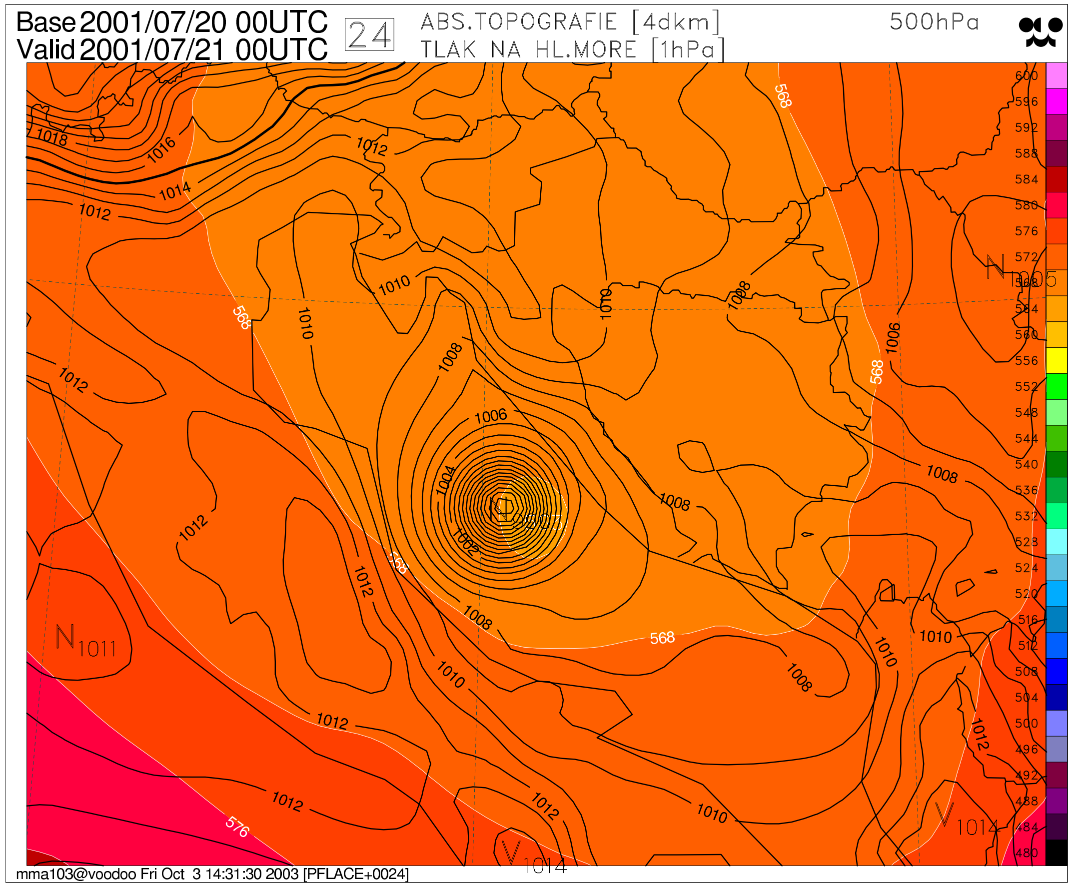
Adriatic storm I. - June 21st, 2001

ALADIN/LACE analyzis



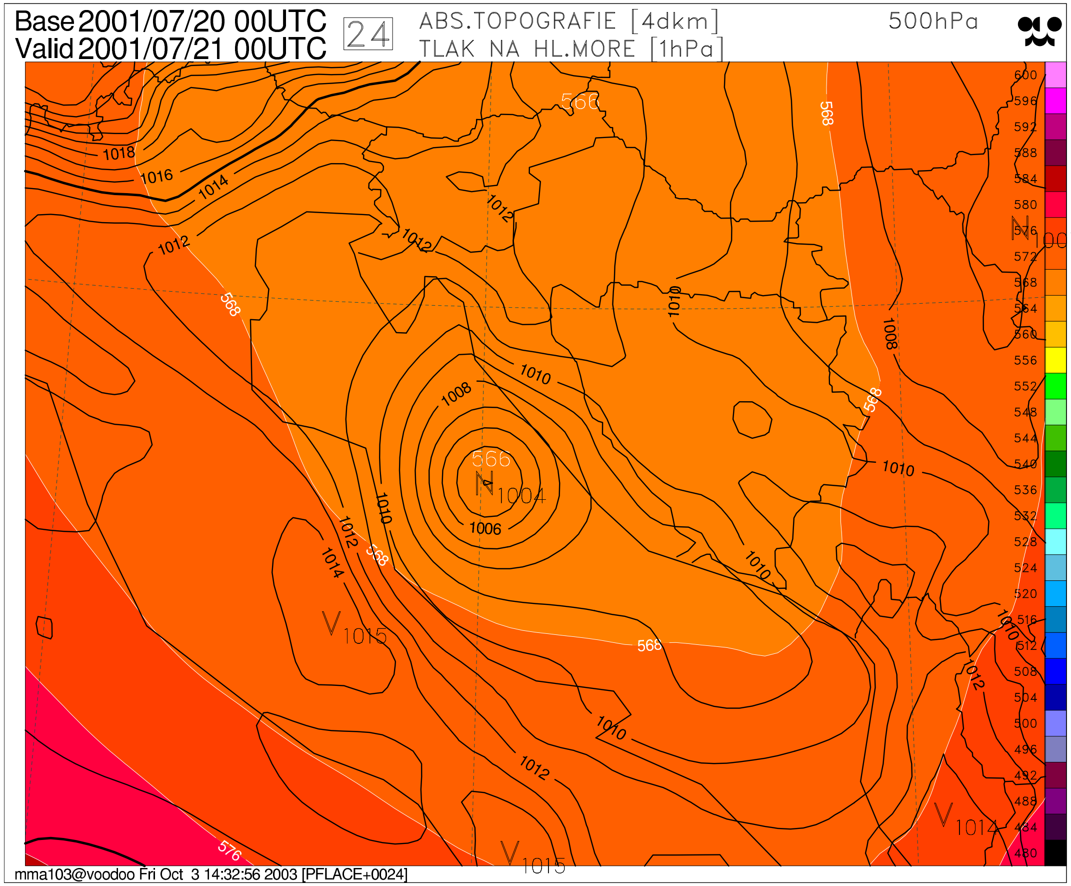
Adriatic storm I. - June 21st, 2001

ALADIN/LACE operational forecast for 24 hours



Adriatic storm I. - June 21st, 2001

ALADIN/LACE forecast for 24 hours with SLHD



Adriatic storm I. - June 21st, 2001

ASCS – Aladin Space Cross Section

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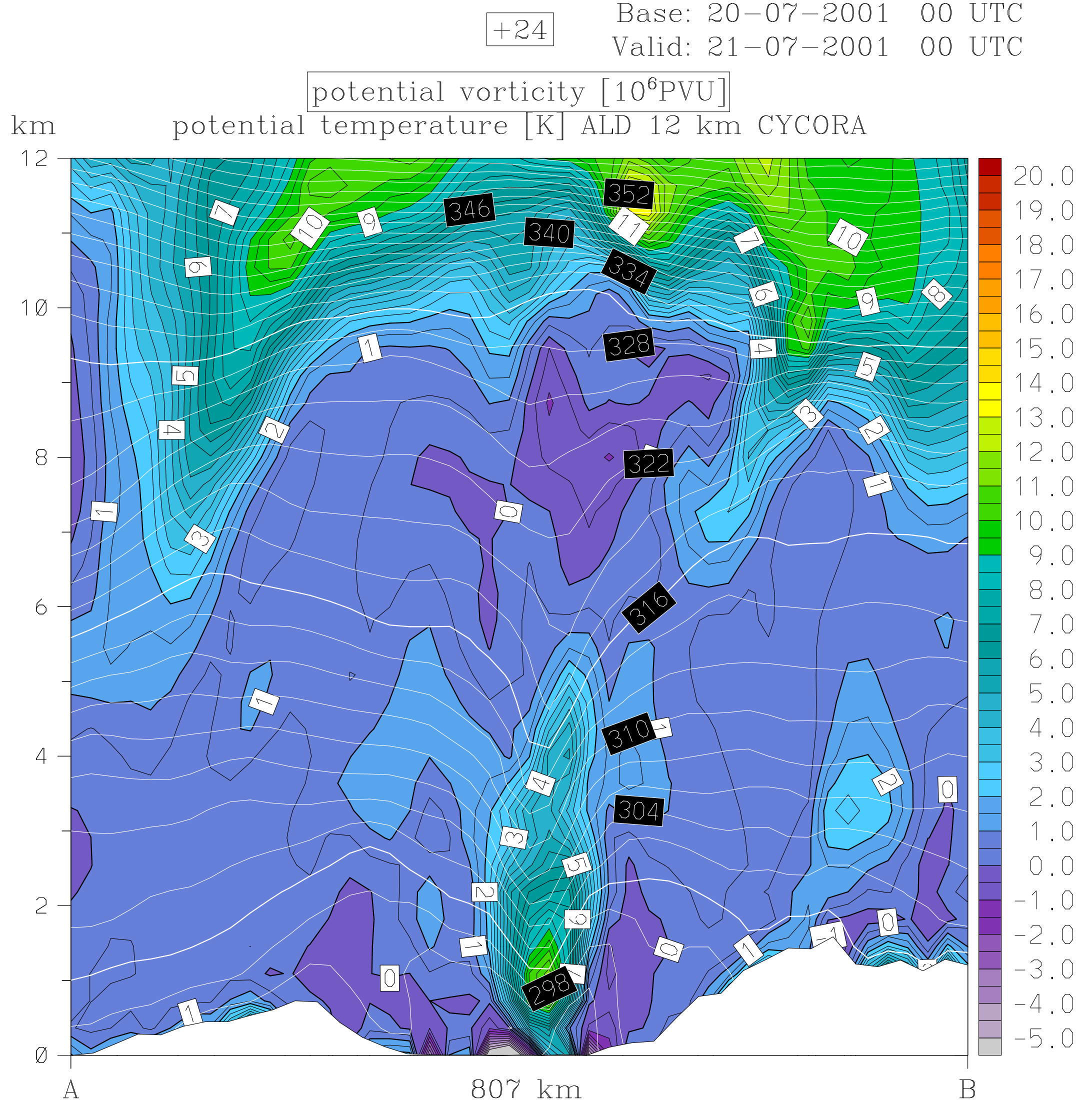
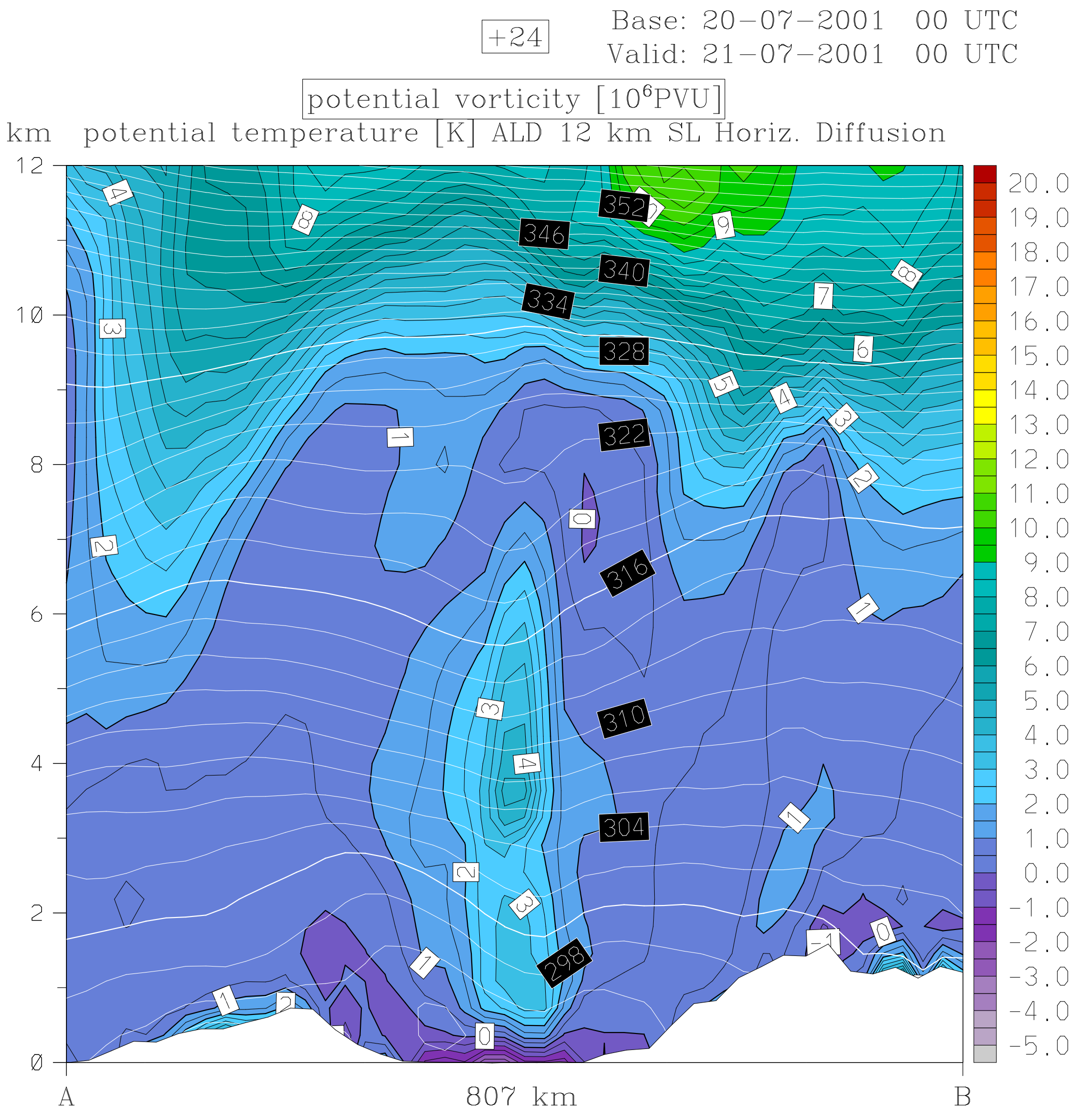
Operational forecast with lin. diffusion

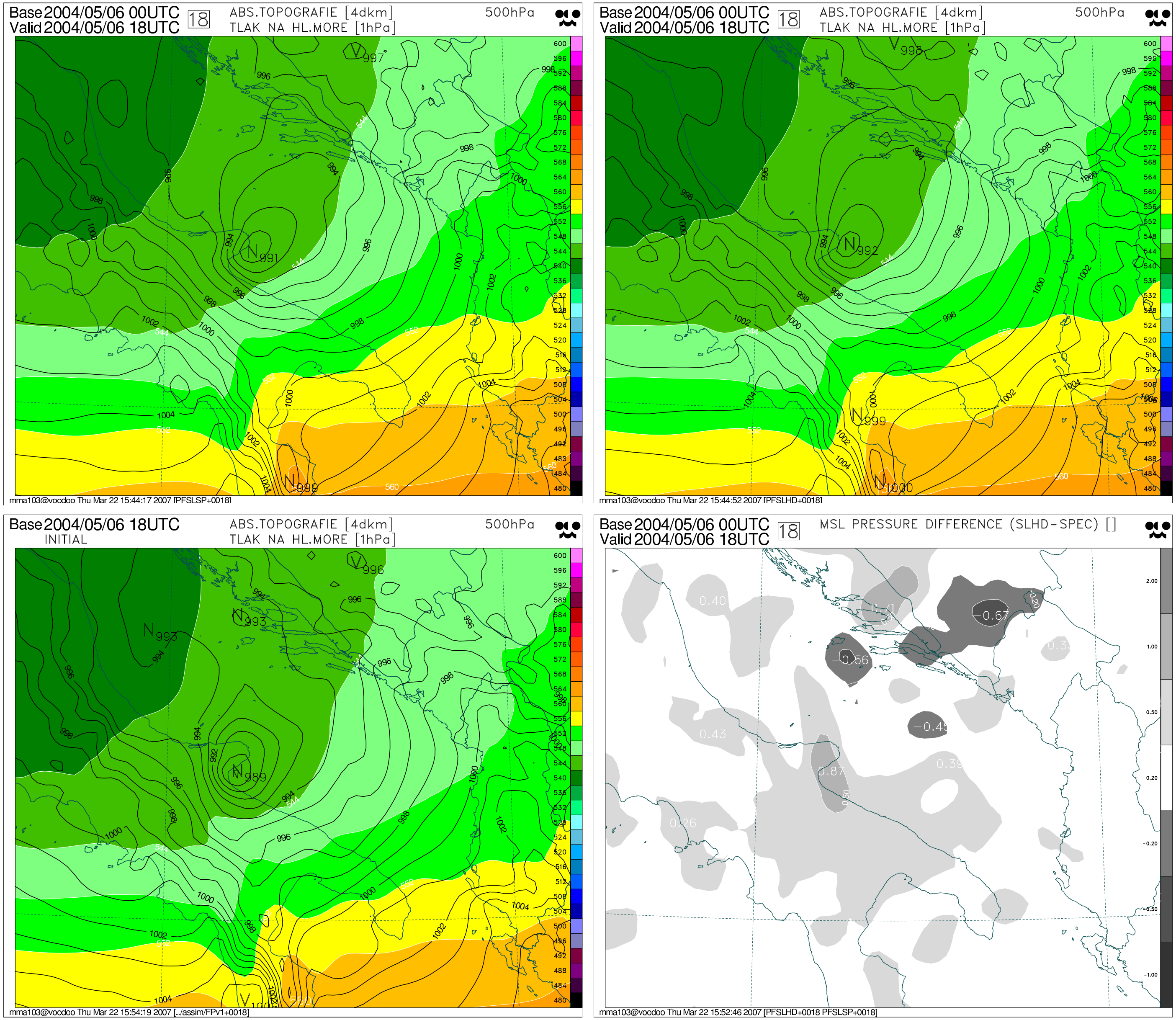


Adriatic storm I. - June 21st, 2001

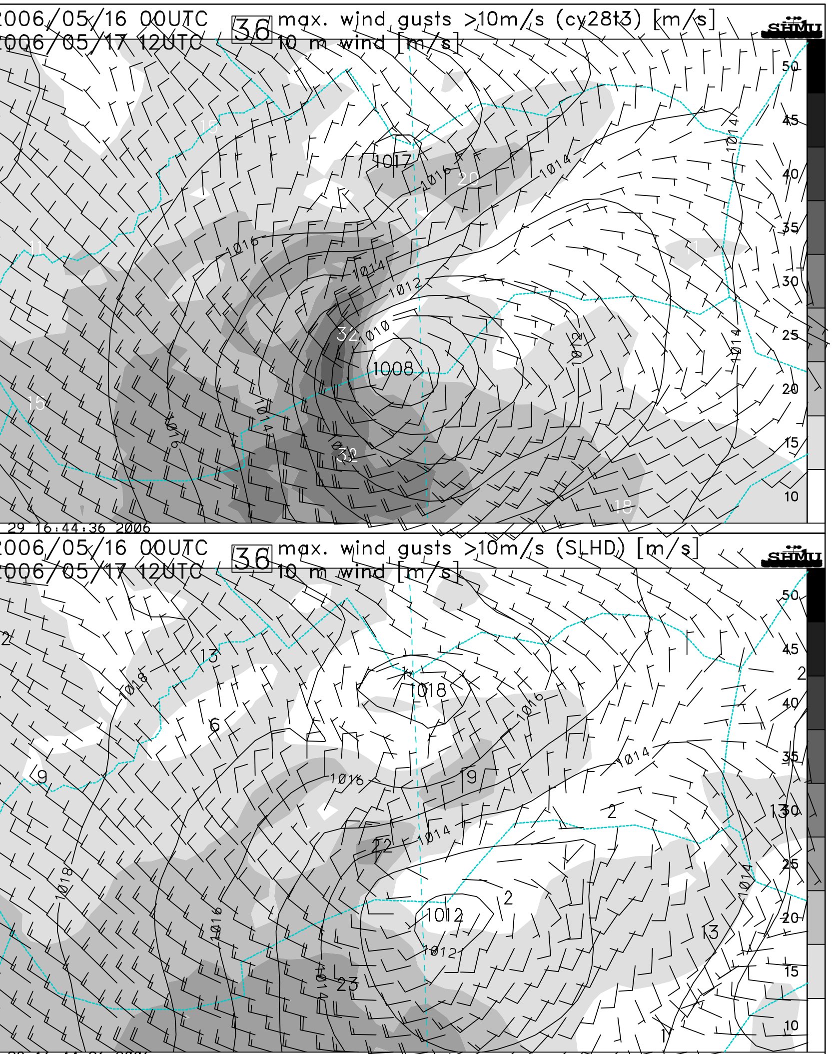
SLHD simulation

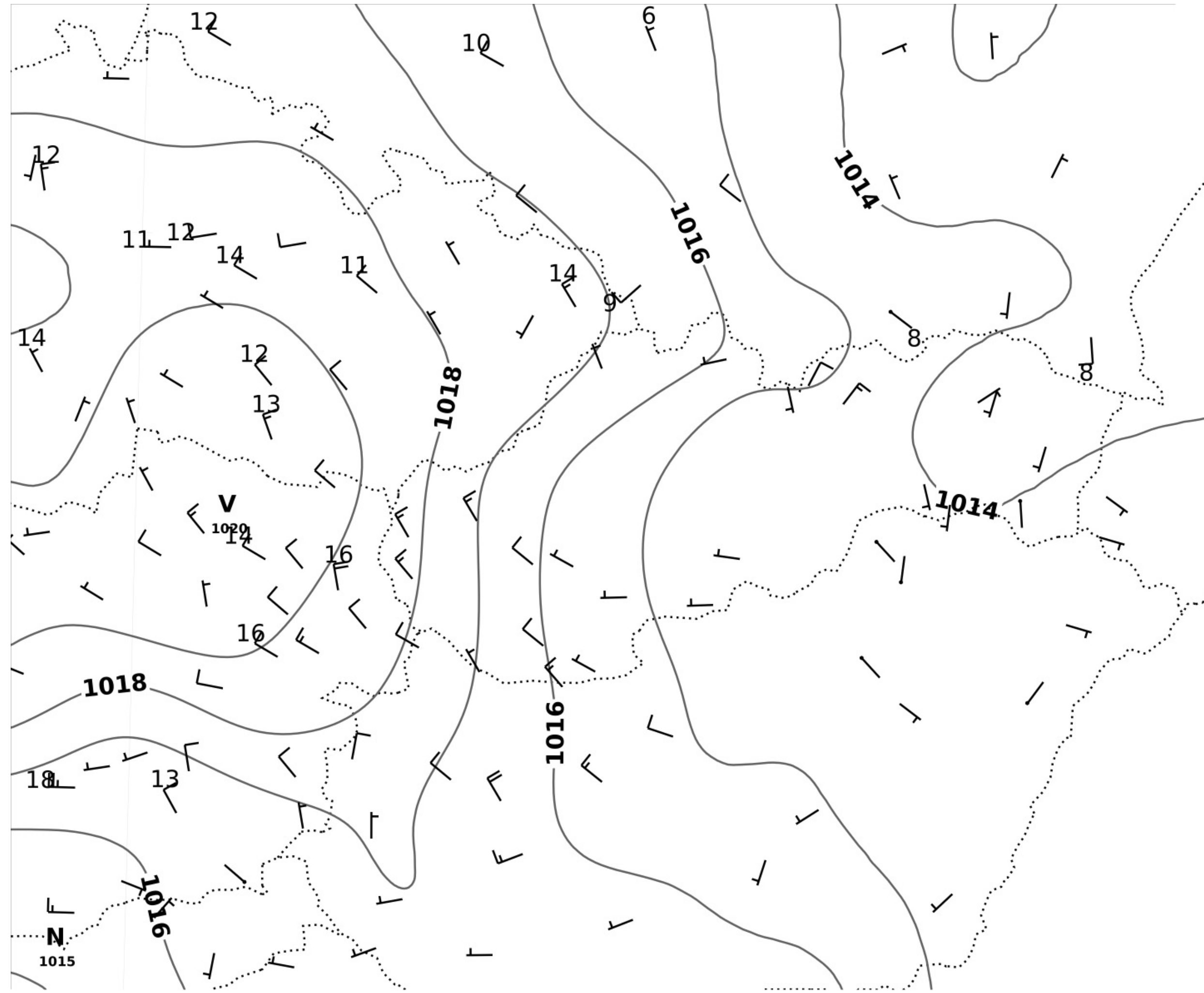


Adriatic storm II - May 6th, 2004



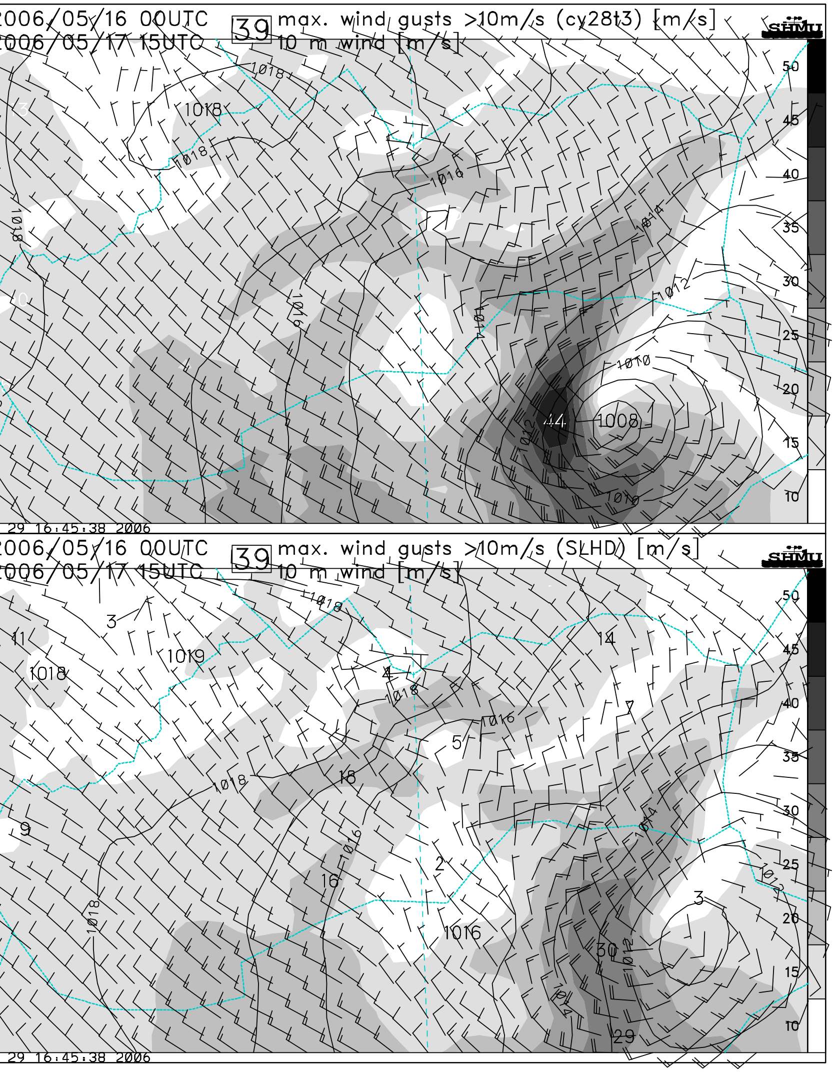
ALADIN/Slovakia forecast for 36 hours

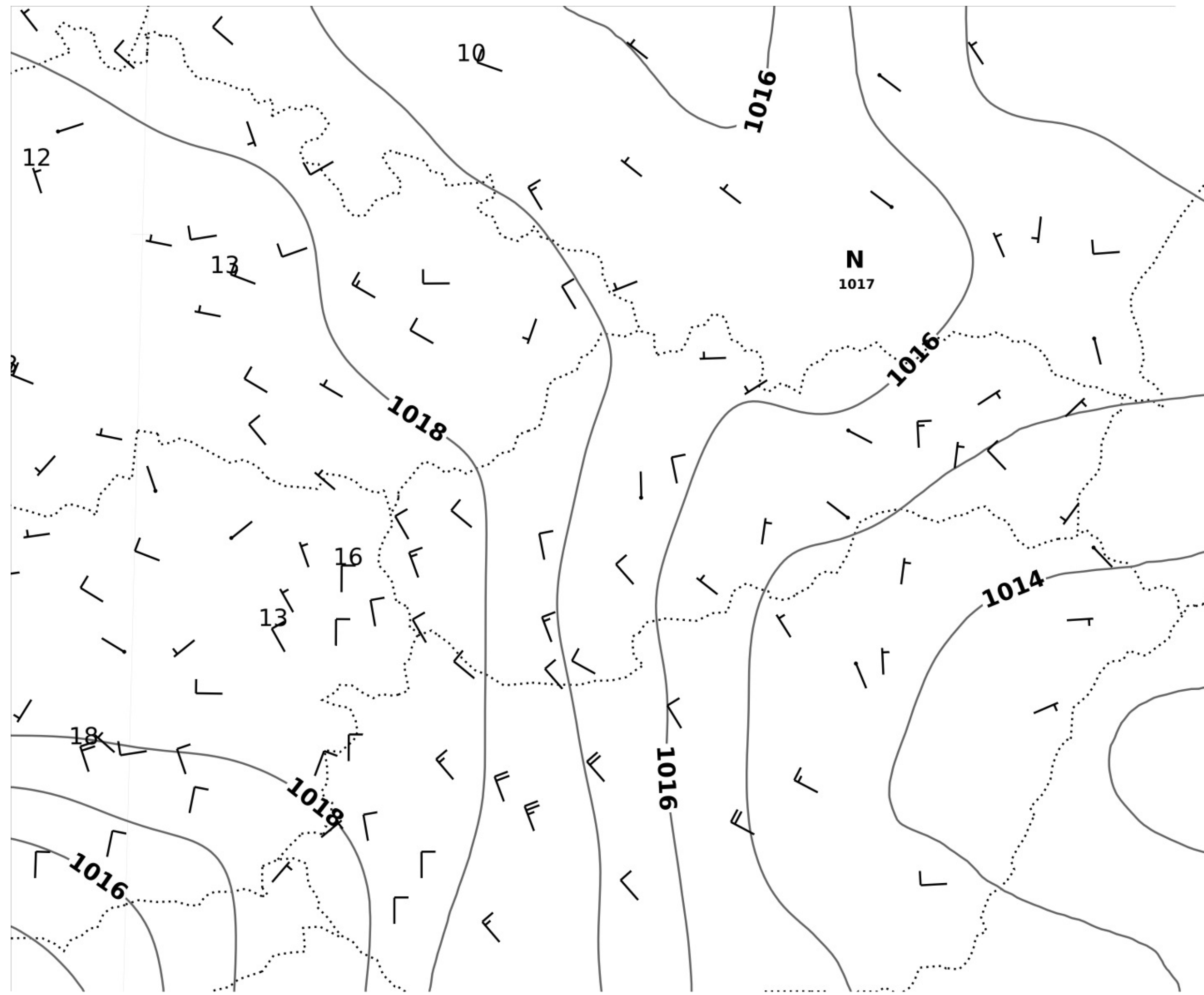




Slovakia - May 17th, 2006

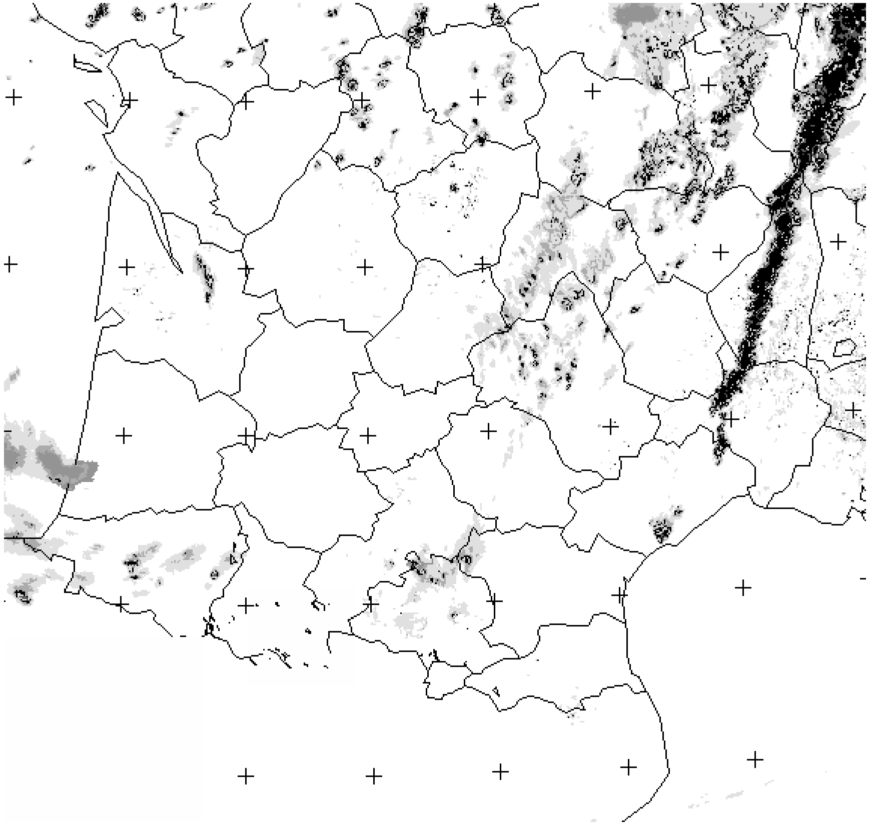
ALADIN/Slovakia forecast for 39 hours

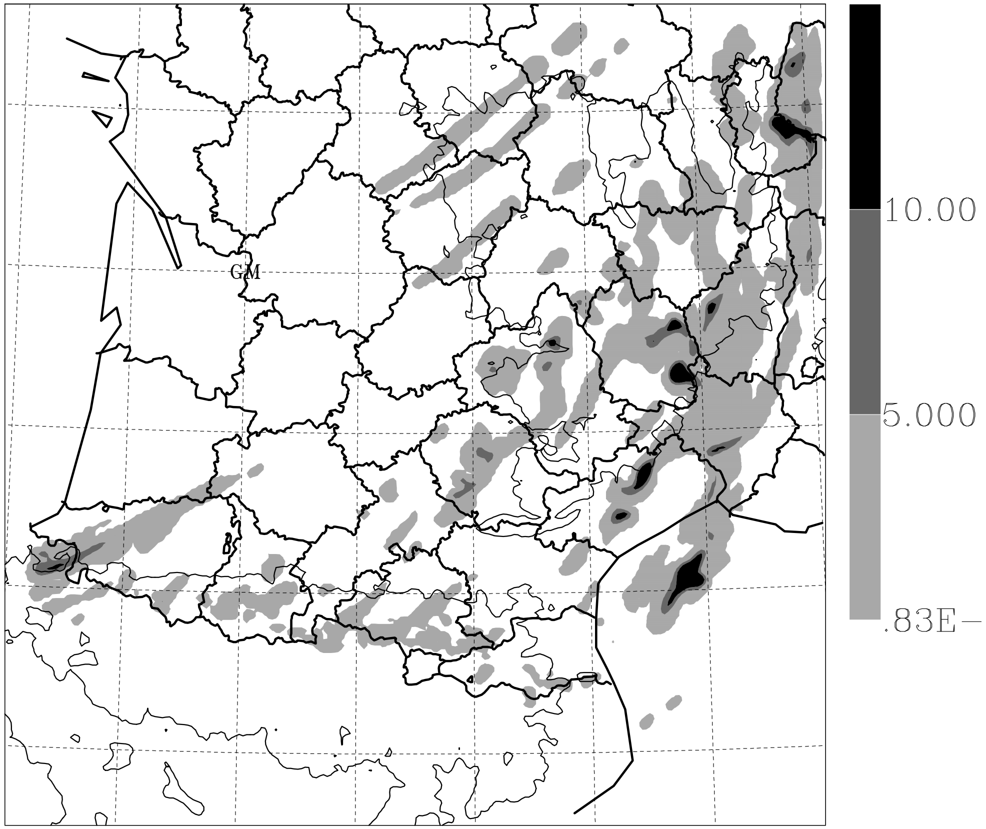




Squall line simulated by AROME

00 +15 UTC May 22nd, 2006





Radar vs. AROME with spectral diffusion

Squall line simulated by AROME

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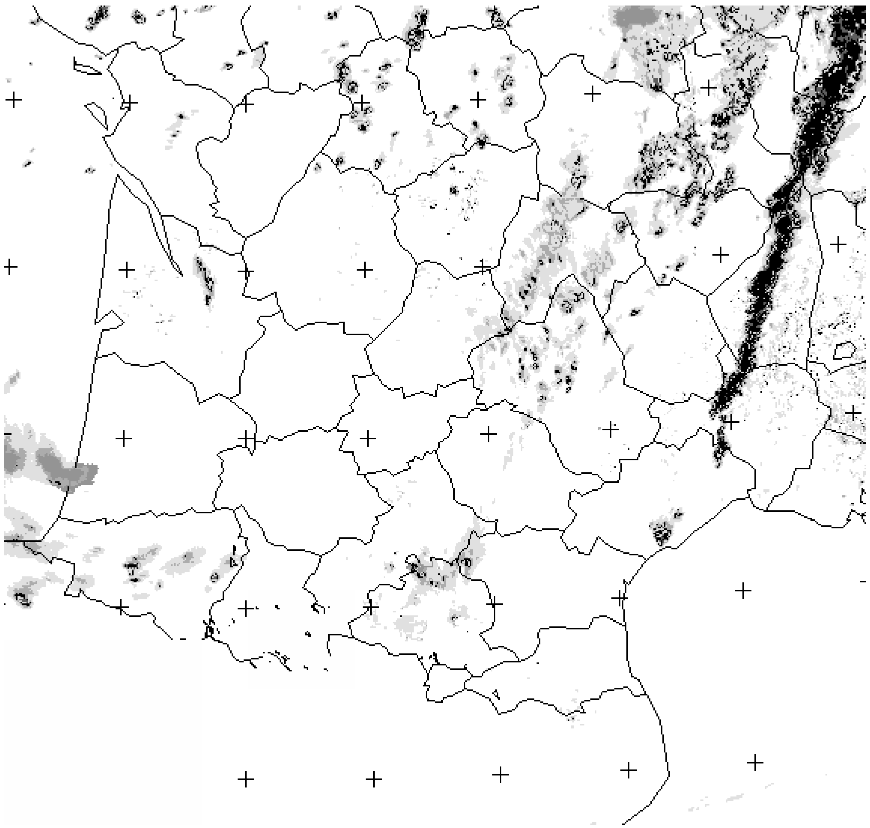

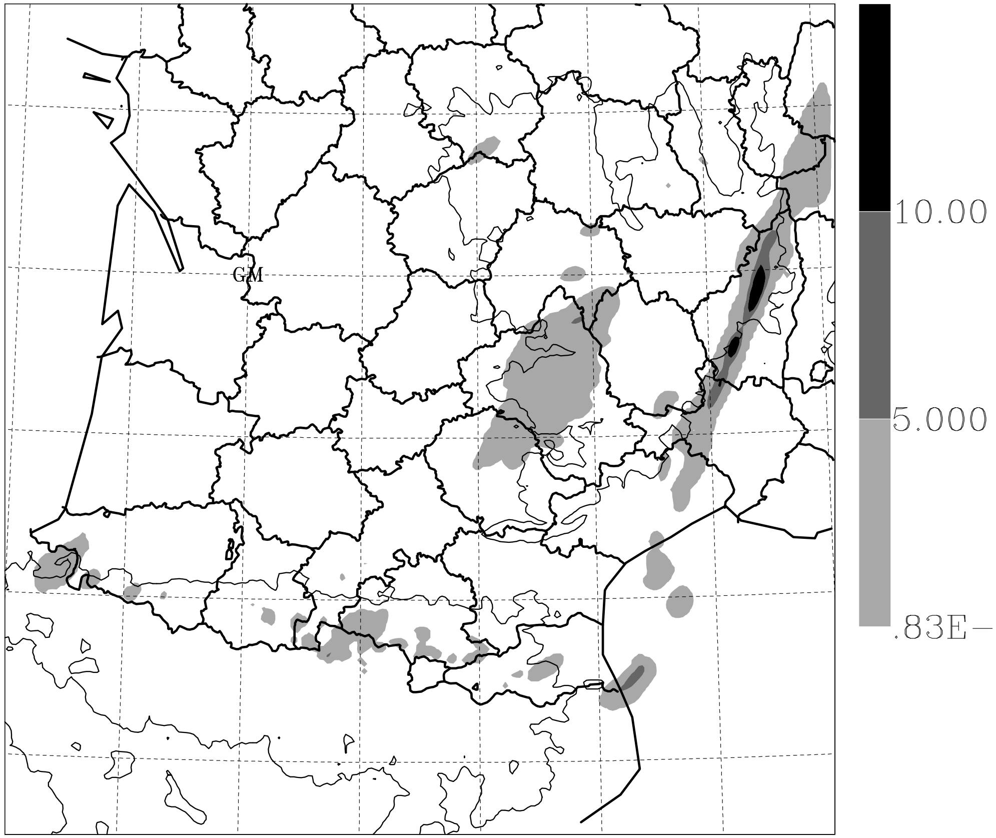

Radar vs. AROME with SLHD

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- Offers elegant solution for BBC condition in NH

SLHD cons

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Realistic simulation of severe weather events needs something better than linear diffusion.